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P4 Study Text
Advanced Financial Management


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## Paper P4

## Advanced Financial Management

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- Comprehensive but concise
- In simple English
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## Paper P4

Advanced Financial Management

## C

## Contents

Page
Syllabus and study guide ..... 1
Formulae and tables ..... 13
Chapter 1: The role and responsibilities of financial managers ..... 19
Chapter 2: Impact of environmental issues on corporate objectives and governance ..... 43
Chapter 3: Capital investment appraisal ..... 55
Chapter 4: DCF: risk analysis ..... 89
Chapter 5: Investing: portfolio theory and the CAPM ..... 113
Chapter 6: Cost of capital ..... 141
Chapter 7: Other aspects of capital investment appraisal ..... 181
Chapter 8: International investment and financing decisions ..... 209
Chapter 9: Mergers and acquisitions ..... 229
Chapter 10: Corporate reconstruction and reorganisation ..... 275
Chapter 11: The money markets. The treasury function ..... 289
Chapter 12: Foreign exchange risk and currency risk management ..... 307
Chapter 13: Interest rate risk. Hedging with FRAs and swaps ..... 329
Chapter 14: Futures and hedging with futures ..... 349
Chapter 15: Options and hedging with options ..... 377
Chapter 16: Option pricing and delta hedging ..... 399
Chapter 17: Financial management and multinationals ..... 421
Practice questions ..... 443
Answers ..... 475
Index ..... 521

## Syllabus and study guide

## Aim

To apply relevant knowledge, skills and exercise professional judgement as expected of a senior financial executive or advisor, in taking or recommending decisions relating to the financial management of an organisation.

## Main capabilities

On successful completion of this paper, candidates should be able to:
A Explain the role and responsibility of the senior financial executive or advisor in meeting conflicting needs of stakeholders
B Evaluate potential investment decisions and assessing their financial and strategic consequences, both domestically and internationally.
C Assess and plan acquisitions and mergers as an alternative growth strategy.
D Evaluate and advise on alternative corporate re-organisation strategies.
E Apply and evaluate alternative advanced treasury and risk management techniques.
F Evaluate the impact of macro economics and recognise the role of international financial institutions in the financial management of multinationals.
G Identify and assess the potential impact of emerging issues in finance and financial management

## Syllabus

A Role and responsibility towards stakeholders
1 Conflicting stakeholder interests
2 The role and responsibility of senior financial executive/advisor
3 Impact of environmental issues on corporate objectives and on governance
$4 \quad$ Financial strategy formulation
5 Ethical issues in financial management

## B Advanced investment appraisal

1 Discounted cash flow techniques and the use of free cash flows
2 Impact of financing on investment decisions and adjusted present values
3 Application of option pricing theory in investment decisions
4 International investment and financing decisions
5 Impact of capital investment on financial reporting

## C Acquisitions and mergers

1 Acquisitions and mergers versus other growth strategies
2 Valuation for acquisitions and mergers
3 Regulatory framework and processes
$4 \quad$ Financing acquisitions and mergers

D Corporate reconstruction and re-organisation
1 Predicting corporate failure
2 Financial reconstruction
3 Business re-organisation
E Treasury and advanced risk management techniques
1 The role of the treasury function in multinationals
2 The use of financial derivatives to hedge against forex risk
3 The use of financial derivatives to hedge against interest rate risk
4 Other forms of risk
5 Dividend policy in multinationals and transfer pricing

## F Economic environment for multinationals

1 Management of international trade and finance
2 Strategic business and financial planning for multinationals
G Emerging issues in finance and financial management
1 Developments in world financial markets
2 Financial engineering and emerging derivative products
3 Developments in international trade and finance

## Approach to examining the syllabus

The P4, Advanced Financial Management, paper builds upon the skills and knowledge examined in the F9, Financial Management, paper. At this stage candidates will be expected to demonstrate an integrated knowledge of the subject and an ability to relate their technical understanding of the subject to issues of strategic importance to the firm. The study guide specifies the wide range of contextual understanding that is required to achieve a satisfactory standard at this level. In particular the ethical and managerial aspects of the role of the senior financial manager or advisor will regularly feature in examination papers.

## Examination Structure

The examination will be a three hour paper in two sections:

Section A has two compulsory questions worth 60 marks in total. This section will normally cover significant issues relevant to the senior financial manager or advisor and will be set in the form of a short case study or scenario. The requirements of the section A questions are such that candidates will be expected to show a comprehensive understanding of issues from across the syllabus. Each question will contain a mix of computational and discursive elements. Normally, approximately 50 per cent of the marks will be apportioned to each of the two elements. A maximum of 40 marks will be available for either question in Section A.

Section B questions are designed to provide a more focused test of the syllabus with, normally, at least one question being wholly discursive. Candidates will be expected to provide answers in a specified form such as a short report or board memorandum commensurate with the professional level of the paper.

Section A: Answer both questions, total 60 marks
Section B: Answer two from three questions,

20 marks each Total 100 marks

Candidates will be provided with a formulae sheet as well as present value, annuity and standard normal distribution tables.

## Study Guide

This study guide provides more detailed guidance on the syllabus. You should use this as the basis of your studies.

## A ROLE AND RESPONSIBILITY TOWARDS STAKEHOLDERS

## 1. Conflicting stakeholder interests

a) Assess the potential sources of the conflict within a given corporate governance/ stakeholder framework informed by an understanding of the alternative theories of managerial behaviour. Relevant underpinning theory for this assessment would be:
i) The Separation of Ownership and Control
ii) Transaction cost economics and comparative governance structures
iii) Agency Theory
b) Recommend, within specified problem domains, appropriate strategies for the resolution of stakeholder conflict and advise on alternative approaches that may be adopted.
c) Compare the emerging governance structures and policies with respect to corporate governance (with particular emphasis upon the European stakeholder and the US/UK shareholder model) and with respect to the role of the financial manager.
2. The role and responsibility of senior financial executive/advisor
a) Advise the board of directors of the firm in setting the financial goals of the business and in its financial policy development with particular reference to:
i) Investment selection and capital resource allocation
ii) Minimising the firm's cost of capital
iii) Distribution and retention policy
iv) Communicating financial policy and corporate goals to internal and external stakeholders
v) Financial planning and control
vi) The management of risk
b) Develop strategies for the achievement of the firm's goals in line with its agreed policy framework.
c) Recommend strategies for the management of the financial resources of the firm such that they are utilised in an efficient, effective and transparent way.
d) Establish an ethical financial policy for the financial management of the firm which is grounded in good governance, the highest standards of probity and is fully aligned with the ethical principles of the Association.
e) Explore the areas within the ethical framework of the firm which may be undermined by agency effects and/or stakeholder conflicts and establish strategies for dealing with them.
f) Provide advice on personal finance to individual as well as groups of investors, covering areas such as investment and financing.
3. Impact of environmental issues on corporate objectives and on governance
a) Assess the issues which may impact upon corporate objectives and governance from:
i) Sustainability and environmental risk
ii) The carbon-trading economy and emissions
iii) The role of the environment agency
iv) Environmental audits and the triple bottom line approach

## 4. Financial strategy formulation

a) Recommend the optimum capital mix and structure within a specified business context and capital asset structure.
b) Recommend appropriate distribution and retention policy.
c) Establish capital investment monitoring and risk management systems.
d) Develop a framework for risk management comparing and contrasting risk mitigation, hedging and diversification strategies.
5. Ethical issues in financial management
a) Assess the ethical dimension within business issues and decisions and advise on best practice in the financial management of the firm.
b) Demonstrate an understanding of the interconnectedness of the ethics of good business practice between all of the functional areas of the firm.
c) Recommend an ethical framework for the development of a firm's financial policies and a system for the assessment of their ethical impact upon the financial management of the firm.

## B ADVANCED INVESTMENT APPRAISAL

1. Discounted cash flow techniques and the use of free cash flows
a) Evaluate the potential value added to a firm arising from a specified capital investment project or portfolio using the net present value model. Project modelling should include explicit treatment of:
i) Inflation and specific price variation
ii) Taxation and the assessment of fiscal risk
iii) Multi-period capital rationing to include the formulation of programming methods and the interpretation of their output.
b) Establish the potential economic return (using internal rate of return and modified internal rate of return) and advise on a project's return margin and its vulnerability to competitive action.
c) Forecast a firm's free cash flow and its free cash flow to equity (pre and post capital reinvestment).
d) Advise, in the context of a specified capital investment programme, on a firm's current and projected dividend capacity.
e) Advise on the value of a firm using its free cash flow and free cash flow to equity under alternative horizon and growth assumptions.

## 2. Impact of financing on investment decisions and adjusted present values

a) Assess the impact of financing upon investment decisions of:
i) Pecking order theory
ii) Static trade-off theory
iii) Agency effects and capital structure
b) Apply the adjusted present value technique to the appraisal of investment decisions that entail significant alterations in the financial structure of the firm, including their fiscal and transactions cost implications.
c) Outline the application of Monte Carlo simulation to investment appraisal. Candidates will not be expected to undertake simulations in an examination context but will be expected to demonstrate an understanding of:
i) Simple model design
ii) The different types of distribution controlling the key variables within the simulation.
iii) The significance of the simulation output and the assessment of the likelihood of project success.
iv) The measurement and interpretation of project value at risk.
3. Application of option pricing theory in investment decisions
a) Demonstrate an understanding of option pricing theory:
i) Determine, using published data, the five principal drivers of option value (value of the underlying, exercise price, time to expiry, volatility and the risk-free rate).
ii) Discuss the underlying assumptions, structure, application and limitations of the Black-Scholes model
b) Evaluate embedded real options within a project, classifying them into one of the real option archetypes.
c) Assess and advise on the value of options to delay, expand, redeploy and withdraw using the Black Scholes model.
4. International investment and financing decisions
a) Assess the impact upon the value of a project of alternative exchange rate assumptions
b) Forecast project or firm free cash flows in any specified currency and determine the project's net present value or firm value under differing exchange rate, fiscal and transaction cost assumptions:
c) Evaluate the significance of exchange controls for a given investment decision and strategies for dealing with restricted remittance.
d) Assess the impact of a project upon a firm's exposure to translation, transaction and economic risk.
e) Assess and advise upon the costs and benefits of alternative sources of finance available within the international financial markets.

## 5. Impact of capital investment on financial reporting

a) Assess the impact of a significant capital investment project upon the reported financial position and performance of the firm taking into account:
i) Alternative financing strategies
ii) Foreign exchange translation
iii) Taxation and double taxation
iv) Capital allowances and the problem of tax exhaustion.

## C ACQUISITIONS AND MERGERS

1. Acquisitions and mergers versus other growth strategies
a) Discuss the arguments for and against the use of acquisitions and mergers as a method of corporate expansion.
b) Evaluate the corporate and competitive nature of a given acquisition proposal.
c) Advise upon the criteria for choosing an appropriate target for acquisition.
d) Compare the various explanations for the high failure rate of acquisitions in enhancing shareholder value.
e) Evaluate, from a given context, the potential for synergy separately classified as:
i) Revenue synergy
ii) Cost synergy
iii) Financial synergy

## 2. Valuation for acquisitions and mergers

a) Outline the argument and the problem of overvaluation.
b) Estimate the potential near-term and continuing growth levels of a firm's earnings using both internal and external measures.
c) Assess the impact of an acquisition or merger upon the risk profile of the acquirer distinguishing:
i) Type 1 acquisitions that do not disturb the acquirer's exposure to financial or business risk
ii) Type 2 acquisitions that impact upon the acquirer's exposure to financial risk
iii) Type 3 acquisitions that impact upon the acquirer's exposure to both financial and business risk.
d) Advise on the valuation of a type 1 acquisition of both quoted and unquoted entities using:
i) 'Book value-plus' models
ii) Market relative models
iii) Cash flow models, including EVATM, MVA
e) Advise on the valuation of type 2 acquisitions using the adjusted net present value model.
f) Advise on the valuation of type 3 acquisitions using iterative revaluation procedures.
g) Demonstrate an understanding of the procedure for valuing high growth start-ups.

## 3. Regulatory framework and processes

a) Demonstrate an understanding of the principal factors influencing the development of the regulatory framework for mergers and acquisitions globally and, in particular, be able to compare and contrast the shareholder versus the stakeholder models of regulation.
b) Identify the main regulatory issues which are likely to arise in the context of a given offer and
i) assess whether the offer is likely to be in the shareholders' best interests
ii) advise the directors of a target company on the most appropriate defence if a specific offer is to be treated as hostile.

## 4. Financing acquisitions and mergers

a) Compare the various sources of financing available for a proposed cash-based acquisition
b) Evaluate the advantages and disadvantages of a financial offer for a given acquisition proposal using pure or mixed mode financing and recommend the most appropriate offer to be made.
c) Assess the impact of a given financial offer on the reported financial position and performance of the acquirer.

## D CORPORATE RECONSTRUCTION AND RE-ORGANISATION

1. Predicting corporate failure
a) Assess the risk of corporate failure within the short to medium term using a range of appropriate financial evaluation methods (this will require an ability to use multivariate techniques such as the Z and Zeta score models).
b) Advise on the application of financial distress models to firms in emerging markets given local regulatory and financial market conditions.
2. Financial reconstruction
a) Assess a company situation and determine whether a financial reconstruction is the most appropriate strategy for dealing with the problem as presented.
b) Assess the likely response of the capital market and/or individual suppliers of capital to any reconstruction scheme and the impact their response is likely to have upon the value of the firm.
c) Recommend a reconstruction scheme from a given business situation, justifying the proposal in terms of its impact upon the reported performance and financial position of the firm.

## 3. Business re-organisation

a) Recommend, with reasons, strategies for unbundling parts of a quoted company.
b) Evaluate the likely financial and other benefits of unbundling.
c) Advise on the financial issues relating to a management buy-out and buy-in.

## E TREASURY AND ADVANCED RISK MANAGEMENT TECHNIQUES

## 1. The role of the treasury function in multinationals

a) Describe the role of the money markets in:
i) Providing short-term liquidity to industry and the public sector
ii) Providing short-term trade finance
iii) Allowing a multinational firm to manage its exposure to FOREX and interest rate risk.
b) Explain the role of the banks and other financial institutions in the operation of the money markets.
c) Describe the characteristics and role of the principal money market instruments:
i) Coupon bearing:
ii) Discount instruments
iii) Derivatives
d) Outline the role of the treasury management function within:
i) The short term management of the firm's financial resources
ii) The longer term maximisation of shareholder value
iii) The management of risk exposure
2. The use of financial derivatives to hedge against forex risk
a) Demonstrate an understanding of the operations of the derivatives market, including:
i) The relative advantages and disadvantages of exchange traded versus OTC agreements.
ii) Key features, such as standard contracts, tick sizes, margin requirements and margin trading.
iii) The source of basis risk and how it can be minimised.
b) Evaluate, for a given hedging requirement, which of the following is the most appropriate strategy, given the nature of the underlying position and the risk exposure:
i) The use of the forward exchange market and the creation of a money market hedge
ii) Synthetic foreign exchange agreements (SAFE's)
iii) Exchange-traded currency futures contracts
iv) Currency swaps
v) FOREX swaps
vi) Currency options
c) Advise on the use of bilateral and multilateral netting and matching as tools for minimising FOREX transactions costs and the management of market barriers to the free movement of capital and other remittances.

## 3. The use of financial derivatives to hedge against interest rate risk

a) Evaluate for a given hedging requirement which of the following is the most appropriate given the nature of the underlying position and the risk exposure:
i) Forward Rate Agreements
ii) Interest Rate Futures
iii) Interest rate swaps
iv) Options on FRA's (caps and collars), Interest rate futures and interest rate swaps.

## 4. Other forms of risk

a) Assess the firm's exposure to political, economic, regulatory and fiscal risk and the strategies available for the mitigation of such risk.
b) Assess the firm's exposure to credit risk, including:
i) Explain the role of, and the risk assessment models used by, the principal rating agencies.
ii) Estimate the likely credit spread over risk free.
iii) Estimate the firm's current cost of debt capital using the appropriate term structure of interest rates and the credit spread.
c) Explain the role of option pricing models in the assessment of default risk, the value of debt and its potential recoverability
5. Dividend policy in multinationals and transfer pricing
a) Determine a firm's dividend capacity and its policy given:
i) The firm's short- and long-term reinvestment strategy
ii) The impact of any other capital reconstruction programmes on free cash flow to equity such as share repurchase agreements and new capital issues
iii) The availability and timing of central remittances
iv) The corporate tax regime within the host jurisdiction
b) Develop company policy on the transfer pricing of goods and services across international borders and be able to determine the most appropriate transfer pricing strategy in a given situation reflecting local regulations and tax regimes.

## F ECONOMIC ENVIRONMENT FOR MULTINATIONALS

## 1. Management of international trade and finance

a) Advise on the theory and practice of free trade and the management of barriers to trade.
b) Demonstrate an up to date understanding of the major trade agreements and common markets and, on the basis of contemporary circumstances, advise on their policy and strategic implications for a given business.
c) Discuss the objectives of the World Trade Organisation.
d) Discuss the role of international financial institutions within the context of a globalised economy, with particular attention to the International Monetary Fund, the Bank of International

Settlements, The World Bank and the principal Central Banks (the Fed, Bank of England, European Central Bank and the Bank of Japan).
e) Assess the role of the international financial markets with respect to the management of global debt, the financial development of the emerging economies and the maintenance of global financial stability.
2. Strategic business and financial planning for multinationals
a) Advise on the development of a financial planning framework for a multinational taking into account:
i) Compliance with national governance requirements (for example the LSE requirements for admission for trading)
ii) The mobility of capital across borders and national limitations on remittances and transfer pricing.
iii) The pattern of economic and other risk exposures in the different national markets
iv) Agency issues in the central coordination of overseas operations and the balancing of local financial autonomy with effective central control.

## G EMERGING ISSUES

1. Developments in world financial markets

Demonstrate awareness, and discuss the significance to the firm, of latest developments in the world financial markets with particular reference to the removal of barriers to the free movement of capital and the international regulations on money laundering
2. Financial engineering and emerging derivative products

Demonstrate awareness, and discuss the significance to the firm, of latest derivative products with particular emphasis on the risks in derivative trading and the application of the following in their management:
i) Value at Risk
ii) Scenario analysis
iii) Stress testing

## 3. Developments in international trade and finance

Demonstrate an awareness of new developments in the macroeconomic environment, establishing their impact upon the firm, and advising on the appropriate response to those developments both internally and externally.

## Formulae and tables

## Formula Sheet

Modigliani and Miller Proposition 2 (with tax)
$K_{e}=K_{e}^{i}+(1-T)\left(K_{e}^{i}-K_{d}\right) \frac{V_{d}}{V_{e}}$

Two asset portfolio
$S_{p}=\sqrt{w_{a}^{2} s_{a}^{2}+w_{b}^{2} s_{b}^{2}+2 w_{a} w_{b} r_{a b} s_{a} s_{b}}$

## The Capital Asset Pricing Model

$E\left(r_{i}\right)=R_{f}+\beta_{i}\left(E\left(r_{m}\right)-R_{f}\right)$

The asset beta formula
$\beta_{\mathrm{a}}=\left[\frac{\mathrm{V}_{\mathrm{e}}}{\left(\mathrm{V}_{\mathrm{e}}+\mathrm{V}_{\mathrm{d}}(1-\mathrm{T})\right)} \beta_{\mathrm{e}}\right]+\left[\frac{\mathrm{V}_{\mathrm{d}}(1-\mathrm{T})}{\left(\mathrm{V}_{\mathrm{e}}+\mathrm{V}_{\mathrm{d}}(1-\mathrm{T})\right)} \beta_{\mathrm{d}}\right]$

## The Growth Model

$P_{o}=\frac{D_{o}(1+g)}{\left(r_{e}-g\right)}$

## Gordon's growth approximation

$g=b r_{e}$

The weighted average cost of capital

WACC $=\left[\frac{V_{e}}{V_{e}+V_{d}}\right] K_{e}+\left[\frac{V_{d}}{V_{e}+V_{d}}\right] K_{d}(1-T)$

## The Fisher formula

$(1+i)=(1+r)(1+h)$

Purchasing power parity and interest rate parity
$\mathrm{S}_{1}=\mathrm{S}_{\mathrm{o}} \times \frac{\left(1+\mathrm{h}_{\mathrm{c}}\right)}{\left(1+\mathrm{h}_{\mathrm{b}}\right)} \quad \mathrm{f}_{\mathrm{o}}=\mathrm{S}_{\mathrm{o}} \times \frac{\left(1+\mathrm{i}_{\mathrm{c}}\right)}{\left(1+\mathrm{i}_{\mathrm{b}}\right)}$

## The Black Scholes Option Pricing Model

$c=P_{a} N\left(d_{1}\right)-P_{e} N\left(d_{2}\right) e^{-r t}$

Where:
$\mathrm{d}_{1}=\frac{\ln \left(\mathrm{P}_{\mathrm{a}} / \mathrm{P}_{\mathrm{e}}\right)+\left(\mathrm{r}+0.5 \mathrm{~s}^{2}\right) \mathrm{t}}{\mathrm{s} \sqrt{\mathrm{t}}}$
$d_{2}=d_{1}^{-} s \sqrt{t}$

The FOREX modified Black and Scholes option pricing model
$c=e^{-r t} F_{0} N\left(d_{1}\right)-X N\left(d_{2}\right)$

Or

$$
p=e^{-r t} X N\left(-d_{2}\right)-F_{0} N\left(-d_{1}\right)
$$

Where:
$d_{1}=\frac{\ln \left(F_{0} / X\right)+s^{2} T / 2}{s \sqrt{T}}$
and

$$
\mathrm{d}_{2}=\mathrm{d}_{1}-\mathrm{s} \sqrt{\mathrm{~T}}
$$

## The Put Call Parity relationship

$$
\mathrm{p}=\mathrm{c}-\mathrm{P}_{\mathrm{a}}+\mathrm{P}_{\mathrm{e}} \mathrm{e}^{-\mathrm{rt}}
$$

## Present value table

> Present value of 1 i.e. $(1+\mathrm{r})^{-\mathrm{n}}$ $\begin{array}{ll}\text { where } & \mathrm{r} \\ & =\text { discount rate } \\ \mathrm{n} & =\text { number of periods until payment }\end{array}$
Periods Discount rate (r)

| $(\mathrm{n})$ | $1 \%$ | $2 \%$ | $3 \%$ | $4 \%$ | $5 \%$ | $6 \%$ | $7 \%$ | $8 \%$ | $9 \%$ | $10 \%$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 0.990 | 0.980 | 0.971 | 0.962 | 0.952 | 0.943 | 0.935 | 0.926 | 0.917 | 0.909 | 1 |
| 2 | 0.980 | 0.961 | 0.943 | 0.925 | 0.907 | 0.890 | 0.873 | 0.857 | 0.842 | 0.826 | 2 |
| 3 | 0.971 | 0.942 | 0.915 | 0.889 | 0.864 | 0.840 | 0.816 | 0.794 | 0.772 | 0.751 | 3 |
| 4 | 0.961 | 0.924 | 0.888 | 0.855 | 0.823 | 0.792 | 0.763 | 0.735 | 0.708 | 0.683 | 4 |
| 5 | 0.951 | 0.906 | 0.863 | 0.822 | 0.784 | 0.747 | 0.713 | 0.681 | 0.650 | 0.621 | 5 |
| 6 | 0.942 | 0.888 | 0.837 | 0.790 | 0.746 | 0.705 | 0.666 | 0.630 | 0.596 | 0.564 | 6 |
| 7 | 0.933 | 0.871 | 0.813 | 0.760 | 0.711 | 0.665 | 0.623 | 0.583 | 0.547 | 0.513 | 7 |
| 8 | 0.923 | 0.853 | 0.789 | 0.731 | 0.677 | 0.627 | 0.582 | 0.540 | 0.502 | 0.467 | 8 |
| 9 | 0.914 | 0.837 | 0.766 | 0.703 | 0.645 | 0.592 | 0.544 | 0.500 | 0.460 | 0.424 | 9 |
| 10 | 0.905 | 0.820 | 0.744 | 0.676 | 0.614 | 0.558 | 0.508 | 0.463 | 0.422 | 0.386 | 10 |
| 11 | 0.896 | 0.804 | 0.722 | 0.650 | 0.585 | 0.527 | 0.475 | 0.429 | 0.388 | 0.350 | 11 |
| 12 | 0.887 | 0.788 | 0.701 | 0.625 | 0.557 | 0.497 | 0.444 | 0.397 | 0.356 | 0.319 | 12 |
| 13 | 0.879 | 0.773 | 0.681 | 0.601 | 0.530 | 0.469 | 0.415 | 0.368 | 0.326 | 0.290 | 13 |
| 14 | 0.870 | 0.758 | 0.661 | 0.577 | 0.505 | 0.442 | 0.388 | 0.340 | 0.299 | 0.263 | 14 |
| 15 | 0.861 | 0.743 | 0.642 | 0.555 | 0.481 | 0.417 | 0.362 | 0.315 | 0.275 | 0.239 | 15 |
| $(\mathrm{n})$ | $11 \%$ | $12 \%$ | $13 \%$ | $14 \%$ | $15 \%$ | $16 \%$ | $17 \%$ | $18 \%$ | $19 \%$ | $20 \%$ |  |
| 1 | 0.901 | 0.893 | 0.885 | 0.877 | 0.870 | 0.862 | 0.855 | 0.847 | 0.840 | 0.833 | 1 |
| 2 | 0.812 | 0.797 | 0.783 | 0.769 | 0.756 | 0.743 | 0.731 | 0.718 | 0.706 | 0.694 | 2 |
| 3 | 0.731 | 0.712 | 0.693 | 0.675 | 0.658 | 0.641 | 0.624 | 0.609 | 0.593 | 0.579 | 3 |
| 4 | 0.659 | 0.636 | 0.613 | 0.592 | 0.572 | 0.552 | 0.534 | 0.516 | 0.499 | 0.482 | 4 |
| 5 | 0.593 | 0.567 | 0.543 | 0.519 | 0.497 | 0.476 | 0.456 | 0.437 | 0.419 | 0.402 | 5 |
| 6 | 0.535 | 0.507 | 0.480 | 0.456 | 0.432 | 0.410 | 0.390 | 0.370 | 0.352 | 0.335 | 6 |
| 7 | 0.482 | 0.452 | 0.425 | 0.400 | 0.376 | 0.354 | 0.333 | 0.314 | 0.296 | 0.279 | 7 |
| 8 | 0.434 | 0.404 | 0.376 | 0.351 | 0.327 | 0.305 | 0.285 | 0.266 | 0.249 | 0.233 | 8 |
| 9 | 0.391 | 0.361 | 0.333 | 0.308 | 0.284 | 0.263 | 0.243 | 0.225 | 0.209 | 0.194 | 9 |
| 10 | 0.352 | 0.322 | 0.295 | 0.270 | 0.247 | 0.227 | 0.208 | 0.191 | 0.176 | 0.162 | 10 |
| 11 | 0.317 | 0.287 | 0.261 | 0.237 | 0.215 | 0.195 | 0.178 | 0.162 | 0.148 | 0.135 | 11 |
| 12 | 0.286 | 0.257 | 0.231 | 0.208 | 0.187 | 0.168 | 0.152 | 0.137 | 0.124 | 0.112 | 12 |
| 13 | 0.258 | 0.229 | 0.204 | 0.182 | 0.163 | 0.145 | 0.130 | 0.116 | 0.104 | 0.093 | 13 |
| 14 | 0.232 | 0.205 | 0.181 | 0.160 | 0.141 | 0.125 | 0.111 | 0.099 | 0.088 | 0.078 | 14 |
| 15 | 0.209 | 0.183 | 0.160 | 0.140 | 0.123 | 0.108 | 0.095 | 0.084 | 0.074 | 0.065 | 15 |

## Annuity table

Present value of an annuity of 1 i.e. $\frac{1-(1+r)^{-n}}{r}$

$$
\begin{array}{lll}
\text { where } & \mathrm{r} & =\text { discount rate } \\
& \mathrm{n} & =\text { number of periods }
\end{array}
$$

| Periods |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| (n) | $1 \%$ | $2 \%$ | $3 \%$ | $4 \%$ | $5 \%$ | $6 \%$ | $7 \%$ | $8 \%$ | $9 \%$ | $10 \%$ |  |
| 1 | 0.990 | 0.980 | 0.971 | 0.962 | 0.952 | 0.943 | 0.935 | 0.926 | 0.917 | 0.909 | 1 |
| 2 | 1.970 | 1.942 | 1.913 | 1.886 | 1.859 | 1.833 | 1.808 | 1.783 | 1.759 | 1.736 | 2 |
| 3 | 2.941 | 2.884 | 20829 | 2.775 | 2.723 | 2.673 | 2.624 | 2.577 | 2.531 | 2.487 | 3 |
| 4 | 3.902 | 3.808 | 3.717 | 3.630 | 3.546 | 3.465 | 3.387 | 3.312 | 3.240 | 3.170 | 4 |
| 5 | 4.853 | 4.713 | 4.580 | 4.452 | 4.329 | 4.212 | 4.100 | 3.993 | 3.890 | 3.791 | 5 |
| 6 | 5.795 | 5.601 | 5.417 | 5.242 | 5.076 | 4.917 | 4.767 | 4.623 | 4.486 | 4.355 | 6 |
| 7 | 6.728 | 6.472 | 6.230 | 6.002 | 5.786 | 5.582 | 5.389 | 5.206 | 5.033 | 4.868 | 7 |
| 8 | 7.652 | 7.325 | 7.020 | 6.333 | 6.463 | 6.210 | 5.971 | 5.747 | 5.535 | 5.335 | 8 |
| 9 | 8.566 | 8.162 | 7.786 | 7.435 | 7.108 | 6.802 | 6.515 | 6.247 | 5.995 | 5.759 | 9 |
| 10 | 9.471 | 8.983 | 8.530 | 8.111 | 7.722 | 7.360 | 7.024 | 6.710 | 6.418 | 6.145 | 10 |
| 11 | 10.37 | 9.787 | 9.253 | 8.760 | 8.306 | 7.887 | 7.499 | 7.139 | 6.805 | 6.495 | 11 |
| 12 | 11.26 | 10.58 | 9.954 | 9.385 | 8.863 | 8.384 | 7.943 | 7.536 | 7.161 | 6.814 | 12 |
| 13 | 12.13 | 11.35 | 10.63 | 9.986 | 9.394 | 8.853 | 8.358 | 7.904 | 7.487 | 7.103 | 13 |
| 14 | 13.00 | 12.11 | 11.30 | 10.56 | 9.899 | 9.295 | 8.745 | 8.244 | 7.786 | 7.367 | 14 |
| 15 | 13.87 | 12.85 | 11.94 | 11.12 | 10.38 | 9.712 | 9.108 | 8.559 | 8.061 | 7.606 | 15 |
| (n) | $11 \%$ | $12 \%$ | $13 \%$ | $14 \%$ | $15 \%$ | $16 \%$ | $17 \%$ | $18 \%$ | $19 \%$ | $20 \%$ |  |
| 1 | 0.901 | 0.893 | 0.885 | 0.877 | 0.870 | 0.862 | 0.855 | 0.847 | 0.840 | 0.833 | 1 |
| 2 | 1.713 | 1.690 | 1.668 | 1.647 | 1.626 | 1.605 | 1.585 | 1.566 | 1.547 | 1.528 | 2 |
| 3 | 2.444 | 2.402 | 2.361 | 2.322 | 2.283 | 2.246 | 2.210 | 2.174 | 2.140 | 2.106 | 3 |
| 4 | 3.102 | 3.037 | 2.974 | 2.914 | 2.855 | 2.798 | 2.743 | 2.690 | 2.639 | 2.589 | 4 |
| 5 | 3.696 | 3.605 | 3.517 | 3.433 | 3.352 | 3.274 | 3.199 | 3.127 | 3.058 | 2.991 | 5 |
| 6 | 4.231 | 4.111 | 3.998 | 3.889 | 3.784 | 3.685 | 3.589 | 3.498 | 3.410 | 3.326 | 6 |
| 7 | 4.712 | 4.564 | 4.423 | 4.288 | 4.160 | 4.039 | 3.922 | 3.812 | 3.706 | 3.605 | 7 |
| 8 | 5.146 | 4.968 | 4.799 | 4.639 | 4.487 | 4.344 | 4.207 | 4.078 | 3.954 | 3.837 | 8 |
| 9 | 5.537 | 5.328 | 5.132 | 4.946 | 4.772 | 4.607 | 4.451 | 4.303 | 4.163 | 4.031 | 9 |
| 10 | 5.889 | 5.650 | 5.426 | 5.216 | 5.019 | 4.833 | 4.659 | 4.494 | 4.339 | 4.192 | 10 |
| 11 | 6.207 | 5.938 | 5.687 | 5.453 | 5.234 | 5.029 | 4.836 | 4.656 | 4.486 | 4.327 | 11 |
| 12 | 6.492 | 6.194 | 5.918 | 5.660 | 5.421 | 5.197 | 4.988 | 4.793 | 4.611 | 4.439 | 12 |
| 13 | 6.750 | 6.424 | 6.122 | 5.842 | 5.583 | 5.342 | 5.118 | 4.910 | 4.715 | 4.533 | 13 |
| 14 | 6.982 | 6.628 | 6.302 | 6.002 | 5.724 | 5.468 | 5.229 | 5.008 | 4.802 | 4.611 | 14 |
| 15 | 7.191 | 6.811 | 6.462 | 6.142 | 5.847 | 5.575 | 5.324 | 5.092 | 4.876 | 4.675 | 15 |
|  |  |  |  |  |  |  |  |  |  |  |  |

## Standard normal distribution table

|  | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.0 | 0.0000 | 0.0040 | 0.0080 | 0.0120 | 0.0160 | 0.0199 | 0.0239 | 0.0279 | 0.0319 | 0.0359 |
| 0.1 | 0.0398 | 0.0438 | 0.0478 | 0.0517 | 0.0557 | 0.0596 | 0.0636 | 0.0675 | 0.0714 | 0.0753 |
| 0.2 | 0.0793 | 0.0832 | 0.0871 | 0.0910 | 0.0948 | 0.0987 | 0.1026 | 0.1064 | 0.1103 | 0.1141 |
| 0.3 | 0.1179 | 0.1217 | 0.1255 | 0.1293 | 0.1331 | 0.1368 | 0.1406 | 0.1443 | 0.1480 | 0.1517 |
| 0.4 | 0.1554 | 0.1591 | 0.1628 | 0.1664 | 0.1700 | 0.1736 | 0.1772 | 0.1808 | 0.1844 | 0.1879 |
| 0.5 | 0.1915 | 0.1950 | 0.1985 | 0.2019 | 0.2054 | 0.2088 | 0.2123 | 0.2157 | 0.2190 | 0.2224 |
| 0.6 | 0.2257 | 0.2291 | 0.2324 | 0.2357 | 0.2389 | 0.2422 | 0.2454 | 0.2486 | 0.2517 | 0.2549 |
| 0.7 | 0.2580 | 0.2611 | 0.2642 | 0.2673 | 0.2703 | 0.2734 | 0.2764 | 0.2794 | 0.2823 | 0.2852 |
| 0.8 | 0.2881 | 0.2910 | 0.2939 | 0.2967 | 0.2995 | 0.3023 | 0.3051 | 0.3078 | 0.3106 | 0.3133 |
| 0.9 | 0.3159 | 0.3186 | 0.3212 | 0.3238 | 0.3264 | 0.3289 | 0.3315 | 0.3340 | 0.3365 | 0.3389 |
| 1.0 | 0.3413 | 0.3438 | 0.3461 | 0.3485 | 0.3508 | 0.3531 | 0.3554 | 0.3577 | 0.3599 | 0.3621 |
| 1.1 | 0.3643 | 0.3665 | 0.3686 | 0.3708 | 0.3729 | 0.3749 | 0.3770 | 0.3790 | 0.3810 | 0.3830 |
| 1.2 | 0.3849 | 0.3869 | 0.3888 | 0.3907 | 0.3925 | 0.3944 | 0.3962 | 0.3980 | 0.3997 | 0.4015 |
| 1.3 | 0.4032 | 0.4049 | 0.4066 | 0.4082 | 0.4099 | 0.4115 | 0.4131 | 0.4147 | 0.4162 | 0.4177 |
| 1.4 | 0.4192 | 0.4207 | 0.4222 | 0.4236 | 0.4251 | 0.4265 | 0.4279 | 0.4292 | 0.4306 | 0.4319 |
| 1.5 | 0.4332 | 0.4345 | 0.4357 | 0.4370 | 0.4382 | 0.4394 | 0.4406 | 0.4418 | 0.4429 | 0.4441 |
| 1.6 | 0.4452 | 0.4463 | 0.4474 | 0.4484 | 0.4495 | 0.4505 | 0.4515 | 0.4525 | 0.4535 | 0.4545 |
| 1.7 | 0.4554 | 0.4564 | 0.4573 | 0.4582 | 0.4591 | 0.4599 | 0.4608 | 0.4616 | 0.4625 | 0.4633 |
| 1.8 | 0.4641 | 0.4649 | 0.4656 | 0.4664 | 0.4671 | 0.4678 | 0.4686 | 0.4693 | 0.4699 | 0.4706 |
| 1.9 | 0.4713 | 0.4719 | 0.4726 | 0.4732 | 0.4738 | 0.4744 | 0.4750 | 0.4756 | 0.4761 | 0.4767 |
| 2.0 | 0.4772 | 0.4778 | 0.4783 | 0.4788 | 0.4793 | 0.4798 | 0.4803 | 0.4808 | 0.4812 | 0.4817 |
| 2.1 | 0.4821 | 0.4826 | 0.4830 | 0.4834 | 0.4838 | 0.4842 | 0.4846 | 0.4850 | 0.4854 | 0.4857 |
| 2.2 | 0.4861 | 0.4864 | 0.4868 | 0.4871 | 0.4875 | 0.4878 | 0.4881 | 0.4884 | 0.4887 | 0.4890 |
| 2.3 | 0.4893 | 0.4896 | 0.4898 | 0.4901 | 0.4904 | 0.4906 | 0.4909 | 0.4911 | 0.4913 | 0.4916 |
| 2.4 | 0.4918 | 0.4920 | 0.4922 | 0.4925 | 0.4927 | 0.4929 | 0.4931 | 0.4932 | 0.4934 | 0.4936 |
| 2.5 | 0.4938 | 0.4940 | 0.4941 | 0.4943 | 0.4945 | 0.4946 | 0.4948 | 0.4949 | 0.4951 | 0.4952 |
| 2.6 | 0.4953 | 0.4955 | 0.4956 | 0.4957 | 0.4959 | 0.4960 | 0.4961 | 0.4962 | 0.4963 | 0.4964 |
| 2.7 | 0.4965 | 0.4966 | 0.4967 | 0.4968 | 0.4969 | 0.4970 | 0.4971 | 0.4972 | 0.4973 | 0.4974 |
| 2.8 | 0.4974 | 0.4975 | 0.4976 | 0.4977 | 0.4977 | 0.4978 | 0.4979 | 0.4979 | 0.4980 | 0.4981 |
| 2.9 | 0.4981 | 0.4982 | 0.4982 | 0.4983 | 0.4984 | 0.4984 | 0.4985 | 0.4985 | 0.4986 | 0.4986 |
| 3.0 | 0.4987 | 0.4987 | 0.4987 | 0.4988 | 0.4988 | 0.4989 | 0.4989 | 0.4989 | 0.4990 | 0.4990 |

This table can be used to calculate $\mathrm{N}\left(\mathrm{d}_{\mathrm{i}}\right)$, the cumulative normal distribution functions needed for the Black-Scholes model of option pricing. If $d_{i}>0$, add 0.5 to the relevant number above. If $\mathrm{d}_{\mathrm{i}}<0$, subtract the relevant number above from $0 \cdot 5$.

# The role and responsibilities of financial managers 

|  | Contents |
| :--- | :--- |
| 1 | Conflicting stakeholder interests |
| 2 | Improving corporate governance |
| 3 | Role and responsibility of the senior financial <br> management executive |
| 4 | Dividend and retentions policy |

## Conflicting stakeholder interests

- The objectives of an organisation
- Primary corporate objective
- Theories supporting differing views of the primary corporate objective
- Separation of ownership from control
- Agency theory
- Transaction cost theory
- Stakeholder theory
- Differing governance models
- Resolving stakeholder conflict


## 1 Conflicting stakeholder interests

### 1.1 The objectives of an organisation

Every organisation exists for a purpose, which can be expressed in a mission statement. For example, an organisation might exist to manufacture electronic equipment, manufacture chemicals, provide a transport service or provide an education to children.

Within this overall purpose, an organisation should have a primary objective.

- For companies, the main objective might be to maximise the wealth of the company's owners, its equity shareholders
- For state-owned organisations, the main objective might be stated in terms of providing a certain standard of public service
- For a charity, the main objective might be to provide maximum aid or support for a particular group of people.

When the main objective of an organisation is not a financial objective, there is always a financial constraint on its objective, such as providing the highest quality of public service with the available finance.

### 1.2 The primary corporate objective

For companies, the main objective might be to maximise the wealth of its owners, the equity shareholders. However, there is some debate about whether a company should also have important objectives with regard to other 'stakeholder groups', such as its employees, its major suppliers, lenders to the company, its customers, the government and even the general public.

Two differing views of the objectives of an organisation are therefore:

- The only significant objective of a company is to maximise the wealth of the equity shareholders. This is achieved by maximising the combined value of dividends and share price growth
- A company should have several objectives, which might be, for example, to increase the wealth of shareholders, to treat employees fairly and well, to treat customers and suppliers in a proper and ethical way, and to take into consideration the needs of society and the need to preserve the environment.


## Corporate social responsibility (CSR)

Corporate social responsibility is a term to describe the view that a company should pursue objectives that are in the interests of stakeholder groups other than shareholders, such as employees and society as a whole (the 'public').

### 1.3 Theories supporting differing views of the primary corporate objective

Several theories have been developed that support differing views about how a company should be governed (corporate governance) and what the primary objective of a company should be. Three such theories are:

- Agency cost theory
- Transaction cost economic theory
- Stakeholder theory.

The differing views about the objectives of a company can give rise to conflicts of interest, as different stakeholder groups argue that the company should be aiming to do something different and achieve different goals.

Agency theory and transaction cost economics both recognise that problems with pursuing the best interests of a company arise because of a separate of company ownership from control.

### 1.4 Separation of ownership from control

Problems with agreeing the aims and objectives of a company exist because of the separation of the ownership of a company from its control.

- Shareholders are the owners of the company.
- However, the company is a separate legal person, and decisions are made by the company in its own name, not in the name of the shareholders.
- The directors and senior managers of a company control the decisions that the company makes.

The legal rights of shareholders to make decisions for the company are very limited (by law, by stock market regulations and by the company's constitution) in all countries. The shareholders therefore rely on the directors and senior managers to govern the company in the interests of the shareholders and other stakeholders, and laws and codes of corporate governance are aimed at achieving this.

### 1.5 Agency theory

Agency theory, which was developed by Jensen and Meckling (1976), is based on the separation of the ownership and control. Jensen and Meckling argued that when directors and managers are appointed to run a company, an agency-principal relationship is created. This agency relationship is a form of contract between a company's owners and its managers, where the owners (as principal) appoint an agent (the managers) to manage the company on their behalf. Within this arrangement, decision-making authority is delegated by the shareholders to the management.

However, Jensen and Meckling argued that this agency relationship creates a serious conflict of interest between the company's owners and managers.

- Most shareholders want to increase their income and wealth over the long-term. The value of their shares depends on the long-term financial prospects for the company. Shareholders are concerned about short-term profits and dividends, but they are even more concerned about long-term profitability and wealth creation.
- The managers run the company on behalf of the shareholders. They have an employment contract and earn a salary. If they do not own shares in the company, they have no direct interest in future returns for shareholders, or in the value of the shares. Unless their remuneration is linked to profits or share values, their main interests are likely to be the size of their remuneration package and their status as company managers.

In an ideal situation, the 'agency contract' between the owners and the managers of a company should ensure that the managers always act in the best interests of the owners. However, it is impossible to arrange the 'perfect contract', because decisions by the managers affect their own personal welfare as well as the interests of the owners.

Agency conflicts are differences in the interests of owners and managers. Some of these conflicts that might have a direct impact on the financial management of a company are as follows:

- Earnings retention. The remuneration of directors and senior managers is often related to the size of the company (for example, annual turnover) rather than its profits. This gives managers an incentive to increase the size of the company, rather than to increase the returns to the company's shareholders. When this happens, companies might invest in capital investment projects where the expected return is quite small, or propose over-priced takeover bids for other companies.
- Time horizon. Shareholders are concerned about the long-term financial prospects of their company because the share price depends on expectations for the long-term future. In contrast, managers might only be interested in the shortterm. This is partly because they might receive annual bonuses based on shortterm performance, and partly because they might not expect to be with the company for more than a few years.

Agency theory is based on the view that the system of corporate governance should be designed to minimise the agency problem, and reduce agency costs. The theory may be summarised as follows:

- In large companies there is a separation of ownership from control. Professional managers are appointed to act as agents for the owners of the company.
- Individuals are driven by self-interest.
- Conflicts of self-interest arise between shareholders and managers.
- Managers, because they are driven by self-interest, cannot be relied on to act in the best interests of the shareholders. This creates problems in the agency relationship between shareholders and management.
- These agency problems create costs for the shareholders.
- The aim should be to minimise these costs, by improving the monitoring of management and/or providing management with incentives to bring their interests closer to those of the shareholders.


### 1.6 Transaction cost theory

Transaction cost theory provides a different explanation of the relationship between the owners of a company and its management, but reaches a similar conclusion as agency theory.

The theory of transaction cost economics (TCE) is closely associated with the work of Oliver Williamson in the 1970s. He suggested that the operating activities of a company can be performed either through market transactions (with external suppliers) or by doing the work in-house. For example, a company could either obtain its raw materials from an external supplier or it could make the materials itself. Similarly, a company could outsource work, or it could hire full-time employees to do the work 'in-house'. In economic terms, the decision about whether to arrange transactions in the open market or whether to do the work in-house should depend on which is cheaper. If a company does the work in-house, it needs a management structure and a hierarchy of authority with senior management at the top. According to transaction cost theory, the structure of a firm and the relationship between the owners of a firm and its management depends on the extent to which transactions are performed in-house.

In transaction cost theory, total costs are defined as the sum of production costs and transaction costs.

- Production costs are the costs that would be incurred by the company in an ideal economic market. In an ideal economic market, production costs are minimised.
- Transaction costs are additional costs incurred whenever the perfect economic market is not achieved. For example, a company might buy goods from a supplier who is not the cheapest available, because it is not aware that a cheaper supplier exists. A company might sell goods on credit to a customer, not knowing that the customer is a high bad debt risk, and that the debt might never be collected.

In this analysis, the only difference between in-house operations and buying the materials or work externally is the difference in transaction costs between the two. Total costs are minimised when transaction costs are minimised. This should determine the optimal size of the firm (and the extent to which it is 'vertically integrated') and the size of the management hierarchy in the firm. As a general rule, it is in the interests of a company's management to carry out transactions internally, and not in the external market. Doing the work internally:

- removes the risks and uncertainties about prices of products and product quality
- removes all the risks and costs of dealing with external suppliers.

Transaction cost theory also considers the implications of human behaviour for the way in which a company is managed and governed. Williamson made two assumptions about behaviour:

- bounded rationality
- opportunism.


## Bounded rationality

Human beings act rationally, but only within the limits of their understanding. This means that a company manager will act rationally in trying to maximise the value of the company for its shareholders, but the limits to his understanding and ability to act rationally might make him act differently. In any business, there is a limit to the amount of information that individuals can remember, understand and deal with. No one is capable of assessing all the possible courses of action and no one can anticipate what will happen in the future. In a competitive market, no one can anticipate with certainty what competitors will do.

## Opportunism

Williamson also argued that individuals will act in a self-interested way, and 'with guile'. They are always be honest and truthful about their intentions. Opportunism, according to Williamson, is the effort to realise individual gains via a lack of candour or truthfulness in transactions. An individual might try to take advantage of an opportunity to gain a benefit at the expense of someone else. Managers are opportunistic by nature. Given the opportunity, they will take advantage of available ways of improving their own benefits and privileges

In terms of transaction cost economics, a problem with opportunism is that external parties - such as contractors and suppliers of goods - cannot always be trusted to act honestly. As a result, there may be a tendency for a company to carry out transactions itself, rather than rely on external suppliers. However, there is also a risk that by taking control of transactions internally, managers will have opportunities to take decisions and actions that are in their own personal interests. This self-interested behaviour needs to be controlled. In this respect, transaction cost theory has similarities with agency theory.

### 1.7 Stakeholder theory

Agency theory makes the assumption that the main objective of a company should be to maximise the wealth of shareholders. Stakeholder theory is different. It is based on the view that the purpose of corporate governance should be to satisfy, as far as possible, the objectives of all key stakeholders - employees, investors, major creditors, customers, major suppliers, the government, local communities and the general public. A role of the company's directors is therefore to consider the interests of all the major stakeholders.

Managers should try to achieve a range of different objectives, not just the aim of maximising the value of the company for its shareholders. This is because different stakeholders each have their own (different) expectations from the company, which the company's management should attempt to satisfy. However, some stakeholders might be more important than others, so that management should give priority to their interests above the interests of other stakeholder groups.

Stakeholder theory also considers the role of companies in society, and the responsibility that they should have towards society as a whole. It might be argued that some companies are so large, and their influence on society is so strong, that they should be accountable to the public for what they do. The general public are taxpayers and as such they provide the economic and social infrastructure within which companies are allowed to operate. In return, companies should be expected to act as corporate citizens and act in ways that benefit society as a whole. This aspect of stakeholder theory is consistent with the arguments in favour of corporate social responsibility.

### 1.8 Differing governance models

The differing views about the responsibilities of companies to shareholders and other stakeholders are evident in differing corporate governance structures and policies.

- The USA and UK are most closely associated with an agency theory view that company management should act in the best interests of the shareholders. The interests of the shareholders are given some protection by law (such as the Sarbanes-Oxley Act in the US) and by corporate governance codes (such as the Combined Code in the UK) However, there is relatively little recognition of the interest of other stakeholders, such as employees, in the governance of companies.
- Parts of Western Europe, notably Germany, have been associated with a stakeholder view of company responsibilities. In Germany there is a two-tier system of boards in large companies, with a management board responsible for company activities and operations, and a supervisory board with responsibility for broader company matters and for supervision of the management board. Many of the supervisory board directors are representatives of the employees, which means that employee interests have a significant influence on the discussions and decision-making of the supervisory boards.

It is also worth noting, however, that within the US and UK there are different views about the extent to which the directors of companies should comply with the demands and expectations of shareholders. Traditional institutional investors and fund managers are broadly supportive of company management, and rely on management to take decisions that are in the best interests of the shareholders as a whole. In contrast, activist funds (often hedge funds) holding only a fairly small shareholding of $1 \%$ to $3 \%$ in a company might try to force the company to adopt strategies they believe will 'extract value' from the company for the benefit of the shareholders - such as selling off parts of the business.

A key point to note is that although shareholders might expect a company's management to take decisions that are in the shareholders' interests, there could be different views about how the interests of the shareholders are best represented. A significant difference occurs between investors who are prepared to hold shares for the longer term, in the expectation that the value of the shares will rise, and those who invest for a much shorter term, hoping to see a profit by 'extracting value' from under-performing companies.

### 1.9 Resolving stakeholder conflict

In your examination, you might be given a question in which you are asked, as a company manager or as a financial management adviser, how conflicts between different shareholder groups and stakeholder groups might be resolved. To deal with this type of problem, you should consider all the possible courses of action that a company might take in a particular situation, and the probable response that each course of action may elicit from each stakeholder group.

In the chapters that follow, the focus of attention will be largely on shareholder wealth maximisation, but it is always important to remember the potential significance of other stakeholder interests in a company, including the interest of the general public and the government.

## Improving corporate governance

- Board of directors
- Financial reporting and the external auditors: the audit committee
- Directors' remuneration
- Internal control and risk management
- Communication with shareholders
- Corporate governance and corporate social responsibility
- Corporate governance in other countries


## 2 Improving corporate governance

Corporate governance can be improved by taking measures to deal with the problems. As indicated earlier, these measures might be included in a voluntary code, or might be made a legal requirement.

In the UK, there is a mixture of voluntary code and legal requirements. For example, listed UK companies are required by the UK Listing Rules to comply with the Combined Code on Corporate Governance or explain their non-compliance in the annual report and accounts. In addition, quoted companies are required by UK law to disclose details of the remuneration of each individual director and to submit a directors' remuneration report to the shareholders for approval, at the annual general meeting of the company.

Measures recommended by the UK Combined Code for achieving good corporate governance relate to:

- the board of directors: the board's responsibilities and the composition of the board
- financial reporting and the independence of the external auditors
- directors' remuneration
- internal control and risk management
- communications between the company and its shareholders, and the rights and responsibilities of shareholders (particularly investment institutions such as pension funds and insurance companies).

The provisions of the UK Combined Code are described briefly below. These are a useful guideline to corporate governance issues generally, and how they might be dealt with in companies in any country.

### 2.1 The board of directors

## Responsibilities of the board

The board of directors should reserve certain decisions for the board as a whole and should not delegate these decision-making powers to the executive management. The decisions reserved for the board would include decisions about major strategic investments.

## Composition of the board

To prevent the board from being dominated by a single individual, the positions of chairman and CEO should not be held by the same individual. The chairman is responsible for leading the board of directors and representing the company as a figurehead, for example in communicating with the shareholders. The CEO is responsible for leading the executive management team of the company.

In addition, the CEO of a listed company should not go on to become the chairman. If this happens, the chairman will not be independent, and may also exert a strong influence over his (or her) successor as CEO.

To prevent the CEO (or chairman) from exerting excessive influence, the board should include a sufficient number of independent non-executive directors (NEDs). In large listed companies, at least half the board, excluding the chairman, should be independent NEDs.

## Board committees

The board should delegate certain responsibilities to committees of the board, which should report back to the main board. The three board committees identified by the Combined Code are a nominations committee (for appointing directors), an audit committee (to communicate with the external auditors, recommend the appointment and annual fees of the auditors, review the need for internal audit function, etc) and remuneration committee (to consider remuneration policy and negotiate the remuneration of individual directors).

The audit and remuneration committees should consist entirely of independent NEDs, to avoid undue influence in these matters by executive directors. The nomination committee should have a majority of independent NEDs.

## Fulfilling responsibilities as directors adequately

The Combined Code states that directors should be able to give enough of their time to the company in order to carry out their responsibilities. However, it does not specify any limit to the number of (non-executive) directorships any individual should hold.

Instead, the Code states that the board as a whole, each of the board committees and all individual directors (including the chairman) should be subject to an annual performance review. In principle, any individual who performs badly may be asked to resign from the board.

In addition, the annual report and accounts should include information about the number of board meetings and committee meetings attended by each individual director. This may influence shareholders when they are asked to vote for the reelection of any individual director at the annual general meeting.

### 2.2 Financial reporting and the external auditors: the audit committee

To reduce the influence of the executive directors on the external auditors, certain powers should be delegated by the board to the audit committee. The powers and responsibilities of the audit committee should include the following:

- Discussing the annual audit plan with the external auditors
- Discussing with the external auditors any significant accounting issues that affect the content of the annual report and accounts
- Reviewing the auditors' performance. Where this is poor, the committee may recommend a change of auditors to the board of directors (which would then propose a change of auditors to the shareholders)
- Recommending the audit fee to the board of directors
- Monitoring the independence of the external auditors from influence by the executive directors. One way of doing this is to monitor the amount of non-audit work carried out for the company by the auditors. The audit committee should ensure that the audit firm does not over-rely on income from the company, either from the audit fee or fees for non-audit work.

There has been much debate about whether there should be a compulsory rotation of audit firms, so that companies are required to change their audit firm at least every five or seven years. This proposal was strongly opposed by companies and audit firms.

An alternative suggestion is to require the rotation of key audit partners, who should not remain as auditor for a particular company for more than a specified number of years. There is no requirement in the Combined Code about audit partner rotation, but the 8th European Union Directive, when introduced, will require the compulsory rotation of key audit partners after no more than seven years.

The board of directors must state in the annual report and accounts that the company is a going concern.

### 2.3 Directors' remuneration

The board should delegate to a remuneration committee (consisting of independent NEDs) responsibilities for:

- remuneration policy for executive directors, and
- negotiating the remuneration of individual executive directors.

In the UK, quoted companies are required by law to present a directors' remuneration report in the annual report and accounts and invite the shareholders to approve the report at the annual general meeting of the company.

### 2.4 Internal control and risk management

The board of directors must review the internal control system, and risk management system of the company, and satisfy themselves that suitable control systems are in place. The board should report to shareholders that they have done SO.

The responsibility for carrying out an annual review of risk management and the internal control system may be delegated to the audit committee.

### 2.5 Communication with shareholders

The Combined Code requires the board of a company to promote good relations and good communications with their shareholders.

In addition, institutional investors have a responsibility for maintaining a dialogue with the company's board of directors.

A new legal requirement has been introduced throughout the European Union that listed companies should prepare an annual business review, setting out the operating and financial position of the company in easy-to-understand language.

### 2.6 Corporate governance and corporate social responsibility

In summary, the aims of good corporate governance should be to reduce the conflicts of interest between the board of directors and the shareholders.

The UK Combined Code does not include any provisions relating to Corporate Social Responsibility. However, some institutional investors seek to promote CSR in companies, and some institutions will not invest in companies that do not meet certain minimum CSR requirements.

### 2.7 Corporate governance in other countries

Regulations and codes of practice about corporate governance vary between different countries, and the regulations are continually changing.

- In the United States, the main focus of attention has been on the reliability of financial reporting and internal controls. Following the collapse of Enron in 2001 and other corporate scandals such as WorldCom, the US introduced legislation in the Sarbanes-Oxley Act 2002. This legislation includes requirements for SECregistered companies to submit annual statements about the accuracy of the financial statements and the adequacy of financial controls (internal controls) in the company. These controls are subject to annual audit.
- Within the European Union, progress towards good corporate governance is being achieved by EU Directives such as the 8th Directive. In addition, some countries have developed their own voluntary codes of governance, such as the Cromme Code in Germany. In Germany, some of the problems of corporate governance are different from those in the UK. For example, German companies have a two-tier board structure. There is a supervisory board of non-executive
directors, led by the company chairman, and there is a management board of executives, led by the CEO. The non-executives on the supervisory board are by no means all independent, and they usually include representatives of major shareholders (who include banks), employees and retired senior executives.


## Role and responsibility of the senior financial management executive

- Advising on financial goals and financial policy development
- Financial strategy formulation
- Minimising the cost of capital
- Financial risk management and monitoring systems
- Ethical issues in financial management


## 3 Role and responsibility of the senior financial executive

### 3.1 Advising on financial goals and financial policy development

A large part of the role of the senior financial manager is advisory, providing advice on financial strategy and policies. In order to achieve its corporate objectives, a company must develop strategies. To achieve an objective of maximising shareholder wealth, financial strategies should be formulated by the board of directors. Financial strategy is often (but not always) targeted towards achieving growth in annual earnings and achieving a return on investment in excess of the cost of the funds used to make the investment.

The main areas where financial managers provide advice to the board of directors or senior operational management are:

- investment selection and capital investment appraisal
- the allocation of capital resources
- working capital efficiency (not covered in the syllabus for this examination paper)
- minimising the company's cost of capital
- dividend policy and retentions policy
- communicating financial policy to internal and external stakeholders in the company.

Financial managers also have some operational responsibilities, which include:

- financial planning and control
- financial risk management systems, including treasury risk management.


### 3.2 Financial strategy formulation

The financial manager should provide advice to the board of directors and senior management about financial strategy. Financial strategy should support the business strategy of the company. You should keep in mind the following aspects of financial strategy as you study this subject.

- Achieving the optimal capital mix for the company (mix of equity capital and debt capital) by minimising the company's weighted average cost of capital (WACC). Capital investments that exceed the WACC will usually add to the value of the company and the wealth of the shareholders.
- Applying a policy for dividend payments and profit retentions that achieves a suitable balance between the expectation of shareholders for annual dividend payments, and the need to retain some profits to reinvest in the business to achieve further growth in profits in the future. (Dividend policy is considered in more detail later in this chapter.)
- Establishing systems to monitor capital spending and for financial risk management.
- Developing a framework for financial risk management, with a variety of strategies for reducing and controlling risks, risk hedging and risk diversification.


### 3.3 Minimising the cost of capital

A financial manager must be able to calculate a cost of capital, and should advise management on what cost of capital should be used to evaluate particular capital expenditure projects. These aspects of the cost of capital are explained in later chapters.

A basic policy objective should be to minimise the company's cost of capital. This is because for a given annual return, the total value of a company is minimised when the weighted average cost of capital is minimised.

## Example

The annual operational cash flows of Company YZ are $\$ 800,000$ and are expected to remain $\$ 800,000$ in perpetuity.

The total value of the company can be estimated by dividing the annual cash flows by the weighted average cost of capital.

- If the WACC is $10 \%$, the total value of the company (equity and debt capital) is \$8,000,000 (\$800,000/0.10).
- If the WACC is $8 \%$, the total value of the company (equity and debt capital) is \$10,000,000 (\$800,000/0.08).
- If the WACC is $5 \%$, the total value of the company (equity and debt capital) is $\$ 16,000,000$ ( $\$ 800,000 / 0.05$ ).
When the total value of a company is maximised, the wealth of its shareholders is also maximised, in terms of dollars per share.

The WACC can be changed by altering the financial gearing of the company, and changing the relative amounts of debt capital.

The differing views on how WACC is affected by financial gearing are described in a later chapter on the cost of capital.

### 3.4 Financial risk management and monitoring systems

Financial managers are responsible for advising operational managers and the directors of a company about capital investment risk, and should establish a system for the management and monitoring of risk.

The treasury department should also be specifically responsible for monitoring and management of treasury risks, such as foreign currency risks and interest rate risks.

## Framework for risk management

A risk management framework that is useful to remember is as follows. (It is based on the COSO risk management framework, developed some years ago by the Commission of Sponsoring Organisations in the US).

- There should be a well-established culture of risk management. Senior management should recognise the importance of risk and the need to balance the desire for high profits and returns with the need to avoid excessive risk exposures.
- There should be systems for identifying and measuring risks, and recognising which controls must be managed and controlled.
- Methods for managing risks should be applied to risks that are considered sufficiently serious, provided that the cost of controlling the risk does not exceed the potential benefit from reducing the risk of losses.
- The methods used to manage risks should be communicated to all managers concerned. Managers should be kept informed about risk as well as returns.
- There should be a regular monitoring of risk management systems, methods and measures, to ensure that they function effectively and achieve their intended purpose.

Risk assessment will be considered in later chapters, for example with respect to risk in capital investment projects. The risk in a proposed capital project should be assessed and the risk assessment should be included in the decision-making process leading to the decision of whether or not to undertake the investment.

After a capital investment has been undertaken, actual spending and actual returns should be monitored (in a 'post investment audit') to establish whether actual spending was kept within the planned limit and whether the expected returns were actually achieved.

### 3.5 Ethical issues in financial management

In pursuing profits and shareholder wealth maximisation, companies should act ethically, and in giving advice financial managers should be conscious of any ethical issues that might be involved in the matter. Business ethics covers aspects of business behaviour such as:

- Honesty and integrity in business dealings with others: honesty is more than just remaining within the law!
- Concern for other stakeholders, such as employees, suppliers and customers.
- Respect for human rights: this might involve avoiding business dealings with unethical suppliers or suppliers who use child labour or slave labour.
- Concern for the environment: the need to reduce or avoid pollution and the need to develop a sustainable business.
- Recognition by large companies of the social responsibilities to the communities in which they exist and operate.

Some companies have expressed their concern for ethical conduct in a formal corporate code of ethics.

## Corporate codes of ethics

A corporate code of ethics is a code of ethical behaviour, issued by the board of directors of a company. The decisions and actions of all employees in the company must be guided by the code. The effectiveness of a code of ethics depends on the leadership of the company - its directors and senior managers. These individuals must be seen to comply with the ethical code; otherwise other employees will see no purpose in complying with the code themselves.

It has been suggested that there are three reasons why companies might develop a code of ethics. These reasons are progressive, which means that companies might begin by having a code of ethics for the first reason, but then progress to the second and third reasons as they gain experience with implementing the code and appreciating its potential benefits.

- Reason 1: Managing for compliance. The company wants to ensure that all its employees comply with relevant laws and regulations, and conduct themselves in a way that the public expects. For example, companies providing a service to the general public need to ensure that their employees are polite and wellbehaved in their dealings with customers.
- Reason 2: Managing stakeholder relations. A code of ethics can help to improve and develop the relations between the company and its shareholders, by improving the trust that shareholders have in the company. The code might therefore include the ethical stance of the company on disclosing information to shareholders and the investing public (openness and transparency) and on respecting the rights of shareholders.
- Reason 3: Creating a value-based organisation. A company might recognise the long-term benefits of creating an ethical culture, and encouraging employees to act and think in a way that is consistent with the values in its code of ethics. (It could be argued that an ethical company, like a well-governed company, is a more likely to be successful in business in the long-term. However, there is no firm evidence to prove this point, and it is therefore a matter of opinion.)

Note on global organisations. Global companies might have difficulty in developing and implementing a code of ethics for the entire organisation worldwide, because of differences in ethical values in different cultures in different parts of the world. A criticism of codes of ethics of global companies is that they often focus on the company's relationships with stakeholders in their 'home country' and do not give enough thought to their operations in other countries.

## The content of a corporate code of ethics

There is no standard format or content for a code of ethics, but a typical code contains:

- general statements about ethical conduct by employees
- specific reference to the company's dealings with each stakeholder group, such as employees, customers, shareholders and local communities.

A code of conduct should specify that compliance with local laws is essential. In addition, employees should comply with the policies and procedures of the company. There might be a statement that any employee who fails to comply with the company's code of conduct will face disciplinary action.

The code might also include an overview of business conduct, and the need to protect the company's reputation and 'good name'. It might also contain statements about the values of the company, such as:

- acting at all times with integrity
- protecting the environment
- the 'pursuit of excellence'
- respect for the individual.

A code of conduct might address its main concerns about its dealings with stakeholder groups and its ethical treatment of each group.

- Employees. A code of ethics might include statements about:
- human rights, including the right of all employees to join legally-authorised organisations such as a trade union or political party
- equal opportunities for all employees, regardless of gender, race, ethnic origin, religion, age, disability or sexual orientation
- refusal to tolerate harassment of employees by colleagues or managers
- concern for the health and safety of employees
- respect for the privacy of confidential information about each employee
- company policy on giving or receiving entertainment or bribes.
- Customers. A code of ethics might include statements about:
- fair dealing with customers
- product safety and/or product quality
- the truthfulness of advertisements
- respect for the privacy of confidential information about each customer.
- Competitors. A code of ethics might include statements about:
- fair dealing with competitors
- the use of techniques for obtaining information about competitors (industrial spying)
- Shareholders. A code of ethics might not include much about shareholders, because the relationship between a company and its shareholders might be contained in a code of corporate governance that the company follows. The key issue with shareholders is to maintain and develop trust and confidence, which might be achieved through disclosure of information (openness and transparency).

Dividend and retentions policy

- Retentions
- Dividend policy
- Monitoring dividend policy


## 4 Dividend and retentions policy

### 4.1 Retentions

Companies retain earnings to reinvest in the business, and many companies rely extensively on retained profits as a source of new finance. There are several reasons why companies might prefer retained earnings to other sources of new capital:

- There are no issue costs with retained earnings, whereas there is a cost of issuing shares and some costs of arranging borrowing.
- No formalities are required to obtain the capital, and holding on to profits as retained earnings is very convenient.
- Shareholders usually expect some proportion of earnings to be retained in the business.


## Retentions and dividend growth: Gordon's growth model

Retained earnings should be invested for growth in future profits, earnings and dividends. The Gordon growth model will be dealt with in more detail in a later chapter. Briefly, however, the rate of annual growth in dividends can be expressed as:
$\mathrm{g}=\mathrm{br}$
where
$g$ is the annual rate of dividend growth
$b$ is the proportion of earnings that is retained for reinvestment rather than paid as dividend, and
$r$ is the rate of return on investments by the company.

For example, if a company retains $40 \%$ of its earnings and the return on company capital investment is $8 \%$, annual growth in earnings and dividends should be $3.2 \%$ ( $40 \% \times 8 \%$ ). If $60 \%$ of earnings are retained, only $40 \%$ of earnings are available for dividends, but the expected rate of annual earnings and dividend growth should be 4.8\%.

### 4.2 Dividend policy

Stock market companies do not usually announce a dividend policy, and inform investors of the dividends they will expect to pay in future years. However, companies often adopt a consistent dividend policy, and investors might base their expectations of a company's future dividends on what the company has done in the past.

According to academic theory (for example, the theory put forward by Modigliani and Miller) the value of a company's shares does not depend on the dividend policy of the company. Shareholders ought to be indifferent about whether a company pays out its after-tax profits as dividends, or re-invests them. If a company does not pay out its profits to shareholders as dividends, it will reinvest the profits for growth in profits, so that future dividends will be larger and the share price will increase.

If a shareholder needs cash, he should not have to rely on dividend payments. He can borrow the money instead, or even sell the shares.

In practice, however, the dividend policy of a company can affect its share price and does matter to many shareholders.

- Some investors want cash returns from their investments, and expect companies in which they hold shares to pay dividends. On the other hand, some investors buy shares to invest in growth, and are not concerned in the short-term about receiving dividends. Investors can buy shares in companies that have a dividend policy consistent with their preferences. (For example, for many years Microsoft did not pay any dividends at all and reinvested all its profits to grow the business. Investors in Microsoft shares knew that they were investing in a highgrowth stock, and dividend payments were not important to them.)
- The tax position of investors might give them a preference for dividends or capital growth, when the tax rate on dividend payments is different from the tax rate on capital gains
- Paying a dividend might send a signal from the company to the market. If the signal is that the board of directors are confident about the future of the company, this might give investors more confidence in the company and boost the share price
- When a company reduces its dividend below the amount of dividends paid in previous years, this could be a signal that the company is in financial difficulty, and the share price is likely to fall.


### 4.3 Monitoring dividend policy

A company's dividend policy can be monitored with a few simple ratios.

- Dividend per share as a percentage of earnings per share. EPS is the maximum amount that a company could pay as dividends out of the year's profits. A low dividend/EPS ratio indicates high earnings retention. A high dividend/EPS ratio indicates low earnings retention - and this could restrict the ability of the company to finance investments in new capital projects for future growth.
- Percentage growth or decline in annual dividends per share.
- Dividends as a proportion of free cash flow to equity. Free cash flow can be defined in several ways, but for the purpose of analysing dividend policy, a useful measure of free cash flow to equity is after deduction of capital spending on investments in replacement non-current assets.


## Example

The following information is available about Company X and Company Y for the past five years:

|  | Earnings after <br> tax <br> million | Company $\mathbf{X}$ <br> Number of <br> shares <br> million | Free cash flow <br> to equity (FCFE) <br> $\$$ million | Dividends per <br> share <br> cents |
| :---: | :---: | :---: | :---: | :---: |
| Year 1 | 34.5 | 100 | 28.0 | 20 |
| Year 2 | 32.3 | 100 | $(25.0)$ | 20 |
| Year 3 | 31.5 | 120 | 12.0 | 21 |
| Year 4 | 40.0 | 120 | 30.1 | 22 |
| Year 5 | 48.0 | 120 | 32.7 | 24 |


|  | Earnings after <br> tax | Company Y <br> Number of <br> shares | Free cash flow <br> to equity (FCFE) <br> \$ million | Dividends per <br> share <br> cents |
| :---: | :---: | :---: | :---: | :---: |
| Year 1 | 27.0 | 40 | 9.6 | 13.5 |
| Year 2 | 24.5 | 40 | $(5.0)$ | 12.3 |
| Year 3 | 26.8 | 40 | $(6.5)$ | 13.4 |
| Year 4 | 29.8 | 40 | 5.2 | 14.9 |
| Year 5 | 31.6 | 40 | 17.3 | 15.8 |

## Required

(a) Compare the dividend policy of Company X and Company Y .
(b) It has been reported that in the past two or three years, Company $X$ has chosen not to invest in some projects that had a good expected NPV. Suggest how dividend policy might explain this state of affairs.

Answer

|  | Dividends/Earnings | Company $\mathbf{X}$ <br> Dividend <br> growth <br> $\%$ | Dividends/FCFE |
| :---: | :---: | :---: | :---: |
| Year 1 | $\%$ | - | $\%$ |
| Year 2 | 58.0 | 0.0 | 71.4 |
| Year 3 | 61.9 | 5.0 | $(80.0)$ |
| Year 4 | 80.0 | 4.8 | 210.0 |
| Year 5 | 66.0 | 9.1 | 87.7 |
|  | 60.0 |  | 88.1 |


|  | Dividends/Earnings | Company Y <br> Dividend <br> growth <br> $\%$ | Dividends/FCFE |
| :---: | :---: | :---: | :---: |
| Year 1 | \% | - | $\%$ |
| Year 2 | 20.0 | $(8.9)$ | 56.3 |
| Year 3 | 20.0 | 8.9 | $(98.4)$ |
| Year 4 | 20.0 | 3.7 | $(82.5)$ |
| Year 5 | 20.0 | 6.0 | 114.6 |
|  | 20.0 |  | 36.5 |

The dividend policy of Company X appears to be to pay a consistent dividend per share from one year to the next, but with some dividend growth. Dividends have risen fairly consistently over the five year period from 20 cents in Year 1 to 24 cents in Year 5. Investors might expect dividends to remain about the same next year - 24 cents if earnings do not rise and a bit more if earnings increase.

In contrast, the dividend policy of company Y is to pay dividends that are a consistent ratio of earnings. Dividends were exactly $20 \%$ of earnings in each year. This means that dividends per share rise and fall at the same rate as fluctuations in earnings.

It is also noticeable that the dividend payout ratio (ratio of dividends to earnings) is much higher in Company X than in Company Y. In Company Y, the payout ratio is very low - the company is reinvesting $80 \%$ of its earnings, and it might therefore be expected that earnings growth and dividend growth should be higher than it has been.

The ratio of dividends to free cash flow fluctuates considerably for both companies. Clearly, dividend policy is not related to their free cash flows.
(b)

It has been reported that in the past two or three years, Company $X$ has chosen not to invest in some projects that had a good expected NPV. This might be explained by a combination of two factors:
(1) The fairly high dividend payout ratio in Company $X$, which indicates that the company is not building up equity reserves by retaining profits
(2) The high ratio of dividend payout to FCFE in Years 4 and 5, following years in which FCFE was negative (Years 2 and 3). Company $X$ has not been generating enough cash flows after dividend payments to finance new capital projects from internal cash flows.

# Impact of environmental issues on corporate objectives and governance 

## Contents

$\begin{array}{ll}1 & \begin{array}{l}\text { Social and environmental footprints: } \\ \text { environmental risk }\end{array} \\ 2 & \begin{array}{l}\text { Sustainability reporting and environmental } \\ \text { audits }\end{array} \\ 3 & \text { Carbon trading }\end{array}$

- Environmental footprint (ecological footprint)
- Carbon neutrality
- Social footprint
- Sustainability and environmental risk


## 1 Social and environmental footprints: environmental risk

The purpose of economic activity is to create economic wealth. It is now recognised, much more than in the past, that economic activity also has an environmental impact and a social effect. An organisation is said to create an 'environmental footprint' and a 'social footprint' - a visible mark on the environment and on society. The social footprint may be either beneficial or damaging. The environmental footprint is almost inevitably damaging.

### 1.1 Environmental footprint (ecological footprint)

An environmental footprint, also called an ecological footprint, is a term that refers to the impact that an entity has on the environment, in terms of:

- the amount of raw materials that it uses to make its products or services, where the raw materials are subject to depletion (see note)
- non-renewable resources that it uses to make its products or services
- the quantity of wastes and emissions that it creates in the process.

Note: Raw materials subject to depletion are raw materials that can be renewed, but where the current total rate of consumption exceeds the total current rate of renewal. Fish stocks and hard wood timber are examples.

In the past, it was accepted that in order to grow, companies (and economic activity as a whole) had to increase its environmental footprint. With the recognition today that the world cannot go on increasing its environmental footprint, many leading companies are looking for ways to reduce the size of their own particular footprint and 'tread more softly'.

Reducing an environmental footprint involves the development and implementation of policies for:

- better (more efficient) resource management, and using different resources
- 'green' procurement policies
- waste minimisation and waste management (for example, policies on reducing pollution and recycling waste).

Countries that consume most environmental resources and create most environmental damage relative to other countries are those with the highest environmental footprint per head of the population.

### 1.2 Carbon neutrality

The effect on the environment of economic activities by individual companies may be measured in terms of emissions of carbon-based pollutants, such as the release of carbon dioxide into the atmosphere. (Carbon is only one of many different pollutants of air, water and land, but carbon dioxide emissions into the atmosphere have attracted extensive interest and concern.)

Some environmentally-conscious companies already measure their impact on carbon pollution, and might have a stated environmental policy of being 'carbon neutral'.

Carbon neutrality exists when a company is able to counterbalance its use of carbon products, and particularly its carbon dioxide emissions, with activities that reduce the amount of carbon dioxide in the atmosphere such as growing trees or plants (which absorb carbon dioxide from the atmosphere). Some companies have also tried to reduce their impact on carbon dioxide pollution by switching to the use of fuel and energy that does not involve carbon consumption.

## Example

The Du Pont Corporation is an example of a large environmentally-conscious company, which has stated as its mission the aim of seeking sustainable growth.

Du Pont listed some of the initiatives it has taken to create a sustainable business:

- Setting a target of zero waste generation and zero waste emissions
- Conserving energy and resources such as oil, coal, natural gas, water and minerals
- Recycling materials to reduce the need for disposals
- Reducing packaging waste
- Making, using, handling and transporting materials safely and in an environmentally-friendly way and in compliance with local regulations
- Managing land efficiently to increase habitats for wildlife
- Developing new products and processes that reduce the environmental risks.


### 1.3 Social footprint

A social footprint is the effect of economic activity on society and people. In general, economic activity is seen as providing benefits for society, although some companies are much more 'people-friendly' than others. Some companies, for example, use child labour and/or pay subsistence-level wages to their workers.

Companies might seek to measure the contribution of their activities towards society in terms of:

- Total numbers employed or increase in the total number of employees
- The proportion of the total work force employed in different parts of the world
- The proportion of the total work force that is female or from different ethnic groups
- Health and safety at work (for example, numbers of employees injured each year per 1,000 of the work force).

Social ecologists argue that the environmental crisis has been caused by companies seeking growth, profits and economic self-interest. Nothing fundamental has changed. Companies are still trying to get bigger and more profitable, even though they use environmental ideology to express their plans and ambitions. They argue that the environmental crisis cannot be averted without a radical change in human society.

### 1.4 Sustainability and environmental risk

For companies there are two aspects to environmental risk:

- There is reputation risk for companies that are considered serious polluters of the environment and companies that are depleting the world's natural resources. Reputation risk is the risk that a company will acquire a bad reputation in the eyes of the general public customers, and so be at a competitive disadvantage to rival producers.
- There is also the risk that by continuing to use up natural resources and pollute the environment, companies might not have a business that can be sustained into the long-term future. Examples of concerns about sustainable business development have included the consumption of the world's fishing stocks by fishing fleets and the destruction of hardwood forests by forestry companies.

One way in which companies might recognise their responsibility towards the environment is to provide information about their environmental and social footprints.

Many companies now produce a voluntary social and environmental report each year (as a separate publication), or include comments on environmental issues in the chairman's statement of the chief executive officer's report in the published annual report and accounts. However, social and environmental reports often do not include quantified information, and reports often concentrate on the 'good' environmental measures taken by the company and ignore the 'bad' polluting and non-sustainable activities.

## Sustainability reporting and environmental audits

- Accounting and the economic model
- The need for objective environmental reporting
- Sustainability reports: triple bottom line reporting
- Environmental audits


## 2 Sustainability reporting and environmental audits

### 2.1 Accounting and the economic model

The capitalist system creates incentives for maximising wealth, and it is based on the assumption that wealth can only be increased through continual economic growth. This is the 'economic model' of society. As a result, growth-seeking economic activities continue, in spite of growing concerns for the environment, and recognition that continual growth in its current form cannot be sustained.

Accounting has developed in support of the economic model. Financial reporting measures the consequences of a company's activities in terms of the use of the assets that it owns and the liabilities for which it has the direct responsibility for payment. Current accounting practice does not allow companies to report the consequences of their actions on external assets that it does not own, and the creation of liabilities for which it does not have to pay directly.

Investment decisions by large companies are made using accounting techniques such as discounted cash flow analysis, which focuses exclusively on economic consequences of investment, and does not measure or evaluate the environmental and social impact.

Financial reporting, management accounting and financial management have all therefore supported the economic model of business.

### 2.2 The need for objective environmental reporting

In the past, companies ignored their consumption of natural resources such as air and water because they assumed that supplies of these items were both limitless and free. This is no longer the case. It can now be argued that whereas companies are increasing economic wealth through growth and the search for profit maximisation, society may well be getting poorer because of the damage that economic activity is having on the environment and society

As companies have become increasingly aware of environmental issues, and begin to accept that economic growth might not be sustainable, they have become more interested in measuring sustainability and environmental impact.

Traditional accounting methods do not provide for this type of measurement, and to the extent that companies (and society) want environmental and social impacts to be measured, traditional accounting is inadequate.

In order to help companies to set targets for achievement and assess their actual performance, there should be measurement. Environmental and social effects should be quantified, because managers find it easier to plan and control using numbers than using more general qualitative assessments.

Some accountancy bodies, including the ACCA, are contributing towards efforts to establish measurement and reporting systems for social and environmental issues, to complement traditional financial reporting.

Alternative measurement and reporting systems that recognise the need for economic activity to be sustainable have therefore been considered, although there is as yet no widely-accepted standard measurement and reporting system.

### 2.3 Sustainability reports: triple bottom line reporting

Sustainability reporting in the form of 'triple bottom line reports' have been developed as a method of presenting quantified and more objective information about the social and environmental effects of a company's activities.

The term 'triple bottom line' was 'invented' in 1994 by J Elkington. Its aim is to encourage companies to recognise social and environmental issues in their business models and reporting systems. This method of reporting is encouraged by the Global Reporting Initiative (GRI), an internationally-recognised body that promotes sustainability reporting.

The 'triple bottom line' gets its name because companies report their performance not simple in terms of profit: they provide key measurements for three aspects of performance:

- economic indicators
- environmental indicators, and
- social indicators.

Triple bottom line reporting is therefore providing a quantitative summary of a company's economic environmental and social performance over the previous year. Economic indicators will include measurements relating to:

- sales revenue
- profits, earnings and earnings per share
- dividends per share
- global market share (as a percentage of the total market)

■ in some industries, such as car production, units of sale worldwide.

Environmental indicators might include measurements relating to:

- reducing the 'intensity' of materials in products and services
- reducing energy intensity
- minimising the release of toxic materials/pollutants
- improving the ability to recycle material
- maximising the use of renewable resources
- extending the life of a product


## Example

One major global company using triple bottom line reporting reported its environmental performance in terms of:

- global energy use, measured in thousand of GWh
- global carbon dioxide emissions, measured in metric tonnes
- production of non-recycled waste, measured in metric tonnes

■ the number of manufacturing sites that had been awarded an ISO 14000 certificate (which is explained later).

The same company reported, as social indicators:
■ its donations to communities and sponsorships, measured in US dollars
■ diversity: the percentage of its employees who were female and the percentage who came from minority groups

- the number of discrimination charges brought against the company during the year
- employee satisfaction, based on a census of employee opinion
- the recordable injury rate per 1,000 employees.


## Weaknesses in triple bottom line reporting

There are several weaknesses with triple bottom line reporting.

- There are no widely-established standards for triple bottom line reporting, and no standard methods for measuring social and environmental impacts. It is therefore difficult to compare the sustainability of one company with the sustainability of another. (The work of the Global Research Initiative or GRI is to standardise measurements for the triple bottom line. It has been publishing Sustainability Reporting Guidelines since 1999. These were updated and amended in 2002.)
- If the social and environmental measures are not subject to independent audit, there might be doubts about the reliability of the data presented in a company's report.


### 2.4 Environmental audits

A social and environmental audit, or simply an environmental audit, can have several meanings.

- It can mean a formal audit of an environmental management system, to check that the system operates effectively.
- It could be an internal check of a particular aspect of the company's environment management system, such as its system for measuring the environmental costs of waste, or the methods used to measure the cost of site contamination at a particular manufacturing site. This audit might be carried out by members of the company's own internal audit team (who might be an environmental expert rather than an accountant).
■ There may be a check on the company's compliance with environmental and social legislation and regulations.
- It could involve a verification of social and environmental information that will be included in a published report, such as an environmental performance report.
- Similarly it might be a check on the accuracy of figures supplied by the company to the government authorities responsible for environment regulation.
■ It could also refer to the checks that the company's external auditors need to carry out on the company's financial statements, as far as they relate to environmental issues. For example, the introduction of new environmental laws might have an impact on the impairment of non-current assets, and a failure by the company to carry out environmental improvements required by law might create a requirement to make an accrual for remedial costs or a provision for the payment of a fine.

An international auditing practice statement has explained environmental audits as follows:
Environmental audits 'can be performed by external or internal experts...at the discretion of the company's management. In practice, persons from various disciplines can qualify to perform "environmental audits". Often the work is performed by a multi-disciplinary team. Normally "environmental audits" are performed at the request of management and are for internal use. They may address various subject matters, including site contamination, or compliance with environmental laws and regulations' (IAPS 1010).

## How can environmental audits contribute to environmental accounting?

At the moment there is no legal requirement in any country for environmental audits. This type of audit is voluntary.

Similarly there is no legal requirement for environmental accounting, although professional accounting bodies such as the ACCA are encouraging more research by academics and practice by companies.

It seems quite possible, however, that environmental audits and environmental accounting will both become more common, as companies become increasingly aware of the problems of sustainability and sustainable growth, and include
environmental objectives within their corporate strategy and investment decisionmaking.
The development of environmental accounting and environmental auditing will depend to a large extent on the development of environmental management systems, and how soon more companies establish environmental management systems. When environmental management systems are established:

- management needs reliable environmental information
- in general, managers prefer information in a quantified/measured form rather than in qualitative and descriptive terms
- as environmental management systems develop, with measurement systems for setting targets and monitoring performance, it seems likely that the need for audits of the information system will be necessary, to reassure management that the information systems are sound.

However, there still seems a long way to go before social and environmental reporting rivals financial reporting (economic reporting) as the main method of reporting by companies.

## Carbon trading

- Origins of carbon trading
- Features of the EU Emissions Trading Scheme
- The significance of carbon trading for companies and financial management


## 3 Carbon trading

### 3.1 Origins of carbon trading

The Kyoto Protocol to cut greenhouse gas emissions was agreed by the governments of about 140 countries and came into force in 2005. (It runs until 2012.) It has not been ratified by the United States or Australia.

In the European Union, the implementation of the Kyoto agreement has been in the form of the European Emissions Trading Directive, which set out a framework for a European Emissions Trading Scheme (EU ETS).

This is more commonly known as a carbon trading scheme, because the scheme applies only to emissions of carbon dioxide $\left(\mathrm{CO}_{2}\right)$ until 2007, although from 2008 the trading scheme might be extended to the other five greenhouse gases.

### 3.2 Features of the EU Emissions Trading Scheme

The emissions trading scheme has two main elements:

- The governments of the EU countries are required to set a cap on total amount of carbon dioxide emissions, by allocating 'rights to pollute' to individual industrial sites/companies.
- The market is then allowed to trade in these 'rights to pollute', so that high polluting companies can buy carbon trading 'credits' on the market, giving them the right to go beyond their allocated pollution limit.

The EU ETS scheme is in two phases. In the first phase lasting until the end of 2007, carbon dioxide emissions in the EU countries are limited or 'capped' for companies in the power-generating and energy-intensive industrial sectors. Each country sets its own total emissions limit for these industries and the government then allocates 'free emissions allowances' or 'rights to pollute' to companies. In the UK, the first phase of the ETS applied to about 1,500 sites.

Companies are allowed to use their 'free emissions allowances', but in practice some companies do not need all the allowances they have been given, and other companies need to make more emissions than they are allowed.

- Companies with surplus allowances or 'credits' can sell them in a carbon trading market, and companies needing more credits can buy them.
- The price for carbon credits depends on demand and supply.
- The market is fairly new, but there are specialist carbon credit traders who act as intermediaries between buyers and sellers of credits.

There is another aspect to the carbon trading market. Companies that invest in projects that are environmentally friendly are given additional carbon credits by the government. The companies receiving these additional credits can sell them in the market.

## Example

A company decides to build a wind farm. It receives carbon credits from the government because the technology to generate power from the wind has a zero carbon output.

This company does not need its carbon credits, because it does not produce any carbon dioxide emissions. The company can therefore sell its carbon credits to another company that does produce $\mathrm{CO}_{2}$ emissions and needs to obtain more credits.

The company building the wind farm is able to use the money it receives from trading in carbon credits to help pay for the cost of its wind farm.

### 3.3 The significance of carbon trading for companies and financial management

Government actions to reduce greenhouse gases have an impact on companies affected by the regulations in their particular country. For many polluting companies, the consequences are higher costs. Higher costs take the form of both measures to reduce emissions and also the cost of buying carbon credits when permitted limits will otherwise be exceeded.

For companies that will receive carbon credits for environmentally-friendly investments, the system of carbon credits means that they can reduce their investment costs by selling the credits they are given.

## Criticisms of carbon trading

Critics of carbon trading argue that the only way to reduce greenhouse gas emissions is through direct government action and regulation, and that the system of offsetting and trading carbon credits has no effect on the total amount of environmental pollution.

The scheme is also criticised on the grounds that some countries have been given fairly 'generous' limits for carbon dioxide emissions (notably China and India) and that in other countries the government has been too generous in awarding the free emissions allowances.

The biggest difficulty for the Kyoto Protocol and measures to cut greenhouse gas emissions has been the refusal of the United States to join the scheme. The US government began discussions on greenhouse gas emissions in 2007, but has so far not taken any direct action.

## Capital investment appraisal

## Contents

1 Discounted cash flow: net present value method
2 The time to recover the capital invested: payback and duration
3 DCF: internal rate of return (IRR) and modified internal rate of return (MIRR)
4 DCF and taxation
5 DCF and inflation
6 Free cash flow

## Discounted cash flow: net present value method

- Introduction
- Net present value (NPV) method of investment appraisal
- Assumptions about the timing of cash flows
- Changes in working capital and cash flows
- Calculating the NPV of a project
- Discount tables
- Annuities
- Present value of a perpetuity


## 1 Discounted cash flow: net present value method

### 1.1 Introduction

An important element of financial management is deciding how to invest funds in capital projects. The basic principle is that a company should not invest capital unless the expected return from the investment is expected to exceed the cost of the funds invested.

Proposed capital expenditure projects can be evaluated using discounted cash flow.
A crucial aspect of DCF is that capital expenditure is assessed by looking at the cash flows from the investment, not at accounting profits.

You should be familiar with the basic rules of DCF from your earlier studies. This chapter contains revision material for what you should already know. However, in your examination, you will be required to take DCF analysis to a more advanced stage. This chapter explains some advanced aspects of capital investment appraisal. Other aspects will be dealt with in later chapters.

### 1.2 Net present value (NPV) method of investment appraisal

With the NPV method of investment appraisal, all the future cash flows from an investment are converted into a present value, by discounting the cash flow at the investment cost of capital. This cost of capital is the return required from the investment.

The discount rate to apply to a cash flow at the end of year n is:
$1 /(1+r)^{n}$
where $r$ is the cost of capital (so that $9 \%=0.09,12.5 \%=0.125$ and so on).
Cash flows at the beginning of the investment, in Year 0, are already stated at their present value. The present value of $\$ 1$ in Year 0 is $\$ 1$. (The discount factor for Year 0 is $1 /(1+r)^{0}$. This has a value of 1.000 for any and every value of $r$.

### 1.3 Assumptions about the timing of cash flows

The following assumptions are used in DCF about the timing of cash flows:

- The cash flows for each year (or time period) are assumed to occur at the end of the year (or time period). For example, if it is estimated that cash flows in Year 4 will be $\$ 100,000$, it is assumed that the $\$ 100,000$ is a cash inflow on the last day of Year 4.
- If a cash flow will occur early during a particular year, it is assumed that it will occur at the end of the previous year. Therefore a cash expenditure early in Year 1 , for example, is assumed to occur in Year 0.

If greater accuracy is required for the timing of cash flows, each year can be divided into shorter time periods, such as three-month periods. The same assumptions about the timing of cash flows will apply, however, regardless of the length of these time periods.

### 1.4 Changes in working capital and cash flows

When there is an increase in working capital, cash inflows are lower than the cash profit in the same period, by the amount of the increase.

Similarly, when there is a reduction in working capital, cash inflows are higher than cash profits in the period, by the amount of the reduction.

This rule should be familiar to you from cash flow statements.
In DCF analysis, instead of adjusting the cash profits in the years when working capital is increased or reduced, the increases or reductions in working capital are treated as separate cash flow items.

## Example

A five-year project will require a working capital investment of $\$ 20,000$ early in Year 1, and an additional working capital investment of $\$ 15,000$ early in Year 2. The adjustments for working capital to the cash flows in each year of the project will be:

| Year | $\$$ |
| :--- | :---: |
| 0 | $(20,000)$ |
| 1 | $(15,000)$ |
| 5 | 35,000 |

There is a cash inflow at the end of the project because working capital is reduced to zero, and cash flows exceed cash profits in that year by the amount of the working capital reduction.

### 1.5 Calculating the NPV of a project

The net present value (NPV) of a project is the net difference between the present value of all the costs incurred and the present value of all the cash flow benefits (savings or revenues).

- If the present value of benefits exceeds the present value of costs, the NPV is positive.
- If the present value of benefits is less than the present value of costs, the NPV is negative.
- The NPV is 0 when the PV of benefits and the PV of costs are equal.

The decision rule is that, ignoring other factors such as risk and uncertainty, and non-financial considerations, a project is worthwhile financially if the NPV is positive or zero. It is not worthwhile if the NPV is negative.

The net present value of an investment project is also a measure of the value of the investment. For example, if a company invests in a project that has a NPV of $\$ 2$ million, the value of the company should increase by $\$ 2$ million.

### 1.6 Discount tables

Discount tables will be provided in your examination. They take away the need to calculate the value of discount factors $\frac{1}{(1+r)^{n}}$. The discount factors in the tables are rounded to three decimal places.

The following example tests your basic understanding of the NPV method and the use of discount tables. It should also demonstrate that when the cost of capital is lower, the NPV is higher. (And when the cost of capital is higher, the NPV is lower.)

## Example

A company is considering an investment in equipment costing $\$ 70,000$. Working capital of $\$ 5,000$ will also be required early in Year 1. The equipment will have a resale value of $\$ 7,000$ at the end of Year 5. The operating profits from the investment, in cash flows, will be:

| Year | Cash flow |
| :--- | ---: |
|  | $\$$ |
| 1 | 25,000 |
| 2 | 20,000 |
| 3 | 30,000 |
| 4 | 20,000 |
| 5 | 3,000 |

## Required

Using discount tables for the discount factors, calculate the NPV of the project if the cost of capital is:
(a) $12 \%$, and
(b) $8 \%$.

## Answer

In this example, the investment in working capital is treated as a cash outflow of $\$ 5,000$ in Year 0 (early in Year 1). When the working capital is recovered at the end of the project (at the end of Year 5, or early in Year 6), this is treated as a Year 5 cash inflow.

The resale value of the equipment is also a cash inflow at the end of Year 5.
The cash flows for each year are therefore calculated as follows:

| Year | Equipment | Working <br> capital | Cash <br> profits | Annual cash <br> flow |
| :--- | ---: | ---: | ---: | ---: |
| 0 | $\$$ | $\$$ | $\$$ | $\$$ |
| 1 | $(70,000)$ | $(5,000)$ |  | $(75,000)$ |
| 2 |  |  | 25,000 | 25,000 |
| 3 |  |  | 20,000 | 20,000 |
| 4 |  |  | 30,000 | 30,000 |
| 5 | 7,000 | 5,000 | 20,000 | 20,000 |
|  |  |  | 3,000 | 15,000 |

These cash flows should be used to calculate the NPV at (a) $12 \%$ and (b) $8 \%$.

| Year | Cash flow | Cost of capital 12\% |  | Cost of capital 8\% |  |
| :--- | ---: | ---: | ---: | ---: | ---: |
|  |  | Discount factor | Present value | Discount factor | Present value |
|  | $\$$ |  | $\$$ |  | $\$$ |
| 0 | $(75,000)$ | 1.000 | $(75,000)$ | 1.000 | $(75,000)$ |
| 1 | 25,000 | 0.893 | 22,325 | 0.926 | 23,150 |
| 2 | 20,000 | 0.797 | 15,940 | 0.857 | 17,140 |
| 3 | 30,000 | 0.712 | 21,360 | 0.794 | 23,820 |
| 4 | 20,000 | 0.636 | 12,720 | 0.735 | 14,700 |
| 5 | 15,000 | 0.567 | $-8,505$ | 0.681 | 10,215 |
|  |  |  | $+5,850$ |  | $+14,025$ |

## Conclusion

The NPV is higher when the cost of capital is lower.

### 1.7 Annuities

An annuity is a constant cash flow for a given number of time periods.
Examples of annuities are:

- \$80,000 each year for years $1-7$
- \$50,000 each year for years 6 - 12
- $\$ 1,000$ each month for months $1-36$.

Annuity discount factors can be used in DCF investment analysis, mainly to make the calculations easier and quicker. Instead of discounting a constant annual cash flow each year by the discount factor for that year, and then adding up the present values of the annual cash flows, we can multiply the constant annual cash flow by an annuity factor.

The annuity factor is simply the sum of the discount factors for all the years in which the constant annual cash flow occurs.

## Annuity discount tables

The cash flows of project might include an annuity. The discount factors for annuities ('annuity factors') are provided in discount tables at the beginning of this text, and in the examination.

An annuity factor for time periods 1 to n is the sum of the discount factors from time period 1 to time period $n$.

## Example

You should refer to the annuity factor table at the beginning of this text to check the discount factors in the following examples.

- The annuity factor for years $1-2$ at a cost of capital of $8 \%=1.783$. This is found from the annuity table, $\mathrm{n}=2$, discount factor $8 \%$. This annuity factor is the sum of the discount factors at $8 \%$ for years 1 , and $2(0.926+0.857)$.
- The annuity factor for years $1-6$ at a cost of capital of $10 \%=4.355$. This is found from the annuity table, $\mathrm{n}=6$, discount factor $=10 \%$. This annuity factor is the sum of the discount factors at $10 \%$ for years $1,2,3,4,5$ and $6(0.909+0.826+$ $0.751+0.683+0.621+0.564)$.
- The annuity factor for years $4-8$ at a cost of capital of $12 \%$ is calculated from annuity tables as follows:

$$
\begin{array}{ll}
\text { Annuity factor at } 12 \% \text {, Years } 1-8 & = \\
\text { Annuity factor at } 12 \% \text {, Years } 1-3 & = \\
\text { Annuity factor at } 12 \% \text {, Years } 4-8 & = \\
\hline
\end{array}
$$

This is the sum of the discount factors at $12 \%$ for each of the years 4 to 8. ( $0.636+$ $0.567+0.507+0.452+0.402=2.566$.)

## Formula for calculating an annuity factor

It is useful for the purpose of financial management to be aware of the formula for an annuity. This is shown at the top of the annuity table:
Annuity factor $=\frac{1-(1+r)^{-n}}{r}$
This formula can be re-arranged, as follows:
Annuity factor $=\frac{1}{r}\left[1-\frac{1}{(1+r)^{n}}\right]$
For example, the annuity factor for years $1-6$ at $10 \%$ is:
$\frac{1}{0.10}\left[1-\frac{1}{(1.10)^{6}}\right]$
$=\frac{0.4355}{0.10}=4.355$

A knowledge of this formula might be useful for calculating an annuity factor when the cost of capital is not a whole number; for example when the cost of capital is $8.5 \%$ or $11.25 \%$.

## Example

The expected cash flows for a project to purchase the patent rights to a product are as follows:

| Year | Annual cash <br> flow | Discount factor at <br> $\mathbf{1 2 \%}$ | Present value |
| :--- | ---: | ---: | ---: |
|  | $\$$ |  | $\$$ |
| 0 | $(100,000)$ |  |  |
| 1 | 10,000 |  |  |
| 2 | 15,000 |  |  |
| $3-15$ | 20,000 |  |  |
| NPV |  |  |  |

If the cost of capital is $12 \%$, the NPV of the project is calculated as follows:

| Years | Annuity factor at <br> $\mathbf{1 2 \%}$ |
| :--- | ---: |
| $1-15$ | 6.811 |
| $1-2$ | $\underline{1.690}$ |
| $3-15$ | $\underline{5.121}$ |


|  | Annual cash <br> flow | Discount factor at <br> $\mathbf{1 2 \%}$ | Present value |
| :--- | ---: | ---: | ---: |
| Year | $\$$ |  | $\$$ |
| 0 | $(100,000)$ | 1.000 | $(100,000)$ |
| 1 | 10,000 | 0.893 | 8,930 |
| 2 | 15,000 | 0.797 | 11,955 |
| $3-15$ | 20,000 | 5.121 | $\underline{102,420}$ |
| NPV |  |  | $+23,305$ |

Annuity factors can be used to calculate a 'breakeven' or minimum annual cash flow for a project.

## Example

A company is considering an investment of $\$ 70,000$ in a project. The project life would be five years.

What must be the minimum annual cash returns from the project to earn a return of at least $9 \%$ per annum, if the returns are a constant amount each year?
Answer

| Investment | $\$ 70,000$ |
| :--- | ---: |
| Annuity factor at 9\%, years 1-5 | 3.890 |

The minimum annual cash flow must have a present value of $\$ 70,000$. The PV of the returns for the five years is the annual return multiplied by the annuity factor.

Minimum annual return required $=\$ 70,000 / 3.890=\$ 17,995$.

### 1.8 Present value of a perpetuity

A perpetuity is a constant annual cash flow 'for ever', or into the long-term future.
In investment appraisal, it might be assumed that there is an annual cash flow in perpetuity whenever a constant annual cash flow is expected for a long time into the future.

The present value of a perpetuity is: $\mathrm{C} / \mathrm{r}$
where:

- C is the constant annual cash flow in perpetuity
- r is the cost of capital: for example $0.08,0.10$ etcetera.


## Example

(1) What is the present value of $£ 2,000$ in perpetuity, starting in Year 1, if the cost of capital is $8 \%$ ?
(2) What is the present value of $£ 5,000$ in perpetuity, starting in Year 4, if the cost of capital is $11 \%$ ?

Answer
(1) $\frac{£ 2,000}{0.08}=£ 25,000$
(2) Year 3 value of perpetuity
$\frac{£ 5,000}{0.11}=£ 45,456$
Year 3 discount factor at $11 \% \quad 0.731$
Year 0 (present value) of perpetuity $£ 33,228$

## Use of perpetuities in financial management

The PV of a perpetuity is used in the dividend valuation model for share valuations, when there are constant annual dividends. Share valuation methods are explained in a later chapter.

## The time to recover the capital invested: payback and duration

- Payback period
- Discounted payback period
- The concept of duration
- Capital investment projects and duration


## 2 The time to recover the capital invested: payback and duration

### 2.1 Payback period

In addition to using NPV as a basis for making investment decisions, a company might also make their decision partly on the basis of how long it will take to recover the capital invested from the cash flows of the project.

The simplest method of measuring the time to recover the capital investment is the payback period. This is the number of years before the cumulative net cash flows from the project are $\$ 0$, and the capital invested has been recovered.

A company might establish an investment decision rule, for example, that projects will not be undertaken unless they have a positive NPV and the payback period is less than, say, five years.

## Example

An investment in a new business venture is expected to have the following cash flows:

| Year | Annual cash <br> flow |
| :--- | ---: |
|  | $\$$ |
| 0 | $(200,000)$ |
| 1 | $(40,000)$ |
| 2 | 30,000 |
| 3 | 120,000 |
| 4 | 150,000 |
| 5 | 100,000 |
| 6 | 50,000 |

The payback period is calculated as follows:

| Year | Annual cash <br> flow | Cumulative <br> cash flows |
| :--- | ---: | ---: |
|  | $\$$ | $\$$ |
| 0 | $(200,000)$ | $(200,000)$ |
| 1 | $(40,000)$ | $(240,000)$ |
| 2 | 30,000 | $(210,000)$ |
| 3 | 120,000 | $(90,000)$ |
| 4 | 150,000 | 60,000 |
| 5 | 100,000 | 160,000 |
| 6 | 50,000 | 210,000 |

Payback occurs during Year 4. Since cash flows are assumed to occur at the payback period, it could be stated as Year 4. However, it is usual to estimate the payback period in years and months.

In this example, the cumulative cash flows at the end of Year 3 are $\$(90,000)$ and during Year 4, the cash flows are $\$ 150,000$. We can therefore estimate that if cash flows during Year 4 occur at a constant rate, the payback period is:

3 years $+[(60,000 / 150,000) \times 12$ months $)=3$ years and 4.8 months, say 3 years 5 months.

The decision whether or not to invest might depend on whether the NPV of the project is positive and whether the payback period of 3 years and 5 months is acceptable.

## Criticisms of the payback method of project evaluation

There are two major criticisms of using the payback method to decide whether or not to undertake a capital investment project.

- The payback period ignores all the cash flows that are expected to occur after the payback period has been reached. The payback period should never be used on its own. If it is used at all, it should be used with a method of project evaluation that assesses the cash flows over the entire life of the project (such as the NPV method).
- The payback method ignores the time value of money. $\$ 1$ in a future year has a lower present value than $\$ 1$ now. A capital investment is not properly paid back when the cumulative cash inflows are $\$ 0$, because the investment needs to earn a return. A calculation of payback should therefore recognise the opportunity cost of investing capital. A discounted payback period is a more appropriate measure of payback than a payback period that ignores the time value of money.


### 2.2 Discounted payback period

A discounted payback period is calculated in the same way as the 'ordinary' payback period, with the exception that the cash flows of the project are converted to their present value. The discounted payback period is the number of years before the cumulative NPV of the project reaches $\$ 0$.

## Example

Using the same example as for the payback period calculation, and assuming a cost of capital of $10 \%$, the discounted payback period is calculated as follows:

| Year | Annual cash <br> flow | Discount <br> factor at $\mathbf{1 0 \%}$ | PV of cash <br> flow | Cumulative <br> NPV |
| :--- | ---: | ---: | ---: | ---: |
|  | $\$$ |  | $\$$ | $\$$ |
| 0 | $(200,000)$ | 1.000 | $(200,000)$ | $(200,000)$ |
| 1 | $(40,000)$ | 0.909 | $(36,360)$ | $(236,360)$ |
| 2 | 30,000 | 0.826 | 24,780 | $(211,580)$ |
| 3 | 120,000 | 0.751 | 90,120 | $(121,460)$ |
| 4 | 150,000 | 0.683 | 102,450 | $(19,010)$ |
| 5 | 100,000 | 0.621 | 62,100 | 43,090 |
| 6 | 50,000 | 0.564 | 28,200 | 71,290 |
| NPV |  |  | $+71,290$ |  |

The discounted payback period is Year 5, and we can estimate it in years and months as:
4 years $+(19,010 / 62,100) \times 12$ months
$=4$ years 4 months.
The discounted period for a capital investment is always longer than the 'ordinary' non-discounted payback period.

One criticism of the discounted payback method of project evaluation is the same as for the non-discounted payback method. It ignores the expected cash flows from the project after the payback period has been reached.

One measure of time to recover an investment that recognises the values of the cash flows over the entire life of the project is duration.

### 2.3 The concept of duration

Duration can be defined as the time to recover one half of the project value.
The concept of duration is widely used by analysts in the bond markets. It is a measure of how long an investor in bonds must wait before his investment in the bond is recovered.

## Duration of bonds

For example, suppose that there are two bonds X and Y , each with a maturity of four years. The returns for the investor on the bonds will be as follows:

|  | Year | Year | Year | Year | Total return |
| :--- | :---: | :---: | :---: | :---: | :---: |
|  | $\$$ | $\$$ | $\$$ | $\$$ | $\$$ |
| Bond X | 10 | 10 | 10 | 110 | 140 |
| Bond Y | 20 | 20 | 20 | 120 | 180 |

For the investor, investing in a bond would be a four-year capital investment.

If the investor is interested in how long he must wait to recover his investment, we could calculate the average time that this will take. Average time can be calculated as the weighted average number of years that this will take.

To calculate a weighted average period for obtaining the investment returns, the cash flows in each year should be given a weighting. For an investment with $n$ years, the weighting for Year 1 should be 1 , for Year 2 it should be 2 and for year 3 it should be 3, and so on. The weighting for the cash flow in Year n is n .

With four-year bonds, the weighting for the return in Year 1 is therefore 4, and so on.

| Year | Return R | Bond X <br> Weighting W | $\mathbf{R} \times \mathbf{W}$ | Return R | Bond $\mathbf{Y}$ <br> Weighting | $\mathbf{R} \times \mathbf{W}$ |
| :--- | ---: | :--- | ---: | ---: | ---: | ---: |
| 1 | 10 | 1 | 10 | 20 | 1 |  |
| 2 | 10 | 2 | 20 | 20 | 2 | 20 |
| 3 | 10 | 3 | 30 | 20 | 3 | 40 |
| 4 | 110 | 4 | 440 | 120 | 4 | 60 |
|  | 140 |  | 500 |  | 180 |  |

The non-discounted average time to recover the investment is:

- For Bond $X, 500 / 140=3.57$ years
- For bond $Y, 600 / 180=3.33$ years.


## Calculating the duration of a bond

The average times calculated above are similar to the calculation of a nondiscounted payback period. The calculation fails to recognise the time value of money by discounting the returns in each year before weighting them.

A value for the duration of a bond, known as Macaulay's duration, is calculated using the present value of the returns in each year.

In the previous example of the two bonds, suppose that the investor's cost of capital is $10 \%$. The value for Macaulay's duration of each bond is calculated as follows:

| Bond X <br> Year | Return R | Discount factor <br> at 10\% | PV of return | Weighting | PV $\times$ W |
| :--- | ---: | :---: | ---: | ---: | ---: |
|  | 10 | 0.909 | 9.09 | 1 | 9.09 |
| 1 | 10 | 0.826 | 8.26 | 2 | 16.52 |
| 2 | 10 | 0.751 | 7.51 | 3 | 22.53 |
| 3 | 110 | 0.683 |  | 75.13 | 4 |

Duration of Bond $X=348.66 / 99.99=3.49$ years.

| Bond Y <br> Year | Return R | Discount factor <br> at 10\% | PV of return | Weighting | PV $\times \mathbf{W}$ |
| :--- | ---: | ---: | ---: | ---: | ---: |
| 1 | 20 | 0.909 |  |  |  |
| 2 | 20 | 0.826 | 18.18 | 1 | 18.18 |
| 3 | 20 | 0.751 | 16.52 | 2 | 33.04 |
| 4 | 120 | 0.683 | 15.02 | 3 | 45.06 |
|  |  |  | -131.96 | 4 | 327.84 |

Duration of Bond $\mathrm{Y}=424.12 / 131.68=3.22$ years.

## The significance of bond duration

The duration of a bond is an indication of the price sensitivity of the bond to a change in market yields on bonds. When there is a rise in interest rate, the fall in price of a bond (as a percentage) is greater for bonds with a higher/longer duration. The actual amount by which bonds will change in price following a change in market interest rates can also be calculated.

Investors in bonds can therefore manage the risk in their bond portfolio by selecting bonds with a suitable duration.

### 2.4 Capital investment projects and duration

Duration can be calculated for a capital investment project in exactly the same way as for a bond. It is a measure of the average time to obtain the returns from the investment. Another way of saying this is that the duration of a project is the time required to cover one half of the value of the investment returns.

Duration can be used in capital investment appraisal to assess the payback on a project. Unlike payback and discounted payback, however, it takes into consideration the total expected returns from the entire project (at their discounted value), not just the returns up to the payback time.

- If the duration of a project is short relative to the life of the project - for example, if the duration is less than half the expected total life of the project - this means that most of the returns from the project will be recovered in the early years.
- If the duration of a project is a large proportion of the total life of the project - for example if the duration is $75 \%$ or more of the total life of the project - this means that most of the returns from the project will be recovered in the later years.
It could therefore be argued that duration is the best available method of assessing the time for an investment to provide its return on the capital invested.


## Example

The same example that was used earlier for the payback period calculation and discounted payback period can be used to calculate the duration of a project. The cash flows and present values are as follows:

| Year | Annual cash <br> flow | Discount <br> facotr at 10\% | PV of cash <br> flow |
| :--- | ---: | ---: | ---: |
|  | $\$$ |  | $\$$ |
| 0 | $(200,000)$ | 1.000 | $(200,000)$ |
| 1 | $(40,000)$ | 0.909 | $(36,360)$ |
| 2 | 30,000 | 0.826 | 24,780 |
| 3 | 120,000 | 0.751 | 90,120 |
| 4 | 150,000 | 0.683 | 102,450 |
| 5 | 100,000 | 0.621 | 62,100 |
| 6 | 50,000 | 0.564 | 28,200 |
| NPV |  |  | $+71,290$ |

To calculate duration for a project, the negative cash flows at the beginning of the project are ignored. Duration is calculated using cash flows from the year that the cash flows start to turn positive.

However, if there are negative cash flows in any year after the cash flows turn positive, such as in the final year of the project, these negative cash flows are included in the calculation of duration (as negative values).

In this example, the cash flows start to turn positive from Year 2, so duration is calculated using the present values of the cash flows from Year 2 to Year 6. The Year 2 cash flow is given a weighting of 2, the Year 3 cash flow a weighting of 3, and so on.

Duration is therefore calculated as follows:

| Year | Annual cash <br> flow | Discount <br> factor at 10\% | PV of cash <br> flow | Weighting | PV $\times$ <br> Weighting |
| :--- | ---: | ---: | ---: | ---: | ---: |
|  | $\$$ |  | $\$$ |  |  |
| 2 | 30,000 | 0.826 | 24,780 | 2 | 49,560 |
| 3 | 120,000 | 0.751 | 90,120 | 3 | 270,360 |
| 4 | 150,000 | 0.683 | 102,450 | 4 | 409,800 |
| 5 | 100,000 | 0.621 | 62,100 | 5 | 310,500 |
| 6 | 50,000 | 0.564 | 28,200 | 6 | 169,200 |
|  |  |  | 307,650 |  | $1,209,420$ |

The duration of the project is $1,209,420 / 307,650=3.93$ years.

DCF: internal rate of return (IRR) and modified internal rate of return (MIRR)

- The investment decision rule with IRR
- Calculating a project IRR
- NPV, IRR and financial management
- Modified internal rate of return (MIRR)


## 3 DCF: Internal rate of return (IRR) and modified rate of return (MIRR)

The internal rate of return (IRR) of a project is the discounted rate of return on the investment.

- It is the average annual investment return from the project.
- Discounted at a cost of capital equal to the IRR, the NPV of the project cash flows must come to 0 .


### 3.1 The investment decision rule with IRR

An organisation might establish the minimum rate of return that it wants to earn on an investment. If other factors such as non-financial considerations and risk and uncertainty are ignored, the investment decision using IRR should be based on the following rule:

- If a project IRR is equal to or higher than the minimum acceptable rate of return, it should be undertaken.
- It the IRR is lower than the minimum required return, it should be rejected.

Since NPV and IRR are both methods of DCF analysis, the same investment decision should normally be reached using either method.

### 3.2 Calculating a project IRR

The IRR of an investment can be calculated by inputting the project cash flows into a financial calculator. An approximate IRR can be calculated using interpolation.

## Calculating IRR by interpolation

To calculate the approximate IRR by interpolation, you must have the NPV for the project at two different discount rates (cost of capital). Ideally, one NPV should be positive and the other NPV should be negative.

If the NPV at $\mathrm{A} \%$ is positive, $+\$ \mathrm{P}$, and if the NPV at a higher cost of capital $\mathrm{B} \%$ is negative, $-\$ \mathrm{~N}$, then:
$\operatorname{IRR}=A \%+\left[\frac{\mathrm{P}}{\mathrm{P}+\mathrm{N}} \times(\mathrm{B}-\mathrm{A}) \%\right]$
Ignore the minus sign for the negative NPV. For example, if $\mathrm{P}=+75$ and $\mathrm{N}=-30$, then $\mathrm{P}+\mathrm{N}=105$.

To calculate the approximate IRR by interpolation, you must have the NPV for the project at two different discount rates (cost of capital). Ideally, one NPV should be positive and the other NPV should be negative.
The interpolation method is only approximate and is not exact. This is because it assumes that the IRR decreases at a constant rate between the two NPVs.


For a 'typical' project, the IRR estimated by the interpolation method is slightly higher than the actual IRR. The interpolation method gives a more accurate estimate of the IRR when:

- both NPVs in the calculation are close to 0 , and
- the NPV at $\mathrm{A} \%$ is positive and the NPV at $\mathrm{B} \%$ is negative.


## Example

A business requires a minimum expected rate of return of $12 \%$ on its investments. A proposed capital investment has the following expected cash flows:

| Year | $\$$ |
| :--- | ---: |
| 0 | $(80,000)$ |
| 1 | 20,000 |
| 2 | 36,000 |
| 3 | 30,000 |
| 4 | 17,000 |

## Required

Calculate the NPV at a cost of capital of $10 \%$ and a cost of capital of $15 \%$. Use these NPV figures to estimate the IRR.

## Answer

| Year | Cash flow | Discount factor <br> at 10\% | Present value <br> at 10\% | Discount <br> factor at 15\% | Present value <br> at 15\% |
| :--- | ---: | ---: | ---: | ---: | ---: |
| 0 | $(80,000)$ | 1.000 | $(80,000)$ | 1.000 | $(80,000)$ |
| 1 | 20,000 | 0.909 | 18,180 | 0.870 | 17,400 |
| 2 | 36,000 | 0.826 | 29,736 | 0.756 | 27,216 |
| 3 | 30,000 | 0.751 | 22,530 | 0.658 | 19,740 |
| 4 | 17,000 | 0.683 | $\underline{11,611}$ | 0.572 | 9,724 |
| NPV |  |  | $\underline{+2,057}$ |  | $(5,920)$ |

The IRR is above $10 \%$ but below $15 \%$.
Using the interpolation method:
The NPV is $+2,057$ at $10 \%$.
The NPV is $-5,920$ at $15 \%$.
The NPV falls by $7,977(2,057+5,920)$ between $10 \%$ and $15 \%$.
The estimated IRR is:
$\operatorname{IRR}=10 \%+\left[\frac{2,057}{(2,057+5,920)} \times(15-10) \%\right]$
$=10 \%+1.3 \%$
= $11.3 \%$

## Recommendation

The project is expected to earn a DCF return below the target rate of $12 \%$, and on financial grounds it is not a worthwhile investment.

### 3.3 NPV, IRR and financial management

In strategic financial management, the NPV method of investment appraisal is more important than the IRR method. This is because the value of an investment or a financial instrument is the present value of the expected future cash flows that it will produce, discounted at a suitable cost of capital.

The expected NPV of a new investment by a company could possibly be taken as an estimate of the change that should result in the total value of the company, if the investment is undertaken, provided that investors are aware of the project and its expected NPV.
However, although the NPV method is used much more often, the IRR method might sometimes be used for project appraisal.

### 3.4 Modified internal rate of return (MIRR)

A criticism of the IRR method is that in calculating the IRR, an assumption is that all cash flows earned by the project can be reinvested to earn a return equal to the IRR.

For example, suppose that a project has an NPV of $+\$ 300,000$ when discounted at the cost of capital of $8 \%$, and the IRR of the project is $14 \%$. In calculating the IRR, an assumption would be that all cash flows from the project will be reinvested as soon as they are received to earn a return of $14 \%$ - even though the company's cost of capital is only $8 \%$.

Modified internal rate of return is a calculation of the return from a project, as a percentage yield, where it is assumed that cash flows earned from a project will be reinvested to earn a return equal to the company's cost of capital. So in the previous example of the project with an NPV of $\$ 300,000$ at a cost of capital of $8 \%$, MIRR would be calculated using the assumption that project cash flows are reinvested when they are received to earn a return of $8 \%$ per year.

## Using MIRR for project appraisal

It might be argued that if a company wishes to use the discounted return on investment as a method of capital investment appraisal, it should use MIRR rather than IRR, because MIRR is more realistic because it is based on the cost of capital as the reinvestment rate.

## Calculating MIRR

The MIRR of a project is calculated as follows:
■ Step 1. Take the negative net cash flows in the early years of the project, and discount these to a present value. The total PV of these cash flows is the PV of the investment phase of the project. If the only year of negative cash flow is Year 0 , the PV of the investment phase is the cash flow in Year 0. However, if there are negative cash flows in Year 1, or Year 1 and 2, discount these to a present value and add them to the Year 0 cash outflow.

- Step 2. Take the cash flows from the year that the project cash flows start to turn positive and compound these to an end-of-project terminal value, assuming that cash flows are reinvested at the cost of capital. For example, if cash flows are positive from Year 1 of a five-year project:
- compound the cash flow in Year 1 to and end-of-year 5 value using the cost of capital as the compound rate
- compound the cash flow in Year 2 to and end-of-year 5 value using the cost of capital as the compound rate
- compound the cash flow in Year 3 to and end-of-year 5 value using the cost of capital as the compound rate
- compound the cash flow in Year 4 to and end-of-year 5 value using the cost of capital as the compound rate
- add the compounded values for each year to the cash flow at the end of Year 5.

The total of the compounded values is the total value of returns during the 'recovery' phase of the project, expressed as an end-of-project value.

- Step 3. The MIRR is then calculated as follows:
$\operatorname{MIRR}=n \sqrt{\frac{A}{B}}-1$
where
$\mathrm{n}=$ the project life in years
A = the end-of-year investment returns during the recovery phase of the project (as calculated in Step 2)
$\mathrm{B}=$ the present value of the capital investment in the investment phase (as calculated in Phase 1).

Study the following example carefully:

## Example

A business requires a minimum expected rate of return of $8 \%$ on its investments. A proposed capital investment has the following expected cash flows and NPV.

| Year | Cash flow | Discount <br> factor at $8 \%$ | Present <br> value |
| :--- | ---: | ---: | ---: |
|  | $\$$ |  | $\$$ |
| 0 | $(60,000)$ | 1.000 | $(60,000)$ |
| 1 | $(20,000)$ | 0.926 | $(18,520)$ |
| 2 | 30,000 | 0.857 | 25,710 |
| 3 | 50,000 | 0.794 | 39,700 |
| 4 | 40,000 | 0.735 | 29,400 |
| 5 | $(10,000)$ | 0.681 | $(6,810)$ |
| NPV |  |  | $\underline{+9,480}$ |

The IRR of the project is about 11.5\% (workings not shown).
The modified internal rate of return (MIRR) is calculated as follows:

## Step 1. The PV of investment in the investment phase

| Year | Cash flow | Discount <br> factor at $8 \%$ | Present <br> value |
| :--- | ---: | ---: | ---: |
|  | $\$$ |  | $\$$ |
| 0 | $(60,000)$ | 1.000 | $(60,000)$ |
| 1 | $(20,000)$ | 0.926 | $(18,520)$ |
|  |  |  | $(78,520)$ |
|  |  |  |  |

Step 2. Reinvest cash flows in the recovery phase at the cost of capital, $8 \%$

| Year | Cash flow | Compound at 8\% <br> to end of Year 5 | Present <br> value |
| :--- | ---: | ---: | ---: |
|  | $\$$ |  | $\$$ |
| 2 | 30,000 | $\times(1.08)^{3}$ | 37,791 |
| 3 | 50,000 | $\times(1.08)^{2}$ | 58,320 |
| 4 | 40,000 | $\times(1.08)^{1}$ | 43,200 |
| 5 | $(10,000)$ |  | $(10,000)$ |
| Total |  |  | 129,311 |

Step 3. Calculate MIRR using the formula
$\operatorname{MIRR}=5 \sqrt{\frac{129,311}{78,520}}-1$
$\operatorname{MIRR}=0.1049$ or $10.49 \%$, say $10.5 \%$.
The IRR is $11.5 \%$, but the MIRR is lower because it assumes a lower reinvestment rate of cash inflows.

## DCF and taxation

- Taxation cash flows in investment appraisal
- Interest costs and taxation
- Timing of tax cash flows for taxation

■ Tax-allowable depreciation (capital allowances)
■ Tax exhaustion

## 4 DCF and taxation

### 4.1 Taxation cash flows in investment appraisal

In capital investment appraisal, cash flows arise due to the effects of taxation.

- When an investment by a company results in higher profits, there will be higher taxation. For example, if taxation on profits is $25 \%$ and a company earns $\$ 10,000$ each year from an investment, the annual pre-tax cash inflow is $\$ 10,000$, but there is a tax payment of $\$ 2,500$ each year.
- Similarly, if an investment results in lower profits, tax is reduced. For example, if an investment causes higher spending of $\$ 5,000$ each year and the tax on profits is $30 \%$, there will be a cash outflow of $\$ 5,000$ but a cash benefit from a reduction in tax payments of $\$ 1,500$.

Tax cash flows should be included in DCF analysis.

### 4.2 Interest costs and taxation

Interest costs are allowable for tax purposes. However, interest cash flows are not included in DCF analysis, because the interest cost is in the cost of capital (discount rate). In DCF analysis, an after-tax cost of capital is used to calculate present values. An after-tax cost of capital is a discount rate that allows for the benefit of the tax relief on interest payments.

### 4.3 Timing for cash flows for taxation

The payment of tax on extra profits or the savings in tax due to lower profits could occur either:

■ in the same year as the profits or losses to which they relate, or

- one year later ('one year in arrears').

Either of these assumptions could be correct. An examination question should specify which assumption you should use.

## Example

The after-tax cost of capital is $8 \%$. A project costing $\$ 60,000$ will be expected to earn cash profits of $\$ 40,000$ in year 1 and $\$ 50,000$ in year 2 . Taxation at $30 \%$ occurs one year in arrears of the profits or losses to which they relate. There will be no tax relief on the initial investment of $\$ 60,000$.

## Required

Calculate the NPV of the project.

## Answer

| Year | Investment | Pre-tax <br> cash profit | Tax at 30\% | Net cash <br> flow | Discount <br> factor at 8\% | PV |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: |
| 0 | $(60,000)$ |  |  | $(60,000)$ | 1.000 | $(60,000)$ |
| 1 |  | 40,000 |  | 40,000 | 0.926 | 37,040 |
| 2 |  | 50,000 | $(12,000)$ | 38,000 | 0.857 | 32,566 |
| 3 |  |  | $(15,000)$ | $(15,000)$ | 0.794 | $(11,910)$ |
|  |  |  |  |  | NPV | $(2,304)$ |
|  |  |  |  |  |  |  |

### 4.4 Tax-allowable depreciation (capital allowances)

When a business buys a non-current asset, depreciation is charged in the financial accounts. However, depreciation in the financial accounts is not an allowable expense for tax purposes.

Instead, the tax rules provide for 'tax-allowable depreciation' or capital allowances, according to rules determined by the government.

Tax-allowable depreciation affects the cash flows from an investment, and the tax effects must be included in the project cash flows.

There are two ways of allowing depreciation for tax purposes.

- straight line method
- reducing balance method.


## Straight-line method

With the straight-line method, the amount allowed for tax purposes is a constant amount each year. The tax-allowable depreciation is calculated as:
$=\frac{\text { (The cost of the asset minus any expected residual value) })}{\text { Expected years of life }}$.

## Example

An asset costs $\$ 80,000$ and has an expected economic life of four years with no residual value.

If depreciation is allowed for tax purposes over four years using the straight-line method, the allowable depreciation would be $\$ 20,000$ each year.

If the rate of tax on profits is $25 \%$, the annual reduction in tax from the capital allowance is $\$ 20,000 \times 25 \%=\$ 5,000$ for four years.

These cash flows should be treated as cash inflows for the project.

## The reducing balance method

With the reducing balance method, the tax allowable depreciation is a constant percentage each year of the written down value (WDV) of the asset as at the beginning of the year. The WDV of the asset is its cost less all accumulated capital allowances to date.

The reducing balance method is much more likely to feature in an examination question on DCF than the straight-line method.

## Example

A company purchases an asset costing $\$ 80,000$. Tax-allowable depreciation is $25 \%$ on a reducing balance basis. Tax on profits is payable at the rate of $30 \%$. Tax is payable/saved one year in arrears.

It should be assumed that the first claim for tax-allowable depreciation is made against the profits of the company for Year 1.

## Required

(a) Calculate the tax-allowable depreciation (capital allowances) that would be claimed against profits of Year 0 to Year 4, the tax saving, and the year of the cash flow benefit from the tax saving.
(b) Calculate the effect on tax if the asset is sold in Year 5 for $\$ 24,000$.

Answer
(a) The capital allowances for Years $1-4$, and their effect on cash flows, are calculated as follows:

| Year | WDV at <br> start of year | Tax allowable <br> depreciation (25\%) | Tax saved <br> $(\mathbf{3 0 \%}$ | Year of cash <br> flow benefit |
| :--- | ---: | ---: | ---: | ---: |
|  | $\$$ | $\$$ | $\$$ |  |
| 1 |  | $\$ 0,000$ | 20,000 | 6,000 |
| 2 | $(80,000-20,000)$ | 60,000 | 15,000 | 4,500 |
| 3 | $(6,000-15,000)$ | 45,000 | 11,250 | 3,375 |
| 4 | $(45,000-11,250)$ | 33,750 | 8,438 | 2,531 |

(b) If the asset is sold in Year 5, it is assumed for DCF purposes that a balancing charge or a balancing allowance will arise.

- There is no annual capital allowance in the year of disposal.
- If the asset is sold for more than its WDV at the beginning of the year of disposal, a balancing charge will arise. This increases taxable profits, and so increases the amount of tax payable (in the following year, since it is assumed that tax payments occur one year in arrear of profits).
- If the asset is sold for less than its WDV at the beginning of the year of disposal, a balancing allowance will arise. This reduces taxable profits, and so reduces the amount of tax payable (in the following year).

In this example:

|  | $\$$ |
| :--- | ---: |
| Sale value in Year 5 | 24,000 |
| WDV at the start of the year | 25,312 |
| Balancing allowance | 1,312 |

Since there is a balancing allowance of $\$ 1,312$ for Year 5, there will be a tax saving of $\$ 1,312 \times 30 \%=\$ 394$ in Year 6, one year in arrears. The effect of capital allowances on the cash flows of the project is therefore as follows:

| Year | WDV at start of year | Capital allowance (25\%) | Tax saved (30\%) = cash flow benefit | Year of cash flow benefit |
| :---: | :---: | :---: | :---: | :---: |
|  | \$ | \$ | \$ |  |
| 1 | 80,000 | 20,000 | 6,000 | 2 |
| 2 | 60,000 | 15,000 | 4,500 | 3 |
| 3 | 45,000 | 11,250 | 3,375 | 4 |
| 4 | 33,750 | 8,438 | 2,531 | 5 |
| 5 | 25,312 |  |  |  |
| 5 Disposal value | $(24,000)$ |  |  |  |
| 5 Balancing allowance | 1,312 |  | 394 | 6 |

The disposal value of the asset in Year 5, $\$ 24,000$, is also a cash flow of the project.

## Example

A company is considering an investment in a non-current asset (fixed asset) costing $£ 80,000$. The project would generate the following cash profits:

| Year | $\boldsymbol{£}$ |
| :--- | ---: |
| 1 | 30,000 |
| 2 | 40,000 |
| 3 | 20,000 |
| 4 | 10,000 |

The asset is eligible for capital allowances each year at $25 \%$, by the reducing balance method. It is expected to have a residual value of $£ 40,000$ at the end of year 4 . The after-tax cost of capital is $9 \%$. The rate of tax on profits is $30 \%$. Taxation cash flows occur one year in arrears.

The first annual capital allowance can be claimed against the company's profits for Year 1.

## Required

Calculate the NPV of the project.

## Answer

## The balancing charge or balancing allowance?

The previous example shows that if the first annual capital allowance is claimed against Year 1 profits, the WDV of the asset at the beginning of year 4 (the year of disposal) will be $£ 33,750$. The disposal value of the asset at the end of year 4 is $£ 40,000$. This means that there is a balancing charge in Year 4, because the disposal value is higher than the WDV.

Balancing charge $=£ 40,000-£ 33,750=£ 6,250$.
There will be extra tax payable in Year 5 for the balancing charge. Extra tax payment $=£ 6,250 \times 30 \%=£ 1,875$.
(If the residual value had been lower than $£ 33,750$, there would have been a balancing allowance and a saving in tax the following year).

Project cash flows and NPV

| Year | Investment | Cash <br> profits | Tax on <br> profits at <br> $\mathbf{3 0 \%}$ | Tax-allowable <br> dep'n - tax <br> saved/ (extratax) | Net <br> cash <br> flow | Disc <br> factor <br> at $9 \%$ | PV |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| $\mathbf{0}$ | $(80,000)$ | $£$ | $£$ | $£$ | $£$ |  | $£$ |
|  |  |  |  |  | $(80,000$ | 1.000 | $(80,000)$ |
| 1 |  | 30,000 |  |  | 30 |  |  |
| 2 |  | 40,000 | $(15,000)$ | 6,000 | 0.917 | 27,510 |  |
| 3 | 20,000 | $(12,000)$ | 31,000 | 0.842 | 26,102 |  |  |
| 4 | 40,000 | 10,000 | $(6,000)$ | 4,500 | 12,500 | 0.772 | 9,650 |
| 5 |  |  | $(3,000)$ | 3,375 | 47,375 | 0.708 | 33,542 |
|  |  |  |  | $(1,875)$ | $(4,875)$ | 0.650 | $(3,169)$ |

### 4.5 Tax exhaustion

It is normally assumed in DCF analysis that when capital allowances are available, the company has sufficient taxable profits against which the capital allowances can be set. This means that the tax benefit from the capital allowances can be claimed as soon as possible.

A different situation would apply if a company does not have sufficient taxable profits, for example if it has accumulated losses. In this situation, the company might be unable to claim the capital allowances as soon as if sufficient taxable profits existed.

If tax allowances are not claimed as soon as possible, the cash flow benefit from the tax savings will be deferred, and this will affect the cash flows and the NPV of the proposed investment project.

## DCF and inflation

- Real cost of capital and money cost of capital
- The DCF rule on inflation


## 5 DCF and inflation

### 5.1 Real cost of capital and money cost of capital

Inflation will affect future cash flows. Future rates of inflation cannot be predicted with certainty, but can be estimated. It is therefore possible to estimate future cash flows to allow for expected inflation. If the future rate of inflation is expected to be $i \%$.

Inflation-adjusted cash flow $=\left[\right.$ Cash flow at today's value $\times(1+\mathrm{i})^{\mathrm{n}}$ ]
where:
$\mathrm{i}=$ the annual rate of inflation, expressed as a proportion ( $4 \%=0.04$, and so on)
$\mathrm{n}=$ the number of years in the future that the cash flow will occur.
For example, if sales revenue at today's prices is expected to be $\$ 100,000$ in four years' time, and the rate of sales price inflation is expected to be $3 \%$ each year, sales revenue in Year 4 at inflation-adjusted prices will be: $\left.\$ 100,000 \times(1.03)^{4}\right]=\$ 112,551$.

There is a real cost of capital and a money cost of capital.

- The money cost of capital is the required return on investment, allowing for inflation
- The real cost of capital is the required rate of return at today's price levels.

There is a mathematical connection between the real cost of capital and the money cost of capital.
$(1+\mathrm{r})(1+\mathrm{i})=(1+\mathrm{m})$
where:
r = real cost of capital
$\mathrm{i}=$ annual rate of inflation
$\mathrm{m}=$ money cost of capital
A cost of capital is normally calculated as and expressed as a money cost of capital. It is much less common to use a real cost of capital for DCF analysis.

## Example: real cost and money cost of capital

The rate of inflation in the future is expected to be 5\% each year. A business has a weighted average cost of capital of $11 \%$. This is its money cost of capital.

The real cost of capital is calculated as follows:
$(1+r)(1.05)=1.11$
$(1+r)=\frac{1.11}{1.05}$
$r=1.057-1=0.05$ or $5.7 \%$

### 5.2 The DCF rule on inflation

When inflation is expected to have a significant effect on the future cash flows of a project, it should be included in the DCF analysis. There are two ways of dealing with expected inflation in DCF. Both methods are acceptable.

- Method 1. Estimate the cash flows of the project at today's prices, with no adjustment for inflation. Discount these cash flows at the real cost of capital.
- Method 2. Calculate the cash flows of the project to allow for inflation. Discount these inflation-adjusted cash flows at the money cost of capital.

Both methods give the same NPV. You might be expected to use either method in your examination.

Method 2 is usually the easiest in practice, especially when items of cost and revenue are expected to increase in price at different annual rates.

## Example: real cost and money cost of capital

A company has a money cost of capital of $15.5 \%$. The expected future rate of inflation is $10 \%$ each year. The real cost of capital is therefore $5 \%$. (This is calculated as $[(1.155 / 1.10)-1])$.

The company is considering an investment for which the expected future cash flows and NPV are as follows:

| Year | Cash flows: <br> today's prices |
| :--- | ---: |
|  | $\$$ |
| 0 | $(60,000)$ |
| 1 | 18,000 |
| 2 | 35,000 |
| 3 | 20,000 |
| 4 | 10,000 |

What is the NPV of the project?

## Answer

| Year | Method 1 - Today's prices: Discount at the real cost of capital |  |  | Method 2 - Inflated prices: <br> Discount at the money cost of capital |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Cash flows: today's prices | Discount factor at 5\% | PV | Cash flows: inflated prices | Discount factor at 15.5\% | PV |
|  | \$ |  | \$ | \$ |  | \$ |
| 0 | $(60,000)$ | 1.000 | $(60,000)$ | $(60,000)$ | 1.000 | $(60,000)$ |
| 1 | 18,000 | 0.952 | 17,136 | 19,800 | 0.866 | 17,147 |
| 2 | 35,000 | 0.907 | 31,745 | 42,350 | 0.750 | 31,762 |
| 3 | 20,000 | 0.864 | 17,280 | 26,620 | 0.649 | 17,276 |
| 4 | 10,000 | 0.823 | 8,230 | 14,641 | 0.562 | 8,228 |
| NPV |  |  | 14,391 |  |  | 14,413 |

These numbers should be exactly the same. They differ slightly due to 'rounding errors' in the arithmetic.

## Example: calculating an NPV with inflation in the cash flow

A business is planning an investment in new equipment that would have an expected useful life of three years. The cash flows from the project have been estimated as follows:


The expected annual rates of inflation for the next three years are:

| Equipment cost/residual value | $5 \%$ |
| :--- | :--- |
| Sales revenue | $4 \%$ |
| Running costs | $7 \%$ |

In addition, the project will require a working capital investment of $\$ 10,000$ immediately, with an increase to $\$ 11,000$ at the end of Year 1 and a further increase to $\$ 12,000$ at the end of Year 2. This estimate allows for inflation. The weighted average cost of capital, which is the money cost of capital, is $10 \%$.

## Required

Calculate the NPV of the project and recommend whether the project should be undertaken.

## Answer

Since different rates of inflation apply to different items of cash flow, the NPV should be calculated by discounting inflation-adjusted cash flows at the money cost of capital.

## Workings

Equipment residual value $=20,000 \times(1.05)^{3}=23,153$
Sales, Year $1=25,000 \times(1.04)=26,000$
Sales, Year $2=25,000 \times(1.04)^{2}=27,040$
Sales, Year $3=20,000 \times(1.04)^{3}=22,497$
Running costs, Year $1=10,000 \times(1.07)=10,700$
Running costs, Year $2=10,000 \times(1.07)^{2}=11,449$
Running costs, Year $3=8,000 \times(1.07)^{3}=9,800$.
These cash flows should be discounted at the money cost of capital, which is $10 \%$.

| Year | 0 | 1 | 2 | 3 | NPV |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | \$ | \$ | \$ | \$ | \$ |
| Equipment | $(60,000)$ |  |  | 23,153 |  |
| Working capital | $(10,000)$ | $(1,000)$ | $(1,000)$ | 12,000 |  |
| Sales revenue |  | 26,000 | 27,040 | 22,497 |  |
| Running costs |  | $(10,700)$ | $(11,449)$ | $(9,800)$ |  |
| Net cash flow | $(70,000)$ | 14,300 | 14,591 | 47,850 |  |
| Discount factor at 10\% | 1.000 | 0.909 | 0.826 | 0.751 |  |
| Present value | $(70,000)$ | 12,999 | 12,052 | 35,935 | $(9,014)$ |

## Conclusion

The project has a negative NPV. The decision should therefore be: - Do not invest.

## Free cash flow

- Valuation of a business using DCF
- Definition of free cash flow
- Free cash flow for the firm
- Calculating free cash flow for the firm
- Free cash flow for equity
- Free cash flow and dividend policy


## 6 Free cash flow

### 6.1 Valuation of a business using DCF

DCF can be used to estimate a present value of the future profitability of a company. Management might want to know the present value of expected profits of a business in future years. One reason for wanting this information is to enable management to put a current value on the business, possibly in order to:

- decide how much to offer for a target company in a takeover bid, or
- decide whether to accept or reject a takeover offer from another company.

The value of a business can be estimated by discounting expected future cash flows from the business over a number of years into the future.

However, the cash flows that should be used for discounting are the expected free cash flows of the business.

### 6.2 Definition of free cash flow

Free cash flow is the amount of cash generated by a company or business during a specified period of time (say one year) minus the cash payments that the company or business is obliged to make. Essential payments include taxation payments and capital expenditure to replace ageing non-current assets ('replacement' capital expenditure). Free cash flow is therefore the amount of cash generated by the company that management are able to decide how to use.

However, there are differing specific definitions of free cash flow. The most appropriate definition to use might differ according to purpose.

### 6.3 Free cash flow for the firm

Free cash flow for the firm is the amount of free cash flow generated by the business as a whole, regardless of the source of finance. It is therefore calculated without deducting interest payments as an essential cash payment.

There are differing views about whether capital expenditure for expanding the business is an essential cash payment.

- One view is that only replacement capital expenditure should be deducted as essential cash payments.
- A different view is that replacement capital expenditure and capital expenditure to grow the business internally (organically) should both be deducted in calculating free cash flows, but that capital spending on acquisitions and cash received from business disposals should be excluded.
- A third view is that all capital expenditure (net of proceeds from disposals) should be included in the calculation of free cash flow.

There is no 'correct' answer. However, free cash flow is the amount of cash that management can decide how to use.

- If it is assumed that expanding the business is a free decision by management, only replacement capital expenditure should be deducted in calculating free cash flow.
- On the other hand, if it is assumed that capital expenditure to pay for expansion of the business is necessary to grow operating cash flows in future years, a better definition of free cash flow would be one where both replacement capital expenditure and capital expenditure to pay for growth and expansion are deducted.


### 6.4 Calculating free cash flow for the firm

There are two methods of calculating free cash flow for the firm, although they are closely connected. One method starts with operating profit and the other starts with cash flow from operations (as given in the cash flow statement)

## Method 1

## Method 2

Operating profit before interest X and tax
Add back: Depreciation (and any X
other non-cash expenditures)

$$
X
$$

Plus or minus

Changes in working capital
X /

Minus: Tax payments on profit
Tax payments on profit
Replacement capital expenditure Free cash flow

| X | Cash flow from operations Minus: | X |
| :---: | :---: | :---: |
| (X) | Tax payments on profit | (X) |
| (X) | Replacement capital expenditure | (X) |
| X | Free cash flow | X |

Cash flow from operations
(X) Tax payments on profit
(Note: You should remember that an increase in working capital reduces cash flow and a reduction in working capital increases cash flow.)

### 6.5 Free cash flow for equity

Free cash flow for equity is the amount of free cash flow after deduction of interest payments.

| Free cash flow for the firm | X |
| :--- | ---: |
| Minus: Interest payments |  |
| Equals: Free cash flow for equity | $(\mathrm{X})$ |

Free cash flows for equity might be used in the valuation of the share capital of a company; whereas free cash flow for the firm would be used for the valuation of the entire business, equity plus debt capital.

## Examples

Some companies explain in their annual report and accounts how they use free cash flow as a measure of performance, and they give their definition of free cash flow.

Cadbury Schweppes has used the following definition (slightly simplified):

| Cash generated by operations | X |
| :---: | :---: |
| Minus: Interest payments | (X) |
| Minus: Taxation payments | (X) |
| Minus: Dividend payments | (X) |
| Minus: All capital expenditure, but excluding spending on share repurchases | (X) |
| Free cash flow | X |

### 6.6 Free cash flow and dividend policy

An examination question might ask you to comment on the ability of a company to finance its capital investment programme from retained profits and free cash flows.

The ability of a company to source capital investments internally depends not only on the amount of cash flows generated, but also on dividend policy. A situation might easily occur where a company wants to finance a new capital investment project and has enough cash from free cash flows, but is unwilling to invest because using the cash in this way would involve cutting the annual dividend payment.

As a senior financial manager you might be required to consider two conflicting financial objectives, and make a recommendation about what the company should do.

## DCF: risk analysis

## Contents

1 Risk and uncertainty in capital investment appraisal
2 Monte Carlo simulation
3 Project value at risk
4 Beginner's guide to normal distribution tables

Risk and uncertainty in capital investment appraisal

- The problem of risk and uncertainty
- Methods of assessing risk and uncertainty
- Expected value of the NPV
- Sensitivity analysis
- Risk modelling
- Fiscal risk


## 1 Risk and uncertainty in capital investment appraisal

### 1.1 The problem of risk and uncertainty

Investment projects are long-term projects, often with a time scale of many years. When the cash flows for an investment project are estimated, the estimates might be incorrect.

Estimates of cash flows might be wrong for two main reasons:

- risk in the investment, and
- uncertainty about the future.


## Risk

Risk exists when the actual outcome from a project could be any of several different possibilities, and it is not possible in advance to predict which of the possible outcomes will actually occur.

The simplest example of risk is rolling a dice. When a dice is rolled, the result will be $1,2,3,4,5$ or 6 . These six possible outcomes are known in advance, but it is not possible in advance to know which of these possibilities will be the actual outcome. With risk assessment, it is often possible to estimate the probabilities of different outcomes. For example, we can predict that the result of rolling a dice will be 1, 2, 3, 4,5 or 6 , each with a probability of $1 / 6$.

Risk can often be measured and evaluated mathematically.

## Uncertainty

Uncertainty exists when there is insufficient information to be sure about what will happen, or what the probability of different possible outcomes might be. For example, a business might predict that sales in three years' time will be $£ 500,000$, but this might be largely guesswork, and based on best-available assumptions about sales demand and sales prices.
Uncertainty occurs due to a lack of sufficient information about what is likely to happen.

It is possible to assess the uncertainty in a project, but with less mathematical precision than for the assessment of risk.

Management should try to evaluate the risk and uncertainty, and take it into account, when making their investment decisions. In other words, investment decisions should consider the risk and uncertainty in investment projects, as well as the expected returns and NPV.

### 1.2 Methods of assessing risk and uncertainty

There are several methods of analysing and assessing risk and uncertainty. Some of these are fairly simple and straightforward. Others are more complex. The available methods include:

- calculating the expected value of the NPV
- sensitivity analysis
- simulation modelling
- calculating a project value at risk (VaR).


### 1.3 Expected value of the NPV

An expected value is a weighted average value, calculated using estimated probabilities of different possible outcomes. To calculate an expected value, the probability of each possible outcome is estimated, and the mean (average) outcome is calculated.

Expected value (EV) $=\Sigma \mathrm{px}$
where:
p represents the probability of each outcome and
$x$ represents the value of each particular outcome.
When expected values are used to assess the risk in capital investment appraisal, $x$ would be the NPV for each possible outcome, and the EV would be the expected value of the project net present value (the EV of the NPV).

The basic decision rule is that an investment project should be undertaken if the expected value of its NPV is positive.

However, a project with a positive EV of NPV might not be undertaken if the risk involved seems too great in relation to the amount of the return expected.

## Example

A company is considering an investment in a project. The project would be a fiveyear project, and would cost $£ 200,000$. The actual returns from the investment are subject to uncertainty, but the following estimates have been prepared for the different possible outcomes:

| Probability | NPV |
| :--- | ---: |
|  | $£$ |
| 0.10 | $(8,000)$ |
| 0.30 | 4,000 |
| 0.40 | 12,000 |
| 0.20 | 20,000 |

The EV of the NPV is calculated as follows:

| Probability | NPV | EV |
| :--- | ---: | ---: |
| $\mathbf{p}$ | $\mathbf{x}$ | $\mathbf{p x}$ |
|  | $£$ | $£$ |
| 0.10 | $(8,000)$ | $(800)$ |
| 0.30 | 4,000 | 1,200 |
| 0.40 | 12,000 | 4,800 |
| 0.20 | 20,000 | 4,000 |
|  | EV of NPV |  |
|  |  | 9,200 |

The EV of the NPV is positive, $+£ 9,200$. The decision would therefore be to undertake the investment, provided that the risk does not seem too great. In this example, there is a $10 \%$ probability that the NPV will be negative, - $£ 8,000$. Management might therefore consider whether they consider the investment worth undertaking, in view of this risk.

## Expected value of cash flows for specific items

An alternative approach to using EVs is to use estimates of probabilities for individual items of cash flow, such as EVs for annual sales revenue from a project and EVs for the expected annual running costs.

## Example

A company is considering an investment in a project where the prospects for winning market share have been estimated as follows:

| Market share |  | Probability |
| :--- | :---: | :---: |
| High | $40 \%$ | 0.10 |
| Medium | $25 \%$ | 0.50 |
| Low | $10 \%$ | 0.40 |

The total value of market sales is expected to be $\$ 1,000,000$ in Year 1, rising to $\$ 6$ million in Year 5.

For the purpose of DCF analysis, we could calculate the EV of sales revenue in each year and use this EV as the cash flow for sales revenue.

EV of market share $=(0.10 \times 40 \%)+(0.50 \times 25 \%)+(0.40 \times 10 \%)=20.5 \%$.
Estimates of sales revenue for the DCF analysis would therefore be $\$ 205,000$ in Year 1 ( $20.5 \%$ of $\$ 1$ million) rising to $\$ 1,230,000$ in Year 5.

## Disadvantages of using expected values

The advantage of using expected values of the NPV is that it provides a weighted average value that allows for all the different possible outcomes.

The disadvantages of using expected values of the NPV are as follows:

- The estimates of probabilities might be subjective, and based on judgement and guesswork.
- The EV of the NPV is not a value for any of the actual possible outcomes. In other words, the EV will not happen.
- An EV is much more reliable for estimating the average outcome from events that will happen repeatedly, many times over. A weighted average is not nearly as suitable for estimating the expected outcome for a once-only investment project.
- Most important of all, an EV does not provide any analysis of the risk or uncertainty. When capital investment projects are evaluated, and a decision is made whether or not to undertake the investment, there should be a thorough analysis of the risk as well as the expected returns.


### 1.4 Sensitivity analysis

Sensitivity analysis is a useful but simple technique for analysing risk and uncertainty in an investment, when it is recognised that estimates of cash flows could be inaccurate, or that events might occur that will make the estimates wrong.

Sensitivity analysis can be used:

- To calculate the effect on the NPV of a given percentage reduction in benefits or a given percentage increase in costs. For example, what would the NPV of the project be if sales volumes were $10 \%$ below estimate, or if annual running costs were $5 \%$ higher than estimate?
- To calculate the percentage amount by which benefits must fall below estimate or costs rise above estimate before the project NPV becomes negative. For example, by how much would sales volumes need to fall below the expected volumes, before the project NPV became negative?
- To assess the effect of an unexpected event occurring in the future that would make the cash flow estimates for the project wrong.
- To assess the effect of a delay so that the expected cash inflows from the project occur later than planned.


## Example

A company is considering the following project:

| Year | Equipment | Income | Running <br> costs | Net cash <br> flow | DCF factor at <br> $\mathbf{1 0 \%}$ | PV |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: |
| 0 | $£$ | $£$ | $£$ | $£$ |  | $£$ |
| 1 | $(8,000)$ |  |  |  | 1.000 | $(8,000)$ |
| 2 |  | 6,000 | 3,500 | 2,500 | 0.909 | 2,273 |
| 3 |  | 8,000 | 4,500 | 3,500 | 0.826 | 2,891 |
| 4 |  | 10,000 | 5,500 | 4,500 | 0.751 | 3,379 |
| NPV |  | 4,000 | 2,500 | 1,500 | 0.683 | 1,025 |

## Required

Estimate the sensitivity of the project to:

- income being lower than estimate, and
- running costs being higher than estimate.


## Answer

We can calculate the PV of the equipment cost, the income and the running costs separately, as follows:

| Year | Equipment <br> PV | Income <br> PV | Running <br> costs <br> PV | NPV |
| :--- | ---: | ---: | ---: | ---: |
|  | $£$ | $£$ | $£$ | $£$ |
| 0 | $(8,000)$ |  |  |  |
| 1 |  | 5,454 | $(3,181)$ |  |
| 2 |  | 6,608 | $(3,717)$ |  |
| 3 |  | 7,510 | $(4,131)$ |  |
| 4 |  | 2,732 | $(1,707)$ |  |
| PV |  | $(8,000)$ | 22,304 | $(12,736)$ |
|  |  |  |  |  |

The sensitivity of the project to errors in the estimates can now be assessed as follows:

- Income. The project would cease to have a positive NPV if income is below the estimate by more than: $\frac{1,568}{22,304}=0.070$ or $7.0 \%$.
- Running costs. The project would cease to have a positive NPV if running costs are above the estimate by more than: $\frac{1,568}{12,736}=0.123$ or $12.3 \%$.

An assessment of risk and uncertainty could then be made on the basis of the likelihood that income will fall below estimate or running costs will increase above the estimate by more than these amounts.

### 1.5 Risk modelling

The risk in an investment can be assessed by constructing a 'model' for the investment, and then considering possible variations in the possible outcomes.
For capital investment appraisal, a risk model might be constructed using a spreadsheet.

Having constructed the model, the risk in the investment can be assessed by testing different scenarios, such as delays in achieving the benefits from an investment, and combinations of variations in costs and benefits.

Risk modelling can be used to consider any aspect of risk. One aspect of risk that can be particularly significant for investment decisions by companies in certain industries is fiscal risk.

### 1.6 Fiscal risk

Fiscal risk is the risk that after a capital investment project has been implemented, the government might increase the rate of tax payable on the profits or cash flows from the project. Higher tax payments could significantly affect the returns from a project.

Fiscal risk varies between countries. Some countries have a reputation for fiscal stability, so that any changes in tax are fairly insignificant. In other countries there is a much higher risk of tax changes. Obviously, companies considering an investment in a country, even their domestic country, should take fiscal risk into consideration when deciding whether or not to invest in a project.

## Example

The oil and natural gas industry is an example of an industry where fiscal risk has had a significant impact on capital expenditure decisions.

Oil companies purchase concessions to explore for oil or gas, and extract any that they find. If they fail to find any oil or gas, they will lose their investment. However, they face a risk that if they are successful in finding and extracting oil, the government will decide to tax their profits. One way of doing this is to charge a windfall tax on the profits of firms in the industry that operate in the country.

In the past the UK government has charged a windfall tax on the profits of companies in the North Sea oil business. The matter was debated in Parliament in 2002, where it was claimed that if the government raised the rate of tax on the profits of offshore oil and gas extraction to $40 \%$ or $50 \%$, further exploration by the oil companies would cease to be viable.

## Reducing fiscal risk

Fiscal risk is unavoidable for companies that pay tax on their profits. However, the risk might sometimes be reduced for large companies. A large company can try to negotiate with the government of a country before deciding whether to invest in the country. They might argue that they will not invest unless the government gives an assurance of fiscal stability.

The risk of an unexpected additional tax (windfall tax) cannot be removed simply by a government's promise, because promises might be broken. Another approach that might be used by some companies is to offer to pay a royalty tax on their output. For example, an oil company might undertake to pay a royalty on every barrel of oil that it extracts under a concession agreement. If the company pays tax in this way, so that the royalty payments increase with the volume of business, the government might be less inclined to charge additional tax on what it sees as excessive profits.

## Monte Carlo simulation

- The nature of Monte Carlo simulation
- Using computers for Monte Carlo simulation
- Monte Carlo simulation and capital investment appraisal
- The significance of the results from a Monte Carlo simulation
- Project volatility


## 2 Monte Carlo simulation

### 2.1 The nature of Monte Carlo simulation

Monte Carlo simulation is a method of measuring an expected outcome by means of generating multiple trials or iterations, in order to determine the expected value of the outcome and also to measure the variability or risk.

This method of determining the EV of an outcome gets its name from the large number of random outcomes that are used to obtain the measurement, which can be compared with the random outcomes that would be expected on the gambling tables of a casino in Monte Carlo.

Monte Carlo simulation depends on the fact that by taking the outcomes of a large number of individual random events (iterations), an accurate measurement of the expected value and probability distribution of the outcome will be obtained.

A simple example might help to make this point more clear.

## Example

Suppose that you roll two dice and calculate the total of the values that turn up on each dice. Each dice has six sides numbered 1 to 6 , so the total from rolling two dice could be anything between $2(1+1)$ and $12(6+6)$.

If you rolled the dice just once, the total could be anywhere in the range 2 to 12 . If you roll the dice again, the total might be different. If you roll the dice enough times, you will get a large number of totals, all in the range 2 to 12 . Some totals will appear more frequently than others. For example, a combined total of 6 for the two dice (which results from $1+5,2+4$ and $3+3$ ) will occur more often than a total of 12 (where both dice must turn up as 6).

By rolling the two dice many times over, an average total value will be obtained. (It should be 7 , if the dice are rolled enough times. However there might be a slight error, and the average obtained from your simulation might be slightly above or below 7.0. The risk of error is reduced by rolling the dice more times - doing more 'iterations' in your simulation.)

The simulation will also produce a probability distribution of total values. This shows the proportion or number of times that each total value was obtained from the two dice. The distribution will look something like this.


The expected value and the probability distribution of the random outcome can be measured. The usual method of calculating the variability in the value is to calculate a standard deviation. In this example, we can calculate the expected value of the total of the two dice (7.0) and we can measure the variability in the total above or below 7.0 as a standard deviation of the expected value. (The standard deviation is about 2.4, but the workings are not shown here.)

### 2.2 Using computers for Monte Carlo simulation

The example of the two dice is a very simple one, and in practice simulation models are used for the analysis of more complex outcomes. The use of Monte Carlo simulation modelling is made possible by computers, which can produce a large number of iterations quickly, to produce a reliable expected value and probability distribution of the outcome.

There are various software packages that might be used for Monte Carlo simulation. Simulation models can be constructed, for example, in a spreadsheet.

Another simplified example might help to illustrate the construction of a simulation model on spreadsheet software such as MS Excel.

Example
Suppose that you want to use a Monte Carlo simulation model to estimate the EV of the total cost of a project, and the standard deviation of this expected value.

The project consists of six activities. You have estimated the minimum cost and the maximum cost for each activity, as follows:

| Activity | Minimum cost <br> M000 | Maximum cost <br> M |
| :--- | ---: | ---: |
| 1 | 10 | 20 |
| 2 | 15 | 21 |
| 3 | 8 | 15 |
| 4 | 16 | 22 |
| 5 | 21 | 28 |
| 6 | $\underline{14}$ | $\underline{18}$ |
| Total | $\underline{84}$ | $\underline{124}$ |

The cost for each activity could be any value in the range between the minimum and maximum cost, and there is an equal probability of each value for cost occurring.

The total cost of the project will therefore be a random variable with a value between $\$ 84,000$ and $\$ 104,000$.

We can use simulation to calculate the EV of the total cost and a standard deviation for this EV.

- The model should be used to generate random values for the cost of each activity.
- The randomly selected values for the six activities should be added, to obtain a total cost for the project.
- This process should be repeated through a large number of iterations.
- The EV of the total project cost is the average of the random total costs produced by all these iterations.
- A standard deviation of the EV of total cost can also be calculated from the data.

A spreadsheet in Excel might be prepared as shown below. The example shows just one iteration to produce a random total cost for the project.

| Cell | A | B | C | D | E | F | G | H |
| :--- | :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 1 | Activity | 1 | 2 | 3 | 4 | 5 | 6 | Total |
| 2 | Min cost (\$000) | 10 | 15 | 8 | 16 | 21 | 14 | 84 |
| 3 | Max cost (\$000) | 20 | 21 | 15 | 22 | 28 | 18 | 124 |
| 4 | Iterations |  |  |  |  |  |  |  |
| 5 | Iteration 1 | 12 | 21 | 9 | 19 | 27 | 14 | 102 |

The Excel spreadsheet package includes a random number selection function $=\operatorname{RAND}($ ). The formula in cell B5 (and B6, B7, B8 and so on for each successive iteration $)$ is $=\operatorname{RAND}()^{*}(20000-10000)+10000$. This will produce a random number anywhere in the range between 10,000 and 20,000 .

The total column H is simply a total of the values in columns B to G for that row. For example, the formula in cell H 5 will be $=\mathrm{SUM}(\mathrm{B5}: \mathrm{G} 5)$.

There should be a row in the spreadsheet for each iteration; therefore the number of iterations required should be decided when the model is constructed. The model can also be programmed to calculate an EV and standard deviation of the EV of the total project cost.

### 2.3 Monte Carlo simulation and capital investment appraisal

The previous example might help to suggest how Monte Carlo simulation might be used in capital investment appraisal. There are many different cash flows in a project whose value might be uncertain, but which can be estimated as a probability distribution. A Monte Carlo simulation model can be created, and the model can be used to calculate the NPV of the project for many iterations, in order to obtain an expected value of the project NPV and a standard deviation of the expected NPV.

## Example

Suppose that you want to use a Monte Carlo simulation model to estimate the NPV of a proposed capital investment project, where there is some uncertainty about the units of sale each year and the variable cost per unit sold.

Probability distributions might be prepared for each of these unknown variables, and random numbers allocated to each possible value, as follows:

| Units per <br> year | Probability | Random <br> numbers | Variable <br> cost | Probability | Random <br> numbers |
| :--- | :---: | :---: | :---: | :---: | :---: |
| 5,000 | 0.15 | $00-14$ | $\$ 5.00$ | 0.25 | $00-24$ |
| 6,000 | 0.20 | $15-34$ | $\$ 5.50$ | 0.60 | $25-84$ |
| 7,000 | 0.30 | $35-64$ | $\$ 6.00$ | 0.15 | $85-99$ |
| 8,000 | 0.25 | $65-89$ |  |  |  |
| 9,000 | 0.10 | $90-99$ |  |  |  |

With a more complex model, a large number of different variables could be given various possible values, with associated probabilities and random number allocations.

The model is then used to produce a large number of possible outcomes and NPVs for the project. For each possible outcome, random values are given to each of the uncertain variables by generating random numbers and deciding the value in accordance with the random number generated.

Alternatively, a model might be constructed based on the assumption that sales units might be anywhere in the range 5,000 to 9,000 each year and the variable cost per unit might be anywhere between $\$ 5.00$ and $\$ 6.00$. Random numbers can be generated for both units of sale and variable cost per unit, in many iterations, using the same method as described above for the project total costs.

For your examination, you are not required to construct a Monte Carlo simulation model for capital investment appraisal and risk analysis, but you are expected to know how a simulation model would be constructed, and to interpret the output obtained from using the model.

### 2.4 The significance of the results from a Monte Carlo simulation

The output from a Monte Carlo simulation is the results from a large number of iterations. These results are used to calculate an expected value (average value) for the item that is being measured, and a standard deviation.

The standard deviation provides a measure of risk. A large standard deviation relative to the average or expected value indicates high risk. The actual result could be significantly higher or lower than the expected value. A small standard deviation relative to the size of the EV indicates lower risk and a greater probability that the actual result will be close to the expected value.

## Risk analysis using the standard deviation

A significant feature of Monte Carlo simulation is that the range of outputs from the many iterations usually follow a normal distribution (provided that the number of iterations is large enough). If it can be assumed that there is a normal distribution of possible results around the expected value, normal distribution analysis can be used to assess probabilities of what the actual outcome might be.

In the previous example of the project activity costs, if the outputs from the iterations follow a normal distribution, the EV of total cost and the standard deviation can be used to assess:

- the probability that the total project cost will not exceed a particular amount, say \$95,000
■ what the maximum total cost of the project should be, at the $95 \%$ or $99 \%$ level of probability.


## Note

If you are not familiar with normal distribution and the analysis of risk with normal distribution, you should read the final section of this chapter. This provides a short beginner's guide to normal distribution analysis.

## How accurate is the estimate of the expected value?

It is also possible to estimate the accuracy of the estimate of the EV of a project NPV using Monte Carlo simulation. The accuracy of the estimated EV can be measured statistically as a total error, as follows:

Error $=(3 \times \sigma) / \sqrt{N}$
where
$\sigma=$ the standard deviation of the expected value of the NPV, obtained from the simulation model
$\mathrm{N}=$ the number of iterations used.

For example, if the EV of the NPV is $+\$ 500,000$, the standard deviation of the EV of the NPV is $\$ 10,000$ and 400 iterations of the simulation model were made to produce these results, the total error in the estimate of the EV of the NPV would be:

Error $=(3 \times 10,000) / \sqrt{ } 400=\$ 1,500$.

For a larger value of N (the larger the number of iterations), the error is smaller. Not surprisingly, this indicates that greater accuracy is obtained from simulation by doing a larger number of iterations of the model.

### 2.5 Project volatility

You might come across the term 'project volatility'. Volatility in this context refers to the possibility that a project's NPV might be much higher or lower than the expected value of the NPV. (In the financial markets, volatility refers to the extent to which share prices, interest rates or exchange rates might fluctuate up and down.)

High volatility indicates much greater variability and so much greater risk.
In financial management, volatility is measured statistically, usually as a standard deviation. The volatility of a capital investment project can be measured as either:

- the standard deviation of the expected NPV of the project, or
- the standard deviation of the PV of the annual net cash flows from the project.

If you are given the volatility of a project in an examination question, check carefully to see whether volatility is defined. If it is not specifically defined in the question, you should assume that it is the standard deviation of the expected NPV of the project.

## Example

Monte Carlo simulation has been used to calculate the expected value of the NPV of a proposed capital investment project and the volatility of the project.

The EV of the NPV is $+\$ 106,000$ and the project volatility, measured as the standard deviation, is $\$ 80,000$.

This indicates that the EV of the NPV is greater than $\$ 0$ ('break-even') by a Z score of:
$106,000 / 80,000=1.325$ standard deviations.
Normal distribution tables show that the probability of a value greater than 1.325 standard deviations below the mean of a normal distribution is $0.5000-0.4079=$ 0.0921 - about $9.2 \%$.


Normal distribution tables can also be used to calculate the following probabilities:

- At the $95 \%$ confidence level, the NPV will not be lower than 1.645 standard deviations below the mean or EV.
$\mathrm{N}(0.95) \sigma=1.645 \times \$ 80,000=\$ 131,600$.
At the $95 \%$ confidence level, the NPV will not be worse than \$106,000 - \$131,600 $=-\$ 25,600$.
- At the $99 \%$ confidence level, the NPV will not be lower than 2.327 standard deviations below the mean or EV.
$\mathrm{N}(0.99) \sigma=2.327 \times \$ 80,000=\$ 186,160$.
At the $99 \%$ confidence level, the NPV will not be worse than $\$ 106,000-\$ 186,160$ $=-\$ 80,160$.


## Project value at risk

- Annual volatility of a project
- The meaning of value at risk and project value at risk
- Calculating project value at risk


## 3 Project value at risk

### 3.1 Annual volatility of a project

Volatility can be measured over differing periods of time. In the previous example of project volatility, the volatility was measured over the entire life of the project.

Volatility can also be measured over shorter time periods, such as annual volatility. In capital investment appraisal, annual volatility is the standard deviation of the PV of annual cash flows from the project.

For any item with a variable value, for which volatility is measured, the statistical volatility increases with the length of the time period over which it is measured. The relationship is as follows:
$\sigma_{\mathrm{L}}=\sigma_{\mathrm{S}} \times \sqrt{ } \mathrm{T}$
where
$\sigma_{L}$ is the volatility (standard deviation) over the longer time period $\sigma_{s}$ is the volatility (standard deviation) over the shorter time period T is the number of short time periods that make up the long time period.

## Example

Expected annual cash flows are $\$ 100,000$ and the volatility of annual cash flows is $\$ 15,000$. Cash flows in any one year are independent of what the cash flows were in the previous years.

Over a five year period, expected cash flows are $\$ 500,000$ and the five-year volatility of cash flows is:
$\$ 15,000 \times \sqrt{ } 5=\$ 33,541$.

### 3.2 The meaning of value at risk and project value at risk

The concept of Value at Risk (VaR) has been developed in the financial services industry, mainly by banks. In banking, Value at Risk can be measured for items such as the risk of losses from credit risk (bad debts) and the risk of losses from a fall in the market value of investments.

For a given period of time, such as one day, one week, one month, three months or one year, and at a given confidence level, Value at Risk can be stated as a single figure. For example, a bank might use its VaR computer model to calculate that on any one day at the $95 \%$ confidence level, the Value at Risk is $\$ 10$ million. This means that at the $95 \%$ level of confidence, the maximum loss the bank will suffer on any one day is $\$ 10$ million.

The concept of VaR can be extended to capital investment appraisal, although at the moment it is not widely used in practice.

A project value at risk is the maximum amount, at a given confidence level, by which the actual NPV from a project will be worse than the expected value of the NPV. It can therefore be used to assess the risk in a capital investment project, which should help management to decide whether or not to invest in the project, taking into consideration the risk as well as the expected return (expected NPV).

### 3.3 Calculating project value at risk

Project value at risk can be calculated in either of two ways:
■ using the standard deviation of the project NPV: this was illustrated by the example in the previous section of this chapter
■ using the annual volatility of the PV of cash flows.

## Example

A simulation model has been used to calculate the expected value of the NPV of a project. This is $\$ 282,000$. The project has an expected life of ten years, and the volatility of the PV of the annual cash flows is $\$ 30,000$.

Normal distribution tables can also be used to calculate the following probabilities:

- At the $95 \%$ confidence level, the project value at risk is:
$\mathrm{N}(0.95) 30,000 \sqrt{10}=1.645 \times \$ 94,868=\$ 156,058$.
At the $95 \%$ confidence level, the NPV will not be worse than $\$ 282,000-\$ 156,058$ $=\$ 125,942$.
- At the $99 \%$ confidence level, the project value at risk:
$\mathrm{N}(0.99) 30,000 \sqrt{10}=2.327 \times \$ 94,868=\$ 220,758$.
At the $99 \%$ confidence level, the NPV will not be worse than $\$ 282,000-\$ 220,758$ $=\$ 61,242$.

This suggests a low level of risk with this capital investment project, because the NPV is expected to be positive even at the $99 \%$ confidence level for project value at risk.

## Beginner's guide to normal distribution tables

- The features of a normal distribution
- A standard normal distribution table
- Using a standard normal distribution table
- Normal distribution tables and confidence levels
- Applications of normal distribution tables in financial management


## 4 Beginner's guide to normal distribution tables

When a measurement is taken of a population or a sample of items, we can calculate an average value for the population or sample.

For example, a business can calculate its average daily sales from a sample of daily sales figures. The actual sales on any day will usually be above or below the average, so there is a range of daily sales figures that the average sales figure represents. Actual daily sales are therefore 'distributed' around the average or mean value. So daily sales might average $\$ 2,400$, but actual daily sales might be anywhere in a range of, say, $\$ 1,200$ to $\$ 3,700$.

It has been found from experience that the distribution of actual values around the average is often very close to a normal distribution. A normal distribution is a distribution that can be defined mathematically, and whose properties are measurable. If a population of items has a distribution that is similar to a normal distribution, the distribution can be analysed statistically using normal distribution tables.

### 4.1 The features of a normal distribution

A normal distribution is a symmetrical distribution of values around an average ('mean'). The distribution of values below the average is mirrored exactly by the distribution of values above the average.

This symmetrical distribution can be drawn as a bell-shaped curve as follows:


This is a graph where the y axis represents probability and the x axis represents the value of the item being measured (such as daily sales). The total area under the curve represents $100 \%$ of total values.

A normal distribution therefore shows that the most likely actual value is the mean value, but the actual value can be above or below the mean. Values close to the mean are more probable (more likely to happen) than values a long way above or below the mean.

Because of the mathematical properties of a normal distribution, it is possible to measure the probability of a value being:

- above a specified amount
- below a specified amount, or
- within a specified range of values.

A key statistical feature of a normal distribution is that the probability of a value being more or less than a specified amount can be measured in terms of standard deviations above or below the average/mean.

- The normal distribution is symmetrical. Because the distribution is symmetrical, there is a $50 \%$ probability that a value will be higher than the average ('above the mean' and a $50 \%$ probability that a value will be less than the average ('below the mean').
- Because of the mathematical properties of the normal distribution, it is also possible to predict the probability that a value will be within a given number of standard deviations above or below the mean. For example:
- there is a probability of $34.13 \%$ that a value will be within a range of one standard deviation below the mean and the mean
- there is a probability of $34.13 \%$ that a value will be within a range of the mean and one standard deviation above the mean
- there is therefore a probability of $68.26 \%(2 \times 34.13 \%)$ that a value will be in a range of one standard deviation below the mean to one standard deviation above the mean.
- For similar reasons we can predict that there is a probability of $15.87 \%$ ( $50 \%$ $34.13 \%$ ) that a value will be more than one standard deviation below the mean and there is also a probability of $15.87 \%$ that a value will be more than one standard deviation above the mean.

These probabilities can be obtained from normal distribution tables. A normal distribution table is provided in the formula sheet in your examination.

### 4.2 A standard normal distribution table

A standard normal distribution table is shown in the introductory pages to this text, in the pages for the formulae and tables.

The table represents the probabilities in one half of a normal distribution. Since the normal distribution is symmetrical, the table represents either the half of the distribution below the mean or the half that is above the mean.

- The rows and columns represent the number of standard deviations above or below the mean.
- The figures in the table are the probabilities that a value will be within that number of standard deviations of the mean.

For example:

- The value for 1 standard deviation is in the row 1.0 and the column 0.00 . The value here is 0.3413 . This shows that in a normal distribution, there is a probability of 0.3413 or $34.13 \%$ of a value between the mean and 1 standard deviation below the mean. There is a similar probability of 0.3413 or $34.13 \%$ of a value between the mean and 1 standard deviation above the mean.
- The value for 1.25 standard deviations is in the row 1.2 and the column 0.05 . The value in the table is 0.3944 . This shows that in a normal distribution, there is a probability of 0.3944 or $39.44 \%$ of a value between the mean and 1 standard deviation below the mean. There is a similar probability of 0.3944 or $39.44 \%$ of a value between the mean and 1 standard deviation above the mean. This also means that the probability that a value will be somewhere between 1.25 standard deviations below the mean and 1.25 standard deviations above the mean is $78.88 \%$.


### 4.3 Using a standard normal distribution table

For any value whose variability can be approximated by a normal distribution, normal distribution tables can be used for probability analysis and so for risk analysis. The following examples show how this can be done:

## Example

A simulation model has been used to calculate the expected value of the NPV of a project. This is $+\$ 30,600$. The standard deviation of the project is $\$ 18,000$.

The variability in the NPV approximates to a normal distribution. (This is usual for the output results from a simulation model.)
(a) What is the probability that the NPV will be greater than $\$ 0$ ?
(b) What is the probability that the NPV will be greater than $\$ 40,000$ ?

## Answer

(a) An NPV of $\$ 0$ is below the mean value of $\$ 31,950$, by $\$ 31,950$. We need to convert this into a number of standard deviations, or 'Z score'.
$\mathrm{Z}=31,950 / 18,000=1.775$ standard deviations.
The probability that the NPV will be between the mean (the EV of the NPV) and a value that is 1.775 standard deviations below the mean is between 0.4616 ( 1.77 standard deviations) and 0.4625 ( 1.78 standard deviations). Since 1.775 is exactly half-way between 1.77 and 1.78 , we can estimate by interpolation that the probability for 1.775 standard deviations is 0.46205 $(0.4616+(0.4625-0.4616) \times 50 \%)$.

This can be rounded to 0.4620 .
The probability that the NPV will be between the mean (the EV of the NPV) and a value that is 1.775 standard deviations below the mean is therefore 0.4620 . The probability that the NPV will be above the mean (the EV of the $\mathrm{NPV})$ is $0.5000(50 \%)$. The probability that the NPV will be higher than $\$ 0$ is therefore 0.9620 or $96.20 \%$.

|  | Probability <br> NPV between $\$ 0$ and the EV(mean) <br> Above the mean <br> Above $\$ 0$ |
| :--- | ---: |
|  | 0.4620 |

(b) An NPV of $\$ 40,000$ is $\$ 8,050$ above the mean value of $\$ 31,950$.

The $Z$ score is $8,050 / 18,000=0.447$.
The probability that the NPV will be between the mean value (EV) and 0.447 standard deviations above the mean is somewhere between 0.1700 ( 0.44 standard deviations) and 0.1736 ( 0.45 standard deviations). We can estimate by interpolation that the probability for 0.447 standard deviations is 0.1726 $(0.1700+(0.1736-0.1700) \times 70 \%)$.

The probability that the NPV will be more than 0.447 standard deviations above the mean is therefore $0.5000-0.1726=0.3274$ or $32.74 \%$.

### 4.4 Normal distribution tables and confidence levels

A normal distribution table can also be used to establish confidence levels. In risk analysis, it is usual to assess risk at the $95 \%$ confidence level or the $99 \%$ confidence level.

- A $95 \%$ confidence level means that there is a $95 \%$ probability.
- A $99 \%$ confidence level means that there is a $99 \%$ probability.


## Confidence levels for values within a range

We can use normal distribution tables to calculate, at a given confidence level, that the value of an item will be within a specified range of values above and below the mean.

For example, suppose that a simulation model produces results showing that the EV of the NPV for a project is $\$ 125,000$ and the standard deviation (project volatility) is $\$ 40,000$. We can predict at the $95 \%$ confidence level that the NPV of the project will be within a specified range around the EV.

To establish a $95 \%$ confidence level for the range around the NPV, we need to identify the range of values below the mean that represent $47.5 \%$ of total probabilities and the range of values above the mean that represent $47.5 \%$ of total probabilities (since $2 \times 47.5 \%=95 \%$ ).

We therefore need to identify the number of standard deviations from the mean that cover $47.5 \%$ of all probabilities. From the normal distribution table, we can identify that this is 1.96 standard deviations.

At a $95 \%$ confidence level, we can therefore predict that the NPV will be somewhere in the range between 1.96 standard deviations below the mean and 1.96 standard deviations above the mean.

Similarly, at a $99 \%$ confidence level, we can predict that the NPV will be somewhere in the range between 2.575 standard deviations below the mean and 2.575 standard deviations above the mean. (This is because there is a probability of 0.4950 that the value will be 2.575 standard deviations below the mean and a probability of 0.4950 that the value will be 2.575 standard deviations above the mean.)

## Example

A simulation model has been used to calculate the expected value of the NPV of a project. This is $+\$ 150,000$. The standard deviation of the project is $\$ 55,000$.

The variability in the NPV approximates to a normal distribution. (This is usual for the output results from a simulation model.)

At the $95 \%$ confidence level, we can predict that the NPV of the project will be in the range of 1.96 standard deviations above and below the mean. $1.96 \times \$ 55,000=$ $\$ 107,800$. At the $95 \%$ confidence level, we can therefore predict that the NPV will be somewhere in the range $\$ 42,800$ to $\$ 257,800$.

At the $99 \%$ confidence level, we can predict that the NPV of the project will be in the range of 2.57 standard deviations above and below the mean. $2.57 \times \$ 55,000=$ $\$ 141,350$. At the $99 \%$ confidence level, we can therefore predict that the NPV will be somewhere in the range $+\$ 8,650$ to $\$ 291,350$.

## Confidence levels for values above or below a specified amount

Confidence levels can also be established to identify the probability that the actual value will be above or below a specified amount. For example, we can establish at the $95 \%$ or $99 \%$ level of confidence that a value will be more than $\$ X$ or less than $\$ X$ (or not more than $\$ \mathrm{X}$ or not less than $\$ \mathrm{X}$ ).

With this type of calculation we are looking at only one side of the normal distribution table (and carrying out a 'one-tailed test'). This differs from confidence levels for a range above and below the mean, where we are looking at both sides of the normal distribution, above and below the mean.


The diagram shows that if we want to establish a $95 \%$ confidence level for a value that is above the mean, we need to calculate the number of standard deviations above the mean for which there is a $45 \%$ probability. There is a $50 \%$ probability that the value will be less than the mean, so taken together we have a $95 \%$ probability level.

From the normal distribution table, we can find that 0.4500 of probabilities are within 1.645 standard deviations of the mean, on one side of the normal distribution table.

At the $95 \%$ level of confidence we can therefore state that the value will be less than an amount equal to 1.645 standard deviations below the mean. We can also state that at the $95 \%$ confidence level, the value will not exceed an amount that is more than 1.645 standard deviations above the mean.

## Example

A simulation model has been used to calculate the expected value of the NPV of a project. This is $+\$ 70,000$. The standard deviation of the project is $\$ 28,000$.

The variability in the NPV approximates to a normal distribution.

At the $95 \%$ confidence level, we can therefore predict that the NPV will be not less than $\$ 46,060(1.645 \times \$ 28,000)$ below the EV of $\$ 70,000$. This means that at the $95 \%$ confidence level the NPV will be not less than $\$ 23,940$.
To establish the $99 \%$ confidence level, we can establish that 0.4900 of probabilities are between the men and about 2.33 standard deviations of the mean. At the $99 \%$ confidence level, we can therefore predict that the NPV will be not less than $\$ 65,240$ $(2.33 \times \$ 28,000)$ below the EV of $\$ 70,000$. This means that at the $99 \%$ confidence level the NPV will be not less than $+\$ 4,760$.

### 4.5 Applications of normal distribution tables in financial management

Normal distributions are found in many 'populations' and can be used for detailed statistical analysis of risk. For your examination, the most likely use of normal distribution tables will be for:

- analysing the output from a Monte Carlo simulation model for the purpose of assessing project risk, or
- the valuation of financial options, which is explained in another chapter.


# Investing: portfolio theory and the CAPM 

|  | Contents |
| :--- | :--- |
| 1 | Measuring investment risk |
| 2 | Portfolio theory |
| 3 | Choosing an investment portfolio |
| 4 | Capital asset pricing model (CAPM) |

## Measuring investment risk

- Risk and return in investments
- What is investment risk?
- Measuring risk as a variance or standard deviation of expected returns
- Assessing the investment risk: coefficient of variation


## 1 Measuring investment risk

### 1.1 Risk and return in investments

Investors invest in shares and bonds in the expectation of making a return. The return that they want from any investment could be described as:

- a return as reward for providing funds and keeping those funds invested, plus
- a return to compensate the investor for the risk.

As a basic rule, an investor will expect a higher return when the investment risk is higher.

### 1.2 What is investment risk?

Investors in bonds, investors in shares and companies all face investment risk.
In the case of bonds, the risks for the investor are as follows:

- The bond issuer may default, and fail to pay the interest on the bonds, or fail to repay the principal at maturity.
- There may be a change in market rates of interest, including interest yields on bonds. A change in yields will alter the market value of the bonds. If interest rates rise, the market value of bonds will fall, and the bond investor will suffer a loss in the value of his investment.

In the examination, you are often told to assume that debt capital is risk-free. (In practice, only government debt denominated in the domestic currency of the government is risk-free).

In the case of equity shares, the risks for the investor are that:

- the company might go into liquidation, or
- (much more significantly) the company's profits might fluctuate, and dividends might also rise or fall from one year to the next.

For investors in equities, the biggest investment risk comes from uncertainty and change from one year to the next in annual profits and dividends. Changes in expected profits and dividends will affect the value of the shares.

When a company invests in a new project, there will be an investment risk. This is the risk that actual returns from the investment will not be the same as the expected returns but could be higher or lower. This investment risk for companies is similar to the investment risk facing equity investors.

### 1.3 Measuring risk as a variance or standard deviation of expected returns

The risk of variations in annual profits and dividends can be measured. When annual returns can differ by a large amount from normal or expected (average) returns, and there are considerable differences in returns from one year to the next, returns are volatile.

High volatility in returns is associated with high investment risk.
Risk can be measured statistically, either from an analysis of historical returns achieved in the past, or from probability estimates of returns in the future. The measurement of risk is a measurement of the volatility of possible returns.

This volatility can be measured as either the variance or standard deviation of expected returns over a given period of time, such as a variance or standard deviation of expected annual returns.

For an investment, we can measure both:

- an expected or average annual return for the time period, and
- a variance or standard deviation of the returns.


## Calculating the variance or standard deviation of expected returns

The variance of expected returns is calculated as $\sum \mathrm{p}(\mathrm{r}-\overline{\mathrm{r}})^{2}$
where:
$p$ is the probability of a given return on the investment
$r$ is the amount of that return
$\overline{\mathrm{r}}$ is the average expected return on the investment. This is the expected value (EV) of the return.

The standard deviation of expected returns is the square root of the variance.

- The symbol for variance is either $\mathrm{s}^{2}$ or $\sigma^{2}$.
- The symbol for standard deviation is either s or $\sigma$.

If you are not familiar with the calculation of a variance, study the following example carefully.

## Example

From historical analysis, it has been established that the probable returns on equity investment A are:

| Probability | Return | Expected value <br> (EV) of return <br> $\mathbf{p r}$ |
| :--- | ---: | ---: |
| $\mathbf{p}$ | $\mathbf{r}$ | $\%$ |
|  | $\%$ | 0.4 |
| 0.2 | 2 | 3.0 |
| 0.5 | 6 | 2.4 |
| 0.3 | 8 | -5.8 |
| $\overline{\mathbf{r}}=$ |  |  |

The variance of the returns is calculated as follows:


The variance of the returns is $4.36 \%$ and the standard deviation (the square root of the variance, s ) $=2.09 \%$.

### 1.4 Assessing the investment risk: coefficient of variation

The expected return from the investment is therefore $5.8 \%$ with a standard deviation of $2.09 \%$. The statistical significance of the standard deviation depends on whether the variability in possible returns has a recognisable statistical distribution, such as a standard normal distribution.

However, even when the standard deviation of returns is not normally distributed, the risk in an investment can be assessed by comparing the size of the risk (standard deviation of returns) with the expected return.

The coefficient of variation is the ratio of the standard deviation to the expected return.

Coefficient of variation $=\frac{\text { Standard deviation of expected returns }}{\text { Expected (value of) returns }}$

A high coefficient of variation indicates high investment risk, because the risk is large in relation to the expected returns.

## Example

From historical analysis, it has been established that the probable returns on equity investment B are:

| Probability | Return |  |
| :--- | ---: | ---: |
| $\mathbf{p}$ | $\mathbf{r}$ | Expected value <br> (EV) of return <br> $\mathbf{p r}$ |
|  | $\%$ | $\%$ |
| 0.4 | $(2)$ | $(0.8)$ |
| 0.3 | 7 | 2.1 |
| 0.3 | 15 | 4.5 |
| $\overline{\mathbf{r}}=$ |  | -5.8 |

The variance of the returns is calculated as follows:


The variance of the returns is $50.16 \%$ and the standard deviation (the square root of the variance, $s$ ) $=7.08 \%$.

The coefficient of variation for Investment $B$ is $7.08 / 5.8=1.22$. This is higher than the coefficient of variation for Investment A in the previous example, which has the same expected return of $5.8 \%$ but a coefficient of variation of $2.09 / 5.8=0.36$.

Clearly, the investment risk with Investment B is higher than the risk with Investment A.

## Portfolio theory

- Introduction to portfolio theory
- Diversification to reduce risk: building a portfolio
- Two-asset portfolios
- Expected return and standard deviation of return for a two-asset portfolio


## 2 Portfolio theory

### 2.1 Introduction to portfolio theory

Portfolio theory is concerned with how investors should build a portfolio of investments that gives them a suitable balance between return and investment risk. The theory may seem complex, but it is not often examined. However, portfolio theory provides a theoretical basis for the capital asset pricing model, which is an important model in financial management.

### 2.2 Diversification to reduce risk: building a portfolio

To a certain extent, an investor can reduce the investment risk - in other words, reduce the volatility of expected returns - by diversifying his investments, and holding a portfolio of different investments.

Creating a portfolio of different investments can reduce the variance and standard deviation of returns from the total portfolio, because if some investments provide a lower-than-expected return, others will provide a higher-than-expected return. Extremely high or low returns are therefore less likely to occur.
(Similarly, a company could reduce the investment risk in its business by diversifying, and building a portfolio of different investments. However, it can be argued that there is no reason for a company to diversify its investments, because an investor can achieve all the diversification he requires by selecting a diversified portfolio of equity investments.)

## Diversification to reduce risk: the correlation of investment returns

The extent to which investment risk can be reduced by building a portfolio of different investments depends on the correlation of the returns from the different investments in the portfolio.

- When returns from different investments in a portfolio are positively correlated, this means that when the return from one of the investments is higher than expected, the returns from the other investments will also be higher than expected. Similarly, when returns from one investment are lower than expected, the returns from all the investments in the portfolio will be lower than expected.
- When returns from two different investments in a portfolio are negatively correlated, this means that when the returns from one of the investments is
higher than expected, the returns from the other investment will be lower than expected.
- When returns from two investments are neither positively nor negatively correlated, this means that when the returns from one investment are higher than expected, the returns from the other investment might be either higher or lower than expected.

Investment risk is reduced by building up a portfolio of investments whose returns are negatively correlated, or where correlation is low.

## Correlation coefficient of investment returns

The correlation of the returns from different investments can be measured statistically by a correlation coefficient, from an analysis of historical data about returns (or from probability estimates of future returns).

A correlation coefficient can range in value from +1 (perfect positive correlation) to - 1 (perfect negative correlation).

A correlation coefficient close to zero indicates very little correlation between investment returns.

Investment risk is reduced most effectively by having investments in a portfolio whose returns are negatively correlated, or where there is not much correlation.

You might be required to calculate the correlation coefficient of the returns from two investments in a portfolio (a two-investment portfolio). You will not be required to make any calculations of correlation for a portfolio of more than two investments, but the same principles apply to larger portfolios.

### 2.3 Two-asset portfolios

One way of calculating the correlation coefficient of returns between two investments in a two-asset portfolio is to begin by calculating the covariance of the returns from the two investments.

Covariance of returns from investment $A$ and investment $B\left(\operatorname{cov}_{\mathrm{A}, \mathrm{B}}\right)$
$=\sum \mathrm{p}\left(\mathrm{r}_{\mathrm{A}}-\overline{\mathrm{r}}_{\mathrm{A}}\right)\left(\mathrm{r}_{\mathrm{B}}-\overline{\mathrm{r}}_{\mathrm{B}}\right)$

## Example

There are two investments in a portfolio, investment A and investment B . The probabilities of annual returns from the investments are as follows:

| Probability | Return from <br> investment A | Return from <br> investment B |
| :---: | ---: | ---: |
|  | $\%$ | $\%$ |
| 0.4 | 5.0 | 12.0 |
| 0.4 | 7.0 | 10.0 |
| 0.2 | 11.0 | 1.0 |

The expected value of the return from investment $A$ is $7 \%$ and the expected value of the return from investment B is $9 \%$.
(For investment A, this is calculated as $(0.4 \times 5 \%)+(0.4 \times 7 \%)+(0.2 \times 11 \%)$. For investment B , the expected return is calculated as $(0.4 \times 12 \%)+(0.4 \times 10 \%)+(0.2 \times$ $1 \%)$. )

The covariance of returns is calculated as follows:

| Probability | $\left(\mathbf{r}_{\mathrm{A}}-\overline{\mathbf{r}}_{\mathrm{A}}\right)$ | $\left(\mathbf{r}_{\mathrm{B}}-\overline{\mathbf{r}}_{\mathrm{B}}\right)$ | $\mathbf{p}\left(\mathbf{r}_{\mathrm{A}}-\overline{\mathbf{r}}_{\mathrm{A}}\right)\left(\mathbf{r}_{\mathrm{B}}-\overline{\mathbf{r}}_{\mathrm{B}}\right)$ |
| :--- | ---: | ---: | ---: |
| $\mathbf{p}$ | $\%$ | $\%$ |  |
| 0.4 | -2.0 | +3.0 | -2.4 |
| 0.4 | 0 | +1.0 | 0.0 |
| 0.2 | +4.0 | -8.0 | -6.4 |
|  |  |  | -8.8 |
|  |  |  |  |

## Correlation coefficient of investment returns

The correlation coefficient for the returns of two investments (A and B) in a two asset portfolio is calculated as:

Correlation coefficient, $\rho_{A, B}=\frac{\text { Covariance }_{A, B}}{\sigma_{A} \sigma_{B}}$

This is the covariance of returns from investments $A$ and $B$, divided by [the standard deviation of returns from investment A multiplied by the standard deviation of returns from investment B].

## Example

Continuing the previous example, the standard deviation of returns for investment A is $2.19 \%$ and the standard deviation of returns for investment B is $4.10 \%$. (The workings are not shown here, but you should be able to calculate them, using the formula described earlier.)

The correlation coefficient for the returns from the two investments, is therefore:

Correlation coefficient, $\rho_{\mathrm{A}, \mathrm{B}}=\frac{-8.8}{(2.19)(4.10)}=-0.98$

## Investment returns that are not correlated

When the returns from two investments are not correlated, the value of the correlation coefficient should be 0 .

### 2.4 Expected return and standard deviation of return for a two-asset portfolio

## Expected return from a two-asset portfolio

The expected return from a two-asset portfolio is the weighted average of the returns from the two investments.
$R_{P}=R_{A} x+R_{B}(1-x)$
where:
$\mathrm{R}_{\mathrm{P}}$ is the expected return from the two-asset portfolio
$R_{A}$ is the expected return from investment $A$
$R_{B}$ is the expected return from investment $B$
x is the proportion of investment A in the portfolio (for example, $40 \%=0.40$ )
$(1-x)$ is the proportion of investment A in the portfolio

## Standard deviation of return for a two-asset portfolio

The overall standard deviation of the return from a two-asset portfolio can be calculated using the following formula, which is given to you in the formula sheet for the examination.
$s p=\sqrt{s_{\mathrm{A}}{ }^{2} \mathrm{x}^{2}+s_{\mathrm{B}}{ }^{2}(1-\mathrm{x})^{2}+2 \mathrm{x}(1-\mathrm{x}) \rho_{\mathrm{A}, \mathrm{B}} s_{\mathrm{A}} s_{\mathrm{B}}}$
where:
$x$ is the proportion of the portfolio consisting of investment $A$ ( $1-x$ ) is the proportion of the portfolio consisting of investment $B$ $s_{A}$ and $s_{B}$ are the standard deviation of returns for investment $A$ and investment $B$ $\rho_{A, B}$ is the covariance of returns for investment $A$ and investment $B$

This formula is included in the formula sheet for your examination, with x and (1$x$ ) represented by $w_{a}$ and $w_{b}$ respectively.

## Example

Returning to the previous example, suppose that the portfolio of an investor consists of $70 \%$ of investment A and $30 \%$ of investment B. The expected return is $7 \%$ for investment A and $9 \%$ for investment B .

The expected return from the portfolio is therefore:
$(7 \% \times 70 \%)+(9 \% \times 30 \%)=7.6 \%$.
The standard deviation of the returns from this portfolio would be:

$$
\sqrt{(2.19)^{2}(0.70)^{2}+(4.10)^{2}(0.30)^{2}+2(0.70)(0.30)(-0.98)(2.19)(4.10)}=\sqrt{0.1676}=0.41 \%
$$

The portfolio therefore has an expected return of $7.6 \%$ with a standard deviation of 0.41\%.

The risk in this portfolio is significantly less than the risk from the two individual investments (whose standard deviations are $2.19 \%$ and $4.10 \%$ ). This is largely due to the fact that the returns from the two investments have strong negative correlation.

## Conclusion: diversification to reduce investment risk

The general principle, even when correlation is not negative, is that:

- diversification within a portfolio reduces risk
- this rule applies to portfolios with many different investments, not just to twoinvestment portfolios.


## Example

A multinational company is considering new investments in three countries. An estimate of the expected returns and the investment risk has been made as follows:

| Country | Expected <br> investment <br> return | Investment <br> risk (= s) |
| :--- | :---: | :---: |
| A | $28 \%$ | $36 \%$ |
| B | $24 \%$ | $29 \%$ |
| C | $20 \%$ | $23 \%$ |

The company has decided that it will invest equal amounts in two of the countries, but will not invest in the third. The returns from the investments in each country are not correlated.

## Required

(a) Calculate the expected return, risk and coefficient of variation for each of the possible investment combinations.
(b) Suggest how this analysis can help the company to decide which investment portfolio to select.

## Answer

The returns from the investments in the different countries would not be correlated, therefore the correlation coefficient is 0 . This means that the final term in the formula for the standard deviation of a two-asset portfolio must also be 0 .

Two-asset portfolio
Country A + Country B
Country A + Country C
Country B + Country C

$$
\begin{aligned}
& (0.5 \times 28 \%)+(0.5 \times 24 \%) \\
& (0.5 \times 28 \%)+(0.5 \times 20 \%) \\
& (0.5 \times 24 \%)+(0.5 \times 20 \%)
\end{aligned}
$$

The standard deviation of the returns from each two-asset portfolio is calculated as follows (allowing for the fact that the coefficient of correlation is 0 for each portfolio):

## Two-asset portfolio

$\begin{array}{ll}\text { Country A + Country B } & {\left[\left(0.5^{2} \times 36^{2}\right)+\left(0.5^{2} \times 29^{2}\right)\right]^{1 / 2}} \\ \text { Country A + Country C } & {\left[\left(0.5^{2} \times 36^{2}\right)+\left(0.5^{2} \times 23^{2}\right)\right]^{1 / 2}} \\ \text { Country B + Country C } & {\left[\left(0.5^{2} \times 29^{2}\right)+\left(0.5^{2} \times 23^{2}\right)\right]^{1 / 2}}\end{array}$

## Standard deviation of

 expected return23.1\%
21.4\%
18.5\%

## Two-asset portfolio

Country A + Country B
23.1/26.0

Country A + Country C 21.4/24.0
Country B + Country C 18.5/22.0

Coefficient of variation
0.89
0.89
0.84

This information might be used by the company's management to make the investment decision and select the preferred two-asset portfolio.

- Investing in Country A and Country B would offer the highest expected return. The investment risk, measured simply as the coefficient of variation, indicates that it would have similar risk to a portfolio of Country A and Country C, for which the expected return is lower.
- Investing in Country B and Country C offers the lowest expected return, although the investment risk (measured by the coefficient of variation) appears to be slightly lower.
On the basis of the available information, it seems probable that the company will choose to invest in either Country A and Country B or in Country B and Country C.

The choice of one portfolio in preference to the other will depend on the 'risk appetite' of the company, and whether it is willing to take on the higher investment risk in the hope of obtaining a higher investment return.

## Choosing an investment portfolio

- Investor preferences
- Efficient portfolios and the efficient frontier
- The market portfolio
- Risk-free investments and choosing an investment portfolio
- The market premium
- The beta factor of a portfolio
- Systematic and unsystematic risk


## 3 Choosing an investment portfolio

### 3.1 Investor preferences

Investors will usually avoid risk, in the sense that if they invest in a risky portfolio, they will expect a higher return than from investing in a less risky portfolio. A higher return is required to compensate the investor for the higher risk.

## Example

Suppose that an investor has a choice of selecting any of the following portfolios:

| Portfolio | Expected return | Risk (= $\sigma$ ) |
| :--- | ---: | ---: |
|  | $\%$ | $\%$ |
| A | 6.0 | 4.0 |
| B | 7.0 | 3.0 |
| C | 9.0 | 5.5 |
| D | 8.0 | 6.0 |

- A normal (risk-averse) investor will not select Portfolio A, because Portfolio B offers a higher return than Portfolio A for less risk
- Similarly, a risk-averse investor will not select Portfolio D, because Portfolio C offers a higher return than Portfolio D for less risk
- It is not clear whether a risk-averse investor would select Portfolio B or Portfolio C, because Portfolio C offers a higher return for greater risk, and Portfolio B offers a lower return but less risk.


### 3.2 Efficient portfolios and the efficient frontier

Investors can assemble a wide variety of different portfolios, each consisting of different investments and in differing proportions. However, normal investors will always select a portfolio that offers a higher expected return or a lower risk than other portfolios that could be assembled.

An efficient portfolio is a portfolio that offers:

- a higher expected return than any other portfolio, for a given level of risk, or
- a lower amount of risk than any other portfolio, for a given size of expected return.

Rational, risk-averse investors will choose an efficient portfolio.

## The efficient frontier of portfolios

In theory, it would be possible to prepare a graph showing every possible portfolio that investors might choose, with the expected return from the portfolio plotted on the $y$ axis and the risk of the portfolio, measured as the standard deviation of its expected returns, on the $x$ axis.

This graph of portfolios would be shown as an egg-shaped grouping of portfolios, with differing expected returns and differing risk.

The efficient portfolios all lie on the top edge. A line can be drawn through these efficient portfolios, to obtain the efficient frontier of investment portfolios. A riskaverse investor will always choose a portfolio on this efficient frontier.


## Investor preferences: indifference theory

The next question is whether investors might choose any investment portfolio on this efficient frontier, or whether there is any particular portfolio that will be preferred by all investors, above all the other portfolios.

In portfolio theory, this question is answered using the concept of indifference curves and investor preferences.

Risk-averse investors are prepared to accept higher risk for a higher investment return, but may also choose a lower expected return for lower risk. An investor's preferences for higher returns or less risk can be illustrated in a graph of indifference curves.

## Investors' indifference curves



In this graph, there are three indifference curves, Curve 1, Curve 2 and Curve 3.
■ Curve 1 shows the combinations of risk and return that are of equal merit or attractiveness to the investor. The investor will be indifferent about choosing any portfolio that lies on this curve.

- Similarly, Curve 2 shows the combinations of risk and return that are of equal merit or attractiveness to the investor. The investor will be indifferent about choosing any portfolio that lies on this curve. However, the investor will prefer a portfolio on Curve 2 rather than a portfolio on Curve 1, because portfolios on Curve 2 will offer a higher return for less risk than portfolios on Curve 1 (or lower risk for the same return).
- Curve 3 also shows the combinations of risk and return that are of equal merit or attractiveness to the investor, and the investor will be indifferent about choosing any portfolio on this curve. However, the investor will prefer a portfolio on Curve 3 rather than a portfolio on Curve 2, because portfolios on Curve 3 will offer a higher return for less risk than portfolios on Curve 2 (or lower risk for the same return).

The conclusion is that investors will select a portfolio for investment that lies on an indifference curve as far to the left on the graph as possible.

### 3.3 The market portfolio

We can combine the efficient frontier of portfolios with investors' indifference curves on the same graph. If we do, the graph will show that there is only one portfolio on the efficient frontier that will maximise the satisfaction of investors. This is at point M on the graph below.

This is the point where an indifference curve as far to the left as possible touches the efficient frontier.

Portfolio M is called the market portfolio.

## Market portfolio



The market portfolio is a portfolio of all investments traded on the stock market, in quantities that reflect their relative overall market value, but excluding risk-free investments.

The reason why the market portfolio contains all investments on the stock market is that if it didn't, there would be some investments that no rational, risk-averse investors would ever buy. When investors are not buying particular investments, the market price of the investments will change. Their price will fall, and their expected return will increase. When this happens, they become more attractive to investors, who will want to buy them.

In other words, market forces of supply and demand will ensure that investors are willing to hold a portfolio of all investments in the market, weighted according to the market valuation of the companies.
(Note: In practice, investors do not hold shares in every company on the stock market. However, major investors do hold a sufficiently large number of different investments so that their portfolio closely resembles the market portfolio.)

### 3.4 Risk-free investments and choosing an investment portfolio

A stock market will have some risk-free investments. These are investments with no investment risk. In practice, government bonds denominated in the domestic currency are classified as risk-free investments.

If a portfolio of risk-free investments is shown on a graph with the efficient frontier of portfolios and the market portfolio, it will be shown as a point on the $y$ axis, where risk is zero. This is shown as Portfolio $G$ in the graph below.

## Risk-free investments and the capital market line



The market portfolio does not include risk-free investments. Investors, however, may choose a portfolio consisting partly of the market portfolio and partly of riskfree investments.

A straight line can be drawn from the portfolio that is $100 \%$ risk-free (Portfolio G) to touch the efficient frontier at a tangent. This will be at the market portfolio M. An investor can select any portfolio on this line, such as Portfolio P , to provide a mixture of risk-free investments and the market portfolio investments.

The line that joins Portfolio G and the market portfolio M is called the capital market line. The capital market line (CML) shows all combinations of risk-free investments and market portfolio investments that investors may select.
(Note: The CML extends beyond Portfolio M. Portfolios on this part of the CML are created when an investor uses all his own money and in addition borrows additional funds (at the risk-free rate of interest) to invest in market portfolio investments. This point is not important.)

### 3.5 The market premium

If an investor invests in a portfolio of risk-free assets, he will receive the risk-free rate of return, which is the interest yield on those risk-free assets.

To compensate an investor for investing in the market portfolio, the expected return must be higher than on risk-free investments. The market premium is the difference between the expected return on the market portfolio and a portfolio of risk-free investments.

Market premium $=\mathrm{R}_{\mathrm{m}}-\mathrm{R}_{\mathrm{f}}$
where:
$\mathrm{R}_{\mathrm{m}}$ is the market rate of return (the expected return on the market portfolio)
$R_{f}$ is the risk-free rate of return.

### 3.6 The beta factor of a portfolio

Any other portfolio on the capital market line must offer an expected return in excess of the risk-free rate.

In the previous diagram, the expected return from Portfolio P can be stated as a formula:

Expected return from Portfolio P:
$R_{p}=R_{f}+\beta\left(R_{m}-R_{f}\right)$
where:
$\mathrm{R}_{\mathrm{f}}$ is the expected return from Portfolio P
$\beta$ is a factor that represents the slope of the capital market line (and the element of risk in the portfolio compared with the risk in the market portfolio). It is called the beta factor.

Beta factors are explained in more detail later, in the section on the capital asset pricing model. At this stage, it is sufficient to note the following points:

- A beta factor can be calculated for every portfolio on the capital market line.
- The market portfolio has a beta factor of 1 .

$$
\mathrm{R}_{\mathrm{m}}=\mathrm{R}_{\mathrm{f}}+1\left(\mathrm{R}_{\mathrm{m}}-\mathrm{R}_{\mathrm{f}}\right)
$$

- A portfolio of risk-free investments has a beta factor of 0 .
$\mathrm{R}_{\mathrm{f}}=\mathrm{R}_{\mathrm{f}}+0\left(\mathrm{R}_{\mathrm{m}}-\mathrm{R}_{\mathrm{f}}\right)$
- A portfolio on the capital market line consisting of $50 \%$ market portfolio investments and $50 \%$ risk-free investments has a beta factor of $0.5[=(50 \% \times 1)+$ $(50 \% \times 0)]$.

A beta factor can also be calculated for individual securities (shares or corporate bonds) in the market portfolio. The beta factor of a portfolio of securities is simply the weighted average of the beta factors of all the securities in the portfolio, with weightings to allow for the different market value of each security in the portfolio.

## Example

A pension fund is adding two new investments to the portfolio of investments in its fund.

The existing pension fund consists of securities with a market value of $\$ 800$ million. These have a combined beta factor of 0.70 .

It is now adding an investment of $\$ 100$ million in domestic government bonds and an investment if $\$ 50$ million in equity shares that have a beta factor of 1.4.

## Required

Calculate the beta factor of the enlarged portfolio after the addition of the two new investments.

## Answer

The government bonds are risk-free and have a beta factor of 0 .

| Securities | Value | Beta factor | Value <br> $\times$ Beta |
| :--- | ---: | ---: | ---: |
|  | $\$ \mathrm{~m}$ |  |  |
| Existing fund | 800 | 0.70 | 560 |
| Government bonds | 100 | 0.00 | 0 |
| New equities | 50 | 1.40 | 70 |
|  | 950 |  | 630 |

Beta of enlarged portfolio $=630 / 950=0.663$.

### 3.7 Systematic and unsystematic risk

It was stated earlier that investors can reduce their investment risk by diversifying. However, not all risk can be eliminated, and there will be some risk that cannot be eliminated by diversification.

- When the economy is weak and in recession, returns from the market as whole are likely to fall. Diversification will not protect investors against falling returns from the market as a whole
- Similarly, when the economy is strong, returns from the market as a whole are likely to rise. Investors in all or most shares in the market will benefit from the general increase in returns.

Therefore there are two types of risk:

- Unsystematic risk, which is risk that is unique to individual investments or securities, that can be eliminated through diversification
- Systematic risk, or market risk. This is risk that cannot be diversified away, because it is risk that affects the market as a whole, and all investments in the market in the same way.


## Implications of systematic and unsystematic risk for portfolio investment

The distinction between systematic risk and unsystematic risk has important implications for investment.

- Investors expect a return on their investment that is higher than the risk-free rate of return (unless they invest $100 \%$ in risk-free investments).
- The higher expected return is to compensate investors for the higher investment risk.
- By diversifying, and investing in a wide range of different securities, investors can eliminate unsystematic risk. This is because if some investments in the portfolio perform much worse than expected, others will perform much better. The good-performing and poor-performing investments 'cancel each other out'.
- In a well-diversified portfolio, the unsystematic risk is therefore zero. Investors should therefore not require any additional return to compensate them for unsystematic risk.
- The only risk for which investors should want a higher return is systematic risk. This is the risk that the market as a whole will perform worse or better than expected.

In the earlier diagrams showing the efficient frontier of portfolios and the capital market line, the risk measurement for portfolios on the efficient frontier and the capital market line is systematic risk only.

## Capital asset pricing model (CAPM)

- Systematic risk in securities
- The beta factor of a security
- Formula for the CAPM
- The beta factor of a small portfolio
- Alpha factor
- Advantages and disadvantages of the CAPM
- Using the CAPM for capital investment appraisal
- International CAPM


## 4 Capital asset pricing model (CAPM)

Concepts that are used in portfolio theory can be applied to an analysis of risk in individual securities, such as the shares of individual companies. The capital asset pricing model (CAPM) establishes a relationship between investment risk and expected return for individual securities.

### 4.1 Systematic risk in securities

Systematic risk is risk that cannot be eliminated by diversifying. Every individual security, with the exception of risk-free securities, has some systematic risk.

Since investors can eliminate unsystematic risk through diversification in a portfolio, their only concern should be with the systematic risk of the securities they hold in their portfolio. The return that they expect to receive should be based on their assessment of systematic risk, rather than total risk (systematic + unsystematic risk) in the security.

The systematic risk of a security can be compared with the systematic risk in the market portfolio as a whole.

- A security might have a higher systematic risk than the market portfolio. This means that when the average market return rises, due perhaps to growth in the economy, the return from the security should rise by an even larger amount. Similarly, if the average market return falls due to a deterioration in business conditions, the return from the security will fall by an even larger amount.
- A security might have a lower systematic risk than the market portfolio, so that when the average market return rises, the return from the security will rise, but by a smaller amount. Similarly, when the average market return falls, the return from the security will also fall, but by a smaller amount.

A risk-free security has no systematic risk, because returns on these securities are unaffected by changes in market conditions.

### 4.2 The beta factor ( $\beta$ ) of a security

The systematic risk for an individual security is measured as a beta factor. This is a measurement of the systematic risk of the security, in relation to the systematic risk of the market portfolio as a whole.

The formula for calculating a security's beta factor is as follows:

Beta factor of Security $S\left(\beta_{\mathrm{S}}\right)=\frac{\text { Systematic risk of Security S }}{\text { Systematic risk of the market as a whole }}$
(The 'market as a whole' is the market portfolio.)

There are two ways of re-stating this formula in statistical terms.
Formula 1: $\beta_{S}=\frac{\operatorname{Cov}_{\mathrm{S}, \mathrm{M}}}{\sigma_{\mathrm{M}^{2}}}$

Formula 2: $\beta_{\mathrm{S}}=\frac{\rho_{\mathrm{S}, \mathrm{M}} \times \sigma_{\mathrm{S}}}{\sigma_{\mathrm{M}}}$
where:
$\operatorname{Cov}_{\mathrm{s}, \mathrm{M}}=$ the covariance of returns from Security S with the returns from the market as a whole
$\sigma_{M}$ is the standard deviation of returns for the market portfolio
$\sigma_{S}$ is the standard deviation of returns for Security $S$
$\rho_{\mathrm{S}, \mathrm{M}}$ is the correlation coefficient for returns from Security $S$ with returns from the market as a whole.

## Example

The annual returns on shares in Company Z have a standard deviation of $3.58 \%$ and the standard deviation of market returns is $2.72 \%$. The correlation coefficient for returns on shares in Company Z and returns for the market as a whole is +0.85 .

The beta factor for shares in Company Z is:
$\beta_{Z}=\frac{0.85 \times 3.58}{2.72}=1.12$

### 4.3 Formula for the CAPM

The beta factor for a risk-free security $=0$.

The beta factor for the market portfolio $=1.0$.

When the beta factor for an individual security is greater than 1 , the increase or fall in its expected return (ignoring unsystematic risk) will be greater than any given increase or decrease in the market return.

When the beta factor for a security is less than 1 , the security is relatively low-risk. The expected increase or decrease in its expected return (ignoring unsystematic risk) will be less than any given increase or decrease in the market return.

The formula for the capital asset pricing model is used to calculate the expected return from a security (ignoring unsystematic risk).
$R_{S}=R_{R F}+\beta_{S}\left(R_{M}-R_{R F}\right)$
where:
$R_{S}$ is the expected return from a security $S$
$R_{R F}$ is the risk-free rate of return
$R_{M}$ is the expected market return
$\beta_{S}$ is the beta factor for security $S$.
(This formula is similar to the formula for the capital market line. It is important, and you need to learn it.)

The expected return from an individual security will therefore vary up or down as the return on the market as a whole goes up or down. The size of the increase or fall in the expected return will depend on:

- the size of the change in the returns from the market as a whole, and
- the beta factor of the individual security.


## Example

The risk-free rate of return is $4 \%$ and the return on the market portfolio is $8.5 \%$. What is the expected return from shares in companies $X$ and $Y$ if:

- the beta factor for company X shares is 1.25
- the beta factor for company Y shares is 0.90 ?


## Answer

Expected return, security $X=4 \%+1.25(8.5-4) \%=9.625 \%$.
Expected return, security $\mathrm{Y}=4 \%+0.90(8.5-4) \%=8.05 \%$.

## Example

The following information is available:
Risk-free rate of return $4.0 \%$
Expected market return $\quad 7.5 \%$
Standard deviation of market return $1.4 \%$
Standard deviation of return from shares in company ABC $\quad 2.5 \%$
Correlation coefficient for returns from shares in Company ABC 0.90 and returns from the market as a whole

## Required

(a) Calculate the beta factor for Company ABC shares.
(b) Estimate the returns currently expected from Company ABC shares.

Answer
(a) $\quad \beta_{\mathrm{ABC}}=\frac{0.90 \times 2.5}{1.4}=1.607$
(b) Expected return, shares in Company $\mathrm{ABC}=4 \%+1.607(7.5-4) \%=9.6245 \%$.

### 4.4 The beta factor of a small portfolio

A portfolio of investments containing just a few securities will not be fully representative of the market portfolio. Its systematic risk will therefore be different from the systematic risk for the market as a whole.

The relationship between the systematic risk of a small portfolio and the systematic risk of the market as a whole can be measured as a beta factor for the portfolio.

A beta factor for a portfolio is the weighted average value of the beta factors of all the individual securities in the portfolio. The weighting allows for the relative proportions of each security in the portfolio.

## Example

A portfolio contains five securities. The proportions of each security in the portfolio and the beta factor of each security are as follows:

| Security | Proportion of the <br> portfolio | Beta factor of <br> the security |
| :--- | ---: | ---: |
| 1 | $10 \%$ | 1.20 |
| 2 | $25 \%$ | 0.90 |
| 3 | $15 \%$ | 0.96 |
| 4 | $30 \%$ | 1.15 |
| 5 | $20 \%$ | 1.06 |

The beta factor for the portfolio is calculated as follows:

| Security | Proportion <br> of the portfolio | Beta factor <br> of the security |  |
| :--- | ---: | ---: | ---: |
|  | p | $\beta$ | $\mathrm{p} \beta$ |
| 1 | $10 \%$ | 1.20 | 0.120 |
| 2 | $25 \%$ | 0.90 | 0.225 |
| 3 | $15 \%$ | 0.96 | 0.144 |
| 4 | $30 \%$ | 1.15 | 0.345 |
| 5 | $20 \%$ | 1.06 | 0.212 |
| Beta factor for the portfolio |  |  | 1.046 |

### 4.5 Alpha factor

When shares yield more or less than their expected return (based on the CAPM), the difference is an abnormal return. This abnormal return might be referred to as the alpha factor. The alpha factor for a security is simply the balancing figure in the following formula:
$R_{S}=R_{R F}+\beta_{S}\left(R_{M}-R_{R F}\right)+\alpha_{S}$

## Example

The return on shares of company A is $11 \%$, but its normal beta factor is 1.10 . The risk-free rate of return is $5 \%$ and the market rate of return is $8 \%$.

There is an abnormal return on the shares:
$11 \%=5 \%+1.10(8-5) \%+\alpha$
$\alpha=2.7 \%$.

## Example

An investor tries to buy shares or bonds for his portfolio that provide a positive abnormal return. He is considering two shares and two bonds for adding to his portfolio.

The required return on shares is measured by the Capital Asset Pricing Model (CAPM). The required return for bonds is measured using a model similar to the CAPM, except that the 'beta' for a bond is measured as the ratio of the duration of the bond in years to the duration of the bond market as a whole.

The following information is available:

| Shares | Expected actual <br> return (\%) | Standard <br> deviation of <br> returns | Correlation <br> coefficient of returns <br> with the market |
| :--- | ---: | ---: | ---: |
| Equity market | 11.0 | 12 | 1.00 |
| Company X | 9.5 | 14 | 0.92 |
| Company Y | 12.0 | 17 | 0.83 |
| Bonds |  |  |  |
| Bond market | Duration (years) | Coupon (\%) | Redemption yield (\%) |
| Bond P | 6.0 | - | 6.2 |
| Bond Q | 5.0 | 7 | 6.4 |
|  | 8.5 | 6 | 6.5 |

The risk-free rate of return is $5 \%$.

## Required

Identify which of these investments currently offers a positive abnormal return.

## Answer

The first step is to calculate beta factors for the shares and the similar factors for the bonds.

For shares, the beta factor is calculated as the correlation coefficient multiplied by the standard deviation of returns for the share, divided by the standard deviation of market returns.

| Security |  | Beta factor |
| :--- | :---: | ---: |
| Share X | $(0.92 \times 14) / 12$ | 1.073 |
| Share Y | $(0.83 \times 17) / 12$ | 1.176 |
| Bond P | $(5.0 / 6.0)$ | 0.833 |
| Bond Q | $(8.5 / 6.0)$ | 1.417 |

We can now calculate the required return for each security (using the CAPM) and compare it with the expected actual returns. The difference is the abnormal return. For bonds, the redemption yield should be used as the measure of return.

| Security |  | Required <br> return | Actual expected <br> return | Abnormal <br> return |
| :--- | ---: | ---: | ---: | ---: |
|  |  | $\%$ | $\%$ | $\%$ |
| Share X | $5 \%+1.073(11-5) \%$ | 11.44 | 9.50 | -1.94 |
| Share Y | $5 \%+1.176(11-5) \%$ | 12.06 | 12.00 | -0.06 |
| Bond P | $5 \%+0.833(6.2-5) \%$ | 6.00 | 6.40 | +0.40 |
| Bond Q | $5 \%+1.417(6.2-5) \%$ | 6.70 | 6.50 | -0.20 |

Only bond P offers a positive abnormal return. If the investor makes investment decisions on the basis of abnormal returns, he will invest in this bond and add it to his portfolio. (However, the abnormal return could be eliminated by a rise in the price of the bond.)

### 4.6 Advantages and disadvantages of the CAPM

The CAPM is based on some simplifying assumptions. For example, the CAPM assumes:

- A perfect capital market
- Uniformity of investor expectations
- All forecasts (expectations) are made in the context of just one time period.

In spite of these simplifying assumptions, the CAPM appears to be reliable in practice.

| Advantages of CAPM | Weaknesses of CAPM |
| :--- | :--- |
| It provides a measurable relationship between | It can be difficult to estimate values for the risk- <br> free rate and market rate of return, and the beta <br> factor for a security. |
| risk and return. | It focuses on systematic risk only and ignores <br> unsystematic risk. Unsystematic risk is <br> significant for an investor who does not have a <br> well-diversified portfolio. |
| It can be used to estimate the cost of capital for |  |
| securities, notably equity shares. | It makes no distinction between the ways in |
| It can be adjapted to establishing a required (risk- |  |
| company. |  |

### 4.7 Using the CAPM for capital investment appraisal

The CAPM can be used by companies for capital investment appraisal. Some types of capital investment projects are more risky than others. For example, the systematic risk of investing in the manufacture of cars may be higher than the systematic risk of investing in a retailing business. Similarly, the risk of investing in one country may be higher than the risk of investing in another country, due to differences in the business environment or economic conditions.

This suggests that the expected returns from particular investments should be adjusted to allow for differences in systematic risk.

If a beta factor for a particular project can be established, a risk-adjusted cost of capital can be applied to the project. This risk-adjusted cost of capital should then be used to calculate the project NPV.

For example, if a project's beta factor is 1.30 , the risk-free rate of return is $4.5 \%$ and the market rate of return is 7\%, the discount rate used to calculate the project NPV would be:

$$
\begin{aligned}
& 4.5 \%+1.30(7-4.50) \% \\
& =7.75 \%, \text { say } 8 \%
\end{aligned}
$$

### 4.8 International CAPM

The capital asset pricing model is usually applied to a single stock market, and the market portfolio represents all the securities that are traded on that market. However, the same general principles of the CAPM can be applied to international investment.

International investors build a portfolio of investments that enables them to invest world-wide. The 'world market' consists of all the main securities, such as company shares, traded in different stock markets throughout the world.
International investors may wish to monitor returns from the 'world market', and may wish to assess returns from individual securities by comparing their return with the expected return from the world market as a whole.

An international capital asset pricing model for individual securities would be:
$R_{S}=R_{R F}+\beta_{S}\left(R_{W M}-R_{R F}\right)$
where $\mathrm{R}_{\mathrm{Wm}}$ is the average return from the world market as a whole.

## Problems with the international CAPM

There are several problems with using an international CAPM.

- A risk-free rate of return has to be selected, but which securities are risk-free? Returns on government securities vary between different countries. A risk-free security for an international CAPM might be the returns on government securities in a stable currency.
- The world market portfolio has to be identified. The world market may be based on an international (world market) stock index, but there may be different views about what should be in the portfolio.
- In practice, it is difficult for many investors to acquire a portfolio of investments that represents the world market. However, many institutional investors do attempt to do this, by:
- investing directly in stock markets around the world
- investing in international (global) companies, and
- possibly also investing in collective investments, such as mutual funds and open-ended investment companies (Oeics).


## Cost of capital

|  | Contents |
| :--- | :--- |
| 1 | Weighted average cost of capital (WACC) |
| 2 | Cost of equity |
| 3 | Cost of debt capital |
| 4 | Calculating the WACC |
| 5 | Cost of capital and gearing |
| 6 | Change in gearing, the WACC and capital <br> investment appraisal <br> 7 |
| Asset beta |  |

Weighted average cost of capital (WACC)

- Cost of equity, cost of debt and the weighted average cost of capital (WACC)
- WACC and capital investment appraisal
- Comparing the cost of equity and the cost of debt


## 1 Weighted average cost of capital (WACC)

### 1.1 Cost of equity, cost of debt and the weighted average cost of capital (WACC)

The cost of capital for investors is the return that investors require from their investment. Companies must be able to make a sufficient return from their own capital investments to pay the returns required by their shareholders and holders of debt capital. The cost of capital for investors therefore establishes a cost of capital for companies.

- For each company there is a cost of equity. This is the return required by its shareholders, in the form of dividends or share price growth
- There is a cost for each item of debt finance. This is the yield required by the lender or bond investor
- When there are preference shares, there is also a cost of preference share capital, which is the dividend yield required by the shareholders.

The cost of capital for a company is the return that it must make on its investments so that it can afford to pay its investors the returns that they require.

The cost of capital for investors and the cost of capital for companies should theoretically be the same. However, they are different because of the differing tax positions of investors and companies.

- The cost of capital for investors is measured as a pre-tax cost of capital. This is a return ignoring taxation.
- The cost of capital for companies recognises that interest costs are an allowable expense for tax purposes, and the cost of debt capital to a company should allow for the tax relief that companies receive on interest payments, reducing their tax payments. The cost of debt capital for companies is measured as an after-tax cost.

The weighted average cost of capital is the average cost of all the sources of capital that a company uses. This average is weighted, to allow for the relative proportions of the different types of capital in the company's capital structure.

## Example: pre-tax and after-tax cost of debt

A company has a bank loan of $\$ 100,000$ on which it pays interest at $8 \%$. The rate of tax on company profits is $25 \%$.

Gross interest on the loan each year is $\$ 8,000$ and the pre-tax cost of the debt is $8 \%$. This means that the company needs to make a profit of $\$ 8,000$ each year before tax and interest in order to cover the interest cost of the debt. Tax relief on the interest is $\$ 2,000$, which means that the tax charge for the company is reduced by $\$ 2,000$ each year. The company therefore needs to make a profit after tax of $\$ 6,000$ to cover the cost of the debt interest, and the after-tax cost of the debt is just $6 \%$. The $\$ 6,000$ after tax profit plus the reduction in the tax charge of $\$ 2,000$ together provide the return of $\$ 8,000$ that is needed to cover the cost of the debt interest.

The after tax cost of debt is simply:

- the pre-tax cost of debt
- multiplied by a factor $(1-\mathrm{t})$ where t is the rate of tax on profits.


### 1.2 WACC and capital investment appraisal

One approach to the evaluation of capital investments by companies is that all their investment projects should be expected to provide a return equal to or in excess of the WACC. If all their investment projects earn a return in excess of the WACC, the company will earn sufficient returns overall to meet the cost of its capital and provide its investors with the returns they require.

This principle is often applied in practice. The general rule is that when capital investment projects are evaluated using the NPV method, the cost of capital to be used is the WACC. This is on the assumption that the capital project will not alter the risk profile of the company's investments and the risk with the new project is similar to the risks with the rest of the company's business operations.

The principle is often applied when the financing for a new capital investment changes the company's WACC. On the assumption that the capital project will not alter the risk profile of the company's investments, the NPV of the new project should be calculated using the new WACC that will exist after the project has been undertaken and financed.

An alternative approach to the evaluation of capital investment projects, which does not use these assumptions, is the adjusted present value method or APV method. (This is explained in another chapter.)

### 1.3 Comparing the cost of equity and the cost of debt

For investors and for companies, the cost of their equity is always higher than the cost of their debt capital. This is because equity investment in a company is always more risky than investment in the debt capital of the same company.

In addition, from a company's perspective, the cost of debt is also reduced by the tax relief on interest payments. This makes debt finance even lower than the cost of equity.
The effect of more debt capital, and higher financial gearing, on the WACC is considered in more detail later.

## Cost of equity

- Methods of calculating the cost of equity
- Cost of equity: the dividend valuation model method
- Cost of equity: the dividend growth model method
- Gordon's growth approximation
- Cost of equity: CAPM method


## 2 Cost of equity

### 2.1 Methods of calculating the cost of equity

There are several methods for estimating the cost of equity in a company. These are the dividend valuation model method, the dividend growth model method and the CAPM method. The three methods are described briefly in this section, though you might be familiar with them already.

### 2.2 Cost of equity: the dividend valuation model method

If it is assumed that future annual dividends are expected to remain constant into the foreseeable future, the cost of equity can be calculated as follows:
$K_{E}=\frac{d}{M V}$
where:
$\mathrm{K}_{\mathrm{E}}$ is the cost of equity
$\mathrm{d}=$ the expected future annual dividend
MV is the share price ex dividend. The ex dividend share price is a price that excludes any dividend that has been declared and is payable in the near future.

## Example

A company's shares are currently valued at $\$ 11.70$ and the company is expected to pay an annual dividend of $\$ 1.40$ per share for the foreseeable future. The cost of equity in the company can therefore be estimated as:
$(1.40 / 11.70)=0.1197$ or $11.97 \%$, say $12 \%$.

### 2.3 Cost of equity: the dividend growth model method

If it is assumed that the annual dividend will grow at a constant percentage rate into the foreseeable future, the cost of equity can be calculated as follows:
$K_{E}=\frac{d(1+g)}{M V}+g$
where:
$K_{E}$ is the cost of equity
$\mathrm{d}=$ the annual dividend for the year that has just ended
g is the annual growth rate in dividends, expressed as a proportion $(8 \%=0.08$, etc.)
MV is the share price ex dividend
$\mathrm{d}(1+\mathrm{g})$ is therefore the expected dividend next year.

## Example

A company's share price is $\$ 11.70$. The company has just paid an annual dividend of $\$ 1.40$ per share, and the dividend is expected to grow by $3 \%$ into the foreseeable future.

The cost of equity in the company can be estimated as follows:
$\mathrm{K}_{\mathrm{E}}=\frac{1.40(1.03)}{11.70}+0.03$
$=0.153=15.3 \%$.

### 2.4 Gordon's growth approximation

A problem with the dividend growth model as a method of calculating the cost of equity is that estimates of future dividend growth might be unreliable.

Gordon's growth approximation is a method of estimating what the future rate of growth might be. It is based on the assumption that a company pays dividends out of profits (earnings). Growth in future profits, and so future growth in dividends is achieved by reinvesting some of the current profits. The reinvested profits earn additional earnings which can then be used to pay higher dividends.

The same principle might be applied to reinvestment of free cash flows rather than reinvestment of profits. However, the concept is the same: dividend growth is achieved by reinvesting some of the returns that could otherwise be paid as current year dividends.

Gordon's growth approximation is an estimate of future dividend growth, expressed by the formula:
$\mathrm{g}=\mathrm{br}_{\mathrm{e}}$
where
$\mathrm{g}=$ the annual rate of dividend growth
$b=$ the proportion of earnings (or free cash flow) reinvested for growth, and
$r_{e}=$ the rate of return on those reinvested earnings (a rate of return on equity since the reinvested earnings represent equity profits).

## Example

A company reported profits after interest and tax of $\$ 6$ million and paid dividends of $\$ 4$ million. This ratio of dividend payments to earnings is fairly typical of the company's dividend policy. The company's cost of equity is $12 \%$.

The proportion of profits reinvested for growth is $0.33(2 / 6)$.
An estimate of the future growth rate in annual dividends, using Gordon's growth approximation, is:
$0.33 \times 0.12=0.04$ or $4.0 \%$.

## Example

Gordon's growth approximation can also be applied to free cash flows, as the following example shows:

A company has a cost of equity of $10 \%$. The company's cash flows for the financial year just ended are as follows:

|  | \$ million |
| :--- | ---: |
| Net cash inflow from operating activities | 84 |
| Interest payments less interest receipts | $(17)$ |
| Taxation paid | $(23)$ |
| Capital expenditure | 44 |
| Financing cash flows: repayment of debt | $(21)$ |
| Increase in cash for the year | $(16)$ |

Free cash flow to equity will be defined as the net cash inflow from operating activities less net interest payments and less payments of tax, but before reinvestment. Here, the free cash flow to equity (FCFE) is $\$ 44$ million.

The rate of reinvestment is assumed to be the total amount of capital expenditure in the year (net of disposal proceeds), which is $\$ 31$ million.

We can estimate the rate of reinvestment of cash flows that could otherwise be paid as dividends as: $21 / 44=0.4773$.

An estimate of the future growth rate in annual dividends, using Gordon's growth approximation, is:
$0.4773 \times 0.10=0.04773$, say $4.8 \%$.

Because the estimate is only an approximation, this growth estimate might be rounded to 5\% per year.

### 2.5 Cost of equity: CAPM method

Another approach to calculating the cost of equity in a company is to use the CAPM and the equity beta for the company's shares.
$K_{E}=R_{R F}+\beta\left(R_{M}-R_{R F}\right)$
where
$\mathrm{K}_{\mathrm{E}}=$ the cost of equity in the company
$\mathrm{R}_{\mathrm{RF}}=$ the risk-free rate of return
$\mathrm{R}_{\mathrm{M}}=$ the return on the market portfolio of securities that are not risk-free $\beta=$ the beta factor for the company's equity.

The CAPM method of estimating the cost of equity is an alternative to a dividendbased estimate using the dividend growth model. The two methods will normally produce differing estimates.

## Example

A company's shares have a current market value of $\$ 13.00$. The most recent annual dividend has just been paid. This was $\$ 2.00$ per share.

## Required

Calculate the cost of equity in this company in each of the following circumstances:
(a) The annual dividend is expected to remain $\$ 2.00$ into the foreseeable future.
(b) The annual dividend is expected to grow by $2 \%$ each year into the foreseeable future
(c) The CAPM is used, the equity beta is 1.20 , the risk-free cost of capital is $5 \%$ and the expected market return is $9 \%$.

Answer
(a) Cost of equity $=\frac{2.00}{25.00}=0.08$ or $8.0 \%$.
(b) Cost of equity $=\frac{2.00(1.02)}{25.00}+0.02=0.1016$ or $10.16 \%$.
(c) Cost of equity $=5 \%+1.20(9-5) \%=9.8 \%$.

## Cost of debt capital

- Cost of variable rate debt (floating rate debt)
- Cost of irredeemable fixed rate debt (perpetual bonds)
- Cost of redeemable fixed rate debt (redeemable fixed rate bonds)
- Cost of preference shares
- The yield curve (term structure of interest rates)
- The yield curve and non-risk-free debt: spreads


## 3 Cost of debt capital

Each item of debt finance for a company has a different cost. This is because debt capital has differing risk, according to whether the debt is secured, whether it is senior or subordinated debt, and the amount of time remaining to maturity.

### 3.1 Cost of variable rate debt (floating rate debt)

The pre-tax cost of variable rate debt (also called floating rate debt), such as the cost of a bank loan, is the current interest rate payable on the debt.

The after-tax cost of variable rate debt is the pre-tax cost multiplied by a factor ( $1-\mathrm{t}$ ), where $t$ is the rate of tax on company profits.

For example, suppose that a company is currently paying interest at $7.5 \%$ on its bank loan of $\$ 10$ million, and the rate of tax on company profits is $30 \%$. The pre-tax cost of the debt is $7.5 \%$ and the after-tax cost is $7.5(1-0.30)=5.25 \%$.

For the purpose of calculating a weighted average cost of capital, the cost of the debt should be its after-tax cost of $5.25 \%$ and its market value (for the purpose of weighting the cost of capital) would be $\$ 10$ million, which is the amount of the loan.

### 3.2 Cost of irredeemable fixed rate debt (perpetual bonds)

The cost of irredeemable fixed rate bonds, which might be described as perpetual bonds, is calculated as follows:

Pre-tax cost

## Post-tax cost

$K_{D}=\frac{i}{M V}$
$K_{D}=\frac{i(1-t)}{M V}$
where:
$\mathrm{K}_{\mathrm{D}}$ is the cost of the debt capital
$i$ is the annual interest payable on each $\$ 100$ (nominal value) of the bonds.
$t$ is the rate of tax on company profits.

MV is the market value of $\$ 100$ nominal value of bonds, excluding any interest currently payable.
For example, suppose that the coupon rate of interest on some irredeemable bonds is $6 \%$ and the market value of the bonds is 103.60 . The tax rate is $25 \%$.
(a) The pre-tax cost of the debt is $6 / 103.60=0.058$ or $5.8 \%$.
(b) The after-tax cost of the bonds is $6(1-0.25) / 103.60=0.043$ or $4.3 \%$.

### 3.3 Cost of redeemable fixed rate debt (redeemable fixed rate bonds)

The cost of redeemable bonds is their redemption yield. This is calculated as the rate of return that equates the present value of the future cash flows payable on the bond (to maturity) with the current market value of the bond. In other words, it is the IRR of the cash flows on the bond to maturity, assuming that the current market price is a cash outflow.

A problem arises with calculating the pre-tax and the after-tax cost of redeemable bonds, because the redemption of the principal at maturity is not an allowable expense for tax purposes. The post-tax cost of redeemable debt could therefore be calculated in either of two ways. Each gives a different cost of capital:

- Method 1. Calculate the pre-tax cost of debt (the IRR of the cash flows ignoring debt) and then apply the factor $(1-t)$ to reach the post-tax cost of debt.
- Method 2. Calculate the post-tax cost of debt as the IRR of the future cash flows, allowing for tax relief on the interest payments and the absence of tax relief on the principal repayment.

If in doubt, use Method 2. However, Method 1 might be more appropriate if it is assumed that the company will replace the redeemable debt at maturity with a new issue of similar debt capital. So either Method 1 or Method 2 might be valid. Read the requirements of the question carefully, to see whether you are given any instructions about which method to use.

## $\bigcirc$ <br> Example

The current market value of a company's 7\% loan stock is 96.25 . Annual interest has just been paid. The bonds will be redeemed at par after four years. The rate of taxation on company profits is $30 \%$.

## Required

Calculate the after-tax cost of the bonds for the company.

## Answer

(a) Method 1

| Year | Cash flow |  | Try 8\% |  | Try 10\% |  |
| :--- | :--- | ---: | :---: | ---: | ---: | ---: |
|  |  |  | Discount factor | PV | Discount factor | PV |
| 0 | Market value | $(96.25)$ | 1.000 | $(96.25)$ | 1.000 | $(96.25)$ |
| 1 | Interest | 7.00 | 0.926 | 6.48 | 0.909 | 6.36 |
| 2 | Interest | 7.00 | 0.857 | 6.00 | 0.826 | 5.78 |
| 3 | Interest | 7.00 | 0.794 | 5.56 | 0.751 | 5.26 |
| 4 | Interest | 7.00 | 0.735 | 5.15 | 0.683 | 4.78 |
| 4 | Redemption | 100.00 | 0.735 | $\underline{73.50}$ | 0.683 | 68.30 |
|  | NPV |  |  | $\underline{+0.44}$ |  | $(5.77)$ |
|  |  |  |  |  |  |  |

Using interpolation, the before-tax cost of the debt is:
$8 \%+\frac{0.44}{(0.44+5.77)} \times(10-8) \%=8.14 \%$

The after-tax cost of the debt is therefore estimated as $8.14 \% \times(100-30)=5.698 \%$ or 5.70\%.
(Note: deciding which cost of capital to try first. If you don't know which cost of capital to try first, calculate the average annual net cash flow as a percentage of the current market value. Here the total annual net cash inflows $=7+7+7+107-96.25$ $=31.75$ which averages 7.9375 each year. As a percentage of the market value 96.25, this is $8.2 \%$. So try $8 \%$ first).
(b) Method 2

It is assumed here that tax savings on interest payments occur in the same year as the interest payments.

| Year | Cash flow |  | Try 6\% |  | Try 5\% |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | Discount factor | PV | Discount factor | PV |
| 0 | Market value | (96.25) | 1.000 | (96.25) | 1.000 | (96.25) |
| 1 | Interest less tax | 4.90 | 0.943 | 4.62 | 0.952 | 4.66 |
| 2 | Interest less tax | 4.90 | 0.890 | 4.36 | 0.907 | 4.44 |
| 3 | Interest less tax | 4.90 | 0.840 | 4.12 | 0.864 | 4.23 |
| 4 | Interest less tax | 4.90 | 0.792 | 3.88 | 0.823 | 4.03 |
| 4 | Redemption | 100.00 | 0.792 | 79.20 | 0.823 | 82.30 |
|  | NPV |  |  | (0.07) |  | +3.41 |

Using interpolation, the after-tax cost of the debt is:
$5 \%+\frac{3.41}{(3.41+0.07)} \times(6-5) \%=5.98 \%$, say $6.0 \%$.

If in doubt, use Method 2.

### 3.4 Cost of preference shares

For irredeemable preference shares, the cost of capital is calculated in the same way as the cost of equity, assuming a constant annual dividend.

For redeemable preference shares, the cost of the shares is calculated in the same way as the pre-tax cost of irredeemable debt. (Dividend payments are not subject to tax relief, therefore the cost of preference shares is calculated ignoring tax, just as the cost of equity ignores tax.)

### 3.5 The yield curve (term structure of interest rates)

The cost of new debt can be estimated by reference to a yield curve.
The cost of fixed-rate debt is commonly referred to as the 'interest yield'. The interest yield on debt capital varies with the remaining term to maturity of the debt.

- As a general rule, the interest yield on debt increases with the remaining term to maturity. For example, it should normally be expected that the interest yield on a fixed-rate bond with one year to maturity/redemption will be lower than the yield on a similar bond with ten years remaining to redemption. Interest rates are normally higher for longer maturities to compensate the lender for tying up his funds for a longer time.
- When interest rates are expected to fall in the future, interest yields might vary inversely with the remaining time to maturity. For example, the yield on a oneyear bond might be lower than the yield on a ten-year bond when rates are expected to fall in the next few months.
- When interest rates are expected to rise in the future, the opposite might happen, and yields on longer-dated bonds might be much higher than on shorter-dated bonds.

Interest yields on similar debt instruments can be plotted on a graph, with the $x$-axis representing the remaining term to maturity, and the $y$-axis showing the interest yield. This type of graph, showing the 'term structure of interest rates', is called a yield curve.

- As indicated above, a normal yield curve slopes upwards, because interest yields are normally higher for longer-dated debt instruments.
- However, on occasions, the yield curve might slope downwards, when it is said to be 'negative' or 'inverse'.
- Sometimes it might slope upwards, but with an unusually steep slope (steeply positive yield curve).


Inverse (negative) yield curve


When the yield curve is inverse, this is usually an indication that the markets expect short-term interest rates to fall at some time in the future.

When the yield curve has a steep upward slope, this indicates that the markets expect short-term interest rates to rise at some time in the future.

Yield curves are widely used in the financial services industry. Two points that should be noted about a yield curve are that:

- Yields are gross yields, ignoring taxation (pre-tax yields).
- A yield curve is constructed for 'risk-free' debt securities, such as government bonds. A yield curve therefore shows 'risk-free yields'.
As the name implies, risk-free debt is debt where the investor has no credit risk whatsoever, because it is certain that the borrower will repay the debt at maturity. Debt securities issued in their domestic currency by the government should always be risk-free: yield curves are therefore constructed for government bonds.


### 3.6 The yield curve and non-risk-free debt: spreads

The interest yield on other debt, such as corporate bonds and loans, is higher than the yield on risk-free debt with the same maturity. For example, the interest rate on a sterling bond of ABC Company with two-years to maturity will be higher than the interest yield on a two-year UK government bond. The higher yield is to compensate investors in corporate bonds for the fact that the debt is not risk-free. The company might default.
'Spread' is the difference between the risk-free rate of return (the yield curve) and the cost of debt for the same maturity that is not risk-free. For example, if the riskfree return on five-year government bonds is $5.4 \%$ and the spread for a company's five-year bonds is 80 basis points, the yield on the company bonds is:

- Yield curve + Spread
- $=5.40 \%+0.80 \%=6.20 \%$.

Note: 1 basis point $=0.01 \%$ and 100 basis points $=1 \%$, so $80 \mathrm{bp}=0.80 \%$.

## The size of spreads

The size of the spread allows for the additional risk in the debt that is not risk-free. The spread is therefore higher for debt that has a higher risk for investors or lenders. Many large companies are given a credit rating by a credit rating agency, such as Moody's, Standard \& Poor's and Fitch. (Strictly, the company's debt is given a credit rating, but it is common to speak of companies having a credit rating rather than the debt having a credit rating.)

- The top credit rating is a 'triple- $\mathrm{A}^{\prime}$ credit rating.
- Spreads are lowest for the top credit ratings, and higher for lower credit ratings.


## Credit ratings

Each credit rating agency uses its own credit rating system. The most well-known are the rating systems of Standard \& Poor's and Moody's. Their ratings for bonds are set out in the table below.

| Standard \& Poor's credit ratings |  | Moody's credit ratings |
| :---: | :---: | :---: |
| Investment grade |  |  |
| AAA | Highest rating | Aaa |
| AA | Still high quality debt | Aa |
| A |  | A |
| BBB |  | Baa |
| Sub-investment grade ('junk') |  |  |
| BB | Major uncertainties about the ability of the borrower to pay interest and repay principal on time | Ba |
| B |  | B |
| CCC |  | Caa |
| CC |  | Ca |
| C |  | C |
| D | In default |  |

Standard \& Poor's credit ratings are also modified by ' + ' and ' - ' signs. A ' + ' sign indicates a better credit rating and a ' - ' indicates a lower credit rating.

- Credit ratings are therefore $\mathrm{AAA}, \mathrm{AA}+\mathrm{AA}, \mathrm{AA}-, \mathrm{A}+, \mathrm{A}, \mathrm{A}-\mathrm{BBB}+, \mathrm{BBB}, \mathrm{BB}-$, $B B+, B B, B B-$ and so on.
- The lowest investment grade credit rating is BBB-.

Moody's credit ratings are modified in a similar way, but using the numbers 1, 2 and 3.

- Credit ratings are therefore Aaa, Aa1, Aa2, Aa3, A1, A2, A3, Baa1, Baa2, Baa3, $\mathrm{Ba} 1, \mathrm{Ba} 2, \mathrm{Ba} 3$ and so on.
- The lowest investment grade rating is Baa3.

Sub-investment grade debt, also called 'junk bonds', is a speculative investment for the lender or bondholder, and yields required by investors are normally much higher than on investment grade debt.

## Spreads and credit ratings

Spreads vary according to:

- the risk characteristics of the industry
- the time remaining to maturity for the debt, and
- the credit rating.


## Example

Yield spreads on US bonds for companies in the construction industry are as follows:

| Spreads: | Years to maturity |  |  |  |  |  |
| :--- | :--- | :--- | :---: | :---: | :---: | :---: |
| Rating | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{5}$ | $\mathbf{7}$ | $\mathbf{1 0}$ |
| AAA/ Aaa | 2 | 4 | 10 | 15 | 20 | 25 |
| AA+/Aa1 | 6 | 10 | 16 | 24 | 30 | 38 |
| AA/Aa2 | 9 | 15 | 24 | 34 | 44 | 55 |
| AA-/Aa3 | 15 | 24 | 30 | 40 | 52 | 64 |
| A+/A1 | 24 | 35 | 45 | 60 | 75 | 88 |
| A/A2 | 32 | 45 | 58 | 78 | 95 | 112 |
| A-/A3 | 45 | 60 | 75 | 100 | 120 | 142 |

This table would show, for example, that if a company wants to issue new sevenyear bonds, and the credit rating for the bonds is expected to be AA-, the company will expect to pay a yield on the bonds that is 52 basis points above the risk-free rate. If the yield curve shows the risk-free rate on US government bonds ('Treasuries') to be $6.6 \%$, the yield on the company's bonds will be $6.6 \%+0.52 \%=$ 7.12\%.

## Calculating the WACC

- Method of calculating the WACC
- WACC and market value


## 4 Calculating the WACC

### 4.1 Method of calculating the WACC

The WACC is a weighted average of the (after-tax) cost of all the sources of capital for the company. It is calculated as follows:

| Source of finance | Market value | $\times$ | Cost | Market value $\times$ Cost |
| :---: | :---: | :---: | :---: | :---: |
|  |  |  | r | $\mathrm{MV} \times \mathrm{r}$ |
| Equity | MVE | $\times$ | $\mathrm{K}_{\mathrm{E}}$ | $\mathrm{MV}_{\mathrm{E}} \times \mathrm{K}_{\mathrm{E}}$ |
| Preference shares | MV | $\times$ | $\mathrm{K}_{\mathrm{P}}$ | $\mathrm{MV}_{\mathrm{P}} \times \mathrm{K}_{\mathrm{P}}$ |
| Debt | MV ${ }_{\text {D }}$ | $\times$ | $K_{\text {D }}$ | $M V_{D} \times K_{\text {d }}$ |
| Total | $\Sigma \mathrm{MV}$ |  |  | $\Sigma \mathrm{MV} \mathrm{x} \mathrm{r}$ |
| WACC = | $\underline{\Sigma M V ~ x ~ r ~}$ |  |  |  |
|  | EMV |  |  |  |

The WACC for a company is found using the method shown above. $K_{D}$ is the aftertax cost of debt capital. If there is more than one source of debt, each with a different cost, you can either:

- have a separate line in the table for each item of debt, or
- calculate a weighted average cost of all debt capital first (after-tax), then have just one line in the table with the total market value of all debt capital and the weighted average after-tax cost of the debt.


## Example

A company has 8 million shares each with a value of $\$ 7.90$, whose cost is $8.4 \%$. It has $6 \%$ bonds with a market value of $\$ 50$ million and an after-tax cost of $3.6 \%$. It has a bank loan of $\$ 10$ million whose after-tax cost is $4.1 \%$. It also has 2 million $8 \%$ preference shares of $\$ 1$ whose market price is $\$ 1.33$ per share and whose cost is $6 \%$. Calculate the WACC.

## Answer

| Source of finance | Market value | Cost | Market value $\times$ Cost |
| :--- | ---: | ---: | ---: |
|  | $\$$ million | $\mathrm{r} \%$ | $\mathrm{MV} \times \mathrm{r}$ |
| Equity | 63.20 | 8.4 | 530.88 |
| Preference shares | 2.66 | 6.0 | 15.96 |
| Bonds | 50.00 | 3.6 | 180.00 |
| Bank loan | 10.00 | 4.1 | 41.00 |
|  | 125.86 |  | 767.84 |

$W A C C=\frac{767.84}{125.86}=6.10 \%$

## Example

You should be prepared in your examination to calculate the cost of equity in a company, given the WACC, the cost of debt and the market value of equity and debt. Here is an example.

A company has a weighted average cost of capital of $8.2 \%$. It has 100 million shares currently valued at $\$ 6$ each, and it has a bank loan of $\$ 200$ million on which the rate of interest is $6.5 \%$. The rate of taxation is $30 \%$.

## Required

Calculate the current cost of equity in the company.

## Answer

The cost of equity can be calculated as the solution to a mathematical problem.
Let the cost of equity be r .
The after-tax cost of debt is $6.5 \% \times(1-0.30)=4.55 \%$.
The WACC , which is $8.2 \%$, is calculated as follows:

| Source of finance | Market <br> value | Cost | Market value <br> $\times$ Cost |
| :--- | ---: | ---: | ---: |
|  | $\$$ million | $\%$ | MV $\times \mathrm{r}$ |
| Equity | 600.0 | r | 600 r |
| Debt | 200.0 | 4.55 | 910.0 |
|  | 800.0 |  | $600 \mathrm{r}+910$ |

$W A C C=8.2 \%=\frac{600 r+910}{800} \%$
$8.2(800)=600 r+910$
$600 \mathrm{r}=5,650$
$r=9.417 \%$
The cost of equity is $9.417 \%$, say $9.4 \%$.

### 4.2 WACC and market value

For a company with constant annual 'cash profits', there is an important connection between WACC and market value. (Note: 'Cash profits' are cash flows generated from operations, before deducting interest costs.)

If we assume that annual cash profits are a constant amount in perpetuity, the total value of a company, equity plus debt capital, is calculated as follows:

Total market value of the company $=\frac{\text { Annual cash profits }}{\text { WACC }}$
From this formula, the following conclusions can be made:

- The lower the WACC, the higher the total value of the company will be (equity + debt capital), for any given amount of annual profits.
- Similarly, the higher the WACC, the lower the total value of the company.

For example, if annual cash profits are $\$ 12$ million, the total market value of the company would be:

- $\$ 100$ million if the WACC is $12 \%$ ( $\$ 12$ million/0.12)
- $\$ 120$ million if the WACC is $10 \%$ ( $\$ 12$ million/0.10)
- $\$ 200$ million if the WACC is $6 \%$ ( $\$ 12$ million/0.06).

The aim should therefore be to achieve a level of financial gearing that minimises the WACC, in order to maximise the value of the company.

Important questions in financial management are:

- How does the WACC change with changes in gearing?
- For each company, is there an 'ideal' level of gearing that minimises the WACC?


## Cost of capital and gearing

- The traditional view of gearing and WACC
- The Modigliani-Miller propositions: ignoring corporate taxation
- The Modigliani-Miller propositions: allowing for corporate taxation


## 5 Cost of capital and gearing

For a given level of annual cash profits before interest and tax, the value of a company (equity + debt) is maximised at the level of gearing where WACC is lowest. This should also be the level of gearing that optimises the wealth of equity shareholders.

The question is therefore:
How does a change in gearing affect the WACC, and is there a level of gearing where the WACC is minimised?

The most important analysis of gearing and the cost of capital, for the purpose of your examination, is the analysis provided by Modigliani and Miller that allows for tax relief on debt interest.

However, the traditional view of WACC and gearing, and Modigliani and Miller's propositions ignoring tax relief on debt are also described briefly.

### 5.1 The traditional view of gearing and WACC

The traditional view of gearing is that there is an optimum level of gearing for a company. This is the level of gearing at which the WACC is minimised.

- As gearing increases, the cost of equity rises. However, as gearing increases, there is a greater proportion of debt capital in the capital structure, and the cost of debt is cheaper than the cost of equity. Up to a certain level of gearing, the effect of having more debt capital has a bigger effect on the WACC than the rising cost of equity, so that the WACC falls as gearing increases.
- However, when gearing rises still further, the increase in the cost of equity has a greater effect than the larger proportion of cheap debt capital, and the WACC starts to rise.

The traditional view of gearing is therefore that an optimum level of gearing exists, where WACC is minimised and the value of the company is maximised.

## Traditional view of gearing and the WACC



### 5.2 The Modigliani-Miller propositions: ignoring corporate taxation

The traditional view of gearing and WACC was challenged by Modigliani and Miller in the 1950s. Initially, their arguments were based on the assumption that corporate taxation, and the tax relief on interest, could be ignored.

You do not need to know Modigliani and Miller's arguments, only the conclusions they reached. They argued that if corporate taxation is ignored, an increase in gearing will have the following effect:

- As the level of gearing increases, there is a greater proportion of cheaper debt capital in the capital structure of the firm.
- However, the cost of equity rises as gearing increases.
- As gearing increases, the net effect of the greater proportion of cheaper debt and the higher cost of equity is that the WACC remains unchanged.
- The WACC is the same at all levels of financial gearing.
- The total value of the company is therefore the same at all levels of financial gearing

Modigliani and Miller therefore reached the conclusion that the level of gearing is irrelevant for the value of a company. There is no optimum level of gearing that a company should be trying to achieve.

## Modigliani-Miller view of gearing and the WACC: no taxation



## Modigliani and Miller's propositions: ignoring taxation

Modigliani and Miller's arguments, ignoring taxation, can be summarised as two propositions.

- Proposition 1. The WACC is constant at all levels of gearing. For companies with identical annual profits and identical business risk characteristics, their total market value (equity plus debt) will be the same regardless of differences in gearing between the companies.
- Proposition 2. The cost of equity rises as the gearing increases. The cost of equity will rise to a level such that, given no change in the cost of debt, the WACC remains unchanged.


## Modigliani-Miller formulae: ignoring taxation

There are three formulae for the Modigliani and Miller theory, ignoring corporate taxation. These are shown below. The letter ' $u$ ' refers to an ungeared company (allequity company) and the letter ' $G$ ' refers to a geared company.

## (1) WACC

The WACC in a geared company and the WACC in an identical but ungeared (all-equity) company are the same:
$\mathrm{WACC}_{G}=\mathrm{WACC}_{\mathrm{U}}$
This formula expresses a part of proposition 1.
(2) Total value of the company (equity plus debt capital)

The total value of an ungeared company is equal to the total value of an identical geared company (combined value of equity + debt capital):
$V_{G}=V_{U}$
This formula expresses another part of proposition 1.

## (3) Cost of equity

The cost of equity in a geared company is higher than the cost of equity in an ungeared company, by an amount equal to:

- the difference between the cost of equity in the ungeared company and the cost of debt $\left(\mathrm{K}_{\mathrm{EU}}-\mathrm{K}_{\mathrm{D}}\right)$
- multiplied by the ratio of the market value of debt to the market value of equity in the geared company (D/E).

$$
\mathrm{K}_{\mathrm{EG}}=\mathrm{K}_{\mathrm{EU}}+\frac{\mathrm{D}}{\mathrm{E}}\left(\mathrm{~K}_{\mathrm{EU}}-\mathrm{K}_{\mathrm{D}}\right)
$$

This formula expresses proposition 2.

## Example

An all-equity company has a market value of $\$ 150$ million and a cost of equity of $10 \%$. It borrows $\$ 50$ million of debt finance, costing $6 \%$, and uses this to buy back and cancel $\$ 50$ million of equity. Tax relief on debt interest is ignored.

## Required

According to Modigliani and Miller, if taxation is ignored, what would be the effect of the higher gearing on (a) the WACC (b) the total market value of the company and (c) the cost of equity in the company?

## Answer

According to Modigliani and Miller:
(a) WACC. The WACC in the company is unchanged, at $10 \%$.
(b) Total value. The total market value of the company with gearing is identical to the market value of the company when it was all equity, at $\$ 150$ million. This now consists of $\$ 50$ million in debt and $\$ 100$ million equity ( $\$ 150$ million - $\$ 50$ million of debt)
(c) Cost of equity. The cost of equity in the geared company is
$10 \%+\left[\frac{50}{100} \times(10-6)\right] \%=12.0 \%$

## Example

A company has $\$ 500$ million of equity capital and $\$ 100$ million of debt capital, all at current market value. The cost of equity is $14 \%$ and the cost of the debt capital is $8 \%$.

The company is planning to raise $\$ 100$ million by issuing new shares. It will use the money to redeem all the debt capital.

## Required

According to Modigliani and Miller, if the company issues new equity and redeems all its debt capital, what will be the cost of equity of the company after the debt has been redeemed? Assume that there is no corporate taxation.

## Answer

In the previous example, the Modigliani-Miller formulae were used to calculate a cost of equity in a geared company, given the cost of equity in the company when it is ungeared (all-equity). This example works the other way, from the cost of equity in a geared company to a cost of equity in an ungeared company. The same formulae can be used.

Using the known values for the geared company, we can calculate the cost of equity in the ungeared company after the debt has been redeemed.
$K_{E G}=K_{E U}+D / E\left[K_{E U}-K_{D}\right]$
$14.0=K_{E U}+100 / 500\left[K_{E U}-8.0\right]$
$1.2 \mathrm{~K}_{\mathrm{EU}}=14.0+1.6$
$\mathrm{K}_{\mathrm{EU}}=\mathbf{1 3 . 0} \%(15.6 / 1.2)$

### 5.3 The Modigliani-Miller view: allowing for corporate taxation

Modigliani and Miller revised their arguments to allow for the fact that there is tax relief on interest. You do not need to know the arguments they used to reach their conclusions, but you must know what their conclusions were. You should also know and be able to apply the formulae described below.
(The formula for the cost of equity is given in the formula sheet in your examination, so you do not need to learn it.)

Modigliani and Miller argued that allowing for corporate taxation and tax relief on interest, an increase in gearing will have the following effect:

- As the level of gearing increases, there is a greater proportion of cheaper debt capital in the capital structure of the firm. However, the cost of equity rises as gearing increases.
- As gearing increases, the net effect of the greater proportion of cheaper debt and the higher cost of equity is that the WACC becomes lower. Increases in gearing result in a reduction in the WACC.
- The WACC is therefore at its lowest at the highest practicable level of gearing. (There are practical limitations on gearing that stop it from reaching very high levels. For example, lenders will not provide more debt capital except at a much higher cost, due to the high credit risk).
- The total value of the company is therefore higher for a geared company than for an identical all-equity company. The value of a company will rise, for a given level of annual cash profits before interest, as its gearing increases.

Modigliani and Miller therefore reached the conclusion that because of tax relief on interest, there is an optimum level of gearing that a company should be trying to achieve. A company should be trying to make its gearing as high as possible, to the maximum practicable level, in order to maximise its value.

## Modigliani-Miller view of gearing and the WACC: with taxation



## Modigliani and Miller's propositions: allowing for taxation

Modigliani and Miller's arguments, allowing taxation, can be summarised as two propositions.

- Proposition 1. The WACC falls continually as the level of gearing increases. In theory, the lowest cost of capital is where gearing is $100 \%$ and the company is financed entirely by debt. (Modigliani and Miller recognised, however, that 'financial distress' factors have an effect at high levels of gearing, increasing the cost of debt and the WACC.) For companies with identical annual profits and identical business risk characteristics, their total market value (equity plus debt) will be higher for a company with higher gearing.
- Proposition 2. The cost of equity rises as the gearing increases. There is a positive correlation between the cost of equity and gearing (as measured by the debt/equity ratio).


## Modigliani-Miller formulae: allowing for taxation

There are three formulae for the Modigliani and Miller theory, allowing for corporate taxation. These are shown below. The letter ' $u$ ' refers to an ungeared company (all-equity company) and the letter ' $\sigma$ ' refers to a geared company.
(1) WACC

The WACC in a geared company is lower than the WACC in an all-equity company, by a factor of $1-\frac{\mathrm{Dt}}{(\mathrm{D}+\mathrm{E})}$.
$W_{A C C}^{G}=W A C C_{U}\left[1-\frac{D t}{(D+E)}\right]$

This formula expresses a part of proposition 1.
(2) Value of a company

The total value of a geared company (equity + debt) is equal to the total value of an identical ungeared company plus the value of the 'tax shield'. This is the market value of the debt in the geared company multiplied by the rate of taxation (Dt).
$\mathrm{V}_{\mathrm{G}}=\mathrm{V}_{\mathrm{U}}+\mathrm{Dt}$
where:
$V_{G}=$ value of geared company
$\mathrm{V}_{\mathrm{U}}=$ value of an identical but ungeared (all-equity) company
$\mathrm{D}=$ market value of the debt in the geared company
$t=$ the rate of taxation on company profits.
This formula expresses another part of proposition 1.
(3) Cost of equity

The cost of equity in a geared company is higher than the cost of equity in an ungeared company, by a factor equal to:

- the difference between the cost of equity in the ungeared company and the cost of debt, $\left(K_{E U}-K_{D}\right)$
- multiplied by the ratio $(1-t) \times \frac{D}{E}$.

$$
\mathrm{K}_{\mathrm{EG}}=\mathrm{K}_{\mathrm{EU}}+(1-t)\left(\mathrm{K}_{\mathrm{EU}}-\mathrm{K}_{\mathrm{D}}\right) \frac{\mathrm{D}}{\mathrm{E}}
$$

This formula expresses proposition 2. It is given to you in your examination, in a formula sheet. Although you do not need to learn the formula, you should become familiar with it, and know how to use it.

## Example

An all-equity company has a market value of $\$ 60$ million and a cost of equity of $8 \%$. It borrows $\$ 20$ million of debt finance, costing $5 \%$, and uses this to buy back and cancel $\$ 20$ million of equity. The rate of taxation on company profits is $25 \%$.

According to Modigliani and Miller:
(a) Market value

The market value of the company after the increase in its gearing will be:

$$
\mathrm{V}_{\mathrm{G}}=\mathrm{V}_{\mathrm{U}}+\mathrm{Dt}
$$

$\mathrm{V}_{\mathrm{G}}=\$ 60$ million $+(\$ 20$ million $\times 0.25)=\$ 75$ million.
The market value of the debt capital is $\$ 20$ million; therefore the market value of the equity in the geared company is $\$ 55$ million ( $\$ 75$ million - $\$ 20$ million).
(b) WACC of the geared company

The WACC of the company after the increase in its gearing is calculated as follows:

$$
\begin{aligned}
& \mathrm{WACC}_{G}=\mathrm{WACC}_{\mathrm{U}}\left[1-\frac{\mathrm{Dt}}{(\mathrm{D}+\mathrm{E})}\right] \\
& \mathrm{WACC}_{\mathrm{G}}=8 \%\left[1-\frac{(\$ 20 \text { million } \times 25 \%)}{(\$ 65 \text { million })}\right]=8 \%(0.9231)=7.38 \%
\end{aligned}
$$

(c) Cost of equity in the geared company

$$
\begin{aligned}
& \mathrm{K}_{\mathrm{EG}}=\mathrm{K}_{\mathrm{EU}}+(1-t)\left(\mathrm{K}_{\mathrm{EU}}-\mathrm{K}_{\mathrm{D}}\right) \frac{\mathrm{D}}{\mathrm{E}} \\
& \mathrm{~K}_{\mathrm{EG}}=8 \%+[(1-0.25)(8-5) 20 / 45]=8 \%+1 \%=9 \%
\end{aligned}
$$

Check: the WACC can now be calculated as follows:

| Source of finance | Market value | Cost | Market value x Cost |
| :--- | ---: | ---: | ---: |
|  | $\$$ million | r | $\mathrm{MV} \times \mathrm{r}$ |
| Equity | 45.00 | 9.00 | 405 |
| Debt (after-tax cost) | 20.00 | 3.75 | 75 |
|  | 65.00 |  | 480 |
|  |  |  |  |

$W \operatorname{WACC}=\frac{480}{65.00}=7.38 \%$

## Change in gearing, the WACC and capital investment appraisal

- Modigliani and Miller with taxation: from one level of gearing to another
- The method
- Relevance for capital investment appraisal


## 6 Change in gearing, the WACC and capital investment appraisal

### 6.1 Modigliani and Miller with taxation: from one level of gearing to another

When a company is considering a major new capital investment project (where the business risk is similar to the risk with the company's other business operations), the method of financing the investment might alter the company's gearing. For example, if a project is financed entirely by new debt capital, its gearing level will increase.

- A change in gearing will alter the cost of equity (Modigliani and Miller proposition 2).
- There might be a change in the cost of debt, where the gearing level rises to such a high level that 'financial distress' concerns make debt capital more expensive. However, at lower levels of gearing it is assumed that the cost of debt is unaffected by changes in the gearing level.
- There will be a reduction in the WACC (Modigliani and Miller proposition 1).

If the project is evaluated using the WACC to estimate the NPV, the new WACC should be used for the NPV evaluation.

This means that when a new capital project will result in a change in gearing, it is necessary to calculate a new WACC before going on to the NPV calculations.

The Modigliani and Miller formulae can be used to do this. The explanation that follows concentrates on the formula for the cost of equity, because this is the formula that you will be given in the examination.

### 6.2 The method

The approach should be as follows:

- Step 1. Start with the company at its original level of gearing. You should be given the value of the company (the value of its equity and the value of its debt capital) and the cost of its equity and debt capital.
- Step 2. Use these values to calculate the value of a comparable ungeared company, and the cost of equity in the ungeared company. Use the Modigliani and Miller formulae to do this.
- You now have the cost of equity in a comparable ungeared company.
- Step 3. Use these values for the ungeared company to work out values for the company at its new level of gearing: total value, value of equity, WACC and cost of equity.


## Example

A company has a total current value of $\$ 100$ million, consisting of $\$ 80$ million equity and $\$ 20$ million of debt capital. The cost of equity is $10 \%$ and the pre-tax cost of the debt capital is $6 \%$. The rate of tax on company profits is $25 \%$.

The company proposes to borrow an additional $\$ 20$ million of debt capital, and use the money to buy back and cancel $\$ 20$ million of its equity.

## Required

According to Modigliani and Miller, what will be the following values for the company at its new level of gearing?
(a) Its total value, divided into a value for the equity and a value for the debt capital
(b) Its WACC
(c) The cost of its equity capital.

This is a long example, but you should work through the solution carefully.

Answer
(a) Total value of the company
(i) Step 1: Value of a similar all-equity company.

We have the current value of the geared company, which is $\$ 100$ million, consisting of $\$ 80$ million equity and $\$ 20$ million debt capital. We can calculate the cost of a similar company that is all-equity financed.
$\mathrm{V}_{\mathrm{G}}=\mathrm{V}_{\mathrm{U}}+\mathrm{Dt}$
100 million $=\mathrm{V}_{\mathrm{U}}+(20$ million $\times 0.25)$
$\mathrm{V}_{\mathrm{U}}=95$ million.
(ii) Step 2: Value of the company at the new level of gearing.

The company will be replacing $\$ 20$ million of equity with $\$ 20$ million of debt capital, so in the new gearing structure, debt capital increases.
The market value of the debt will be $\$ 20$ million $+\$ 20$ million $=\$ 40$ million.

We can calculate the total cost of the company at its new gearing level, using the same MM formula.
$\mathrm{V}_{\mathrm{G}}=\mathrm{V}_{\mathrm{U}}+\mathrm{Dt}$
$\mathrm{V}_{\mathrm{G}}=95$ million $+(40$ million $\times 0.25)$
$=105$ million.

The total value of the company at the new gearing level will be $\$ 105$ million. Of this, $\$ 40$ million will be debt capital; therefore the value of the remaining equity will be $\$ 65$ million.
(b) WACC
(i) Step 1: WACC of a similar all-equity company.

The WACC of the company at its current level of gearing is calculated as follows:

| Source of finance | Market value | Cost | Market value x Cost |
| :--- | ---: | ---: | ---: |
|  | $\$$ million | r | $\mathrm{MV} \times \mathrm{r}$ |
| Equity | 80.00 | 0.10 | 8.00 |
| Debt (after-tax cost) | 20.00 | 0.045 | 0.90 |
|  | 100.00 |  | -8.90 |

$W A C C=8.9 / 100=0.089$ or $8.9 \%$.
We can use the MM formula for WACC to calculate what the WACC would be in a similar ungeared company.

The WACC of a similar ungeared company is:
$0.089=W^{\prime}$ ACC $_{U} \times\left[1-\frac{20 \times 0.25}{(20+80)}\right]$
$0.089=0.95 \mathrm{WACC}_{\mathrm{u}}$
$\mathrm{WACC}_{\mathrm{u}}=0.09368$
(ii) Step 2: WACC of the company at the new level of gearing.

Having established the WACC in an all-equity company, we can now use the same formula to calculate the WACC in the company at its new level of gearing, with $\$ 40$ million debt and $\$ 65$ million of equity.
(The value of equity and debt capital at the new level of gearing were calculated in (a)).
$\mathrm{WACC}_{\mathrm{G}}=0.09368 \times\left[1-\frac{40 \times 0.25}{(40+65)}\right]$
WACC $_{G}=0.09368 \times 0.90476$
$W_{A C C}^{G}=0.085=8.5 \%$.
(c) Cost of equity
(i) Step 1: Cost of equity of a similar all-equity company.

A similar approach is taken for calculating the cost of equity. We start by calculating the cost of equity in a similar all-equity company, using the MM formula for Proposition 2. We know the value of $\mathrm{K}_{\mathrm{EG}}$ and we need to calculate a value for $\mathrm{K}_{\mathrm{EU}}$.

In the original geared company, the value of equity is $\$ 80$ million and the value of debt capital is $\$ 20$ million.
The cost of equity in a similar all-equity company is calculated as follows:

$$
\begin{aligned}
& \mathrm{K}_{\mathrm{EG}}=\mathrm{K}_{\mathrm{EU}}+(1-t)\left(\mathrm{K}_{\mathrm{EU}}-\mathrm{K}_{\mathrm{D}}\right) \frac{\mathrm{D}}{\mathrm{E}} \\
& 10=\mathrm{K}_{\mathrm{EU}}+\left[(1-0.25)\left(\mathrm{K}_{\mathrm{EU}}-6\right) 20 / 80\right] \\
& 10=\mathrm{K}_{\mathrm{EU}}+0.1875 \mathrm{~K}_{\mathrm{EU}}-1.125 \\
& 1.1875 \mathrm{~K}_{\mathrm{EU}}=11.125 \\
& \mathrm{~K}_{\mathrm{EU}}=9.3684 .
\end{aligned}
$$

(ii) Step 2: Cost of equity of the company at the new level of gearing.

Having calculated the cost of equity in a similar all-equity company, we can now calculate the cost of equity in the company at its new level of gearing. Debt capital is $\$ 40$ million and equity is $\$ 65$ million.
(The value of equity and debt capital at the new level of gearing were calculated in (a).)

$$
\begin{aligned}
& \mathrm{K}_{\mathrm{EG}}=\mathrm{K}_{\mathrm{EU}}+(1-t)\left(\mathrm{K}_{\mathrm{EU}}-\mathrm{K}_{\mathrm{D}}\right) \frac{\mathrm{D}}{\mathrm{E}} \\
& \mathrm{~K}_{\mathrm{EG}}=9.3684+(1-0.25)(9.3684-6) 40 / 65 \\
& \mathrm{~K}_{\mathrm{EG}}=9.3684+1.5546=10.923, \text { say } 10.9 \% .
\end{aligned}
$$

### 6.3 Relevance for capital investment appraisal

The Modigliani and Miler formulae can be used to re-calculate the cost of equity and the WACC in a company where the level of gearing changes, provided there is no change in the overall business risk and the company is therefore similar in all respects except for its gearing.

When a company plans a new capital investment that will alter its gearing, without affecting its business risk profile, the MM formulae can be used to calculate the cost of equity and WACC at the new level of gearing. The new WACC can then be used as the discount rate for calculating the NPV of the proposed project.

If you are given an examination question of this type in your examination, you might prefer to use the formula for Proposition 2 (cost of equity) because this is given to you in the formula sheet in the examination, and you do not need to learn it.

Study the following example carefully.

## Example

A US company has a current total market value of $\$ 12$ billion, consisting of $\$ 10$ billion of equity and $\$ 2$ billion of debt capital. The debt capital will mature in four years' time. The current weighted average cost of capital is $6.5 \%$.

The company is considering a new investment costing $\$ 3$ billion, which it would finance entirely by $\$ 3$ billion of new ten-year bonds.

The yield curve for US government bonds (Treasuries) shows that the risk-free cost of four-year debt is $4 \%$ and the risk-free cost of ten-year debt is $4.25 \%$. The credit rating on the company's current debt capital is AA, but if the new bond issue takes place there is a $75 \%$ probability that all the company's debt will be re-rated to AAand a $25 \%$ probability that all the company's debt will be re-rated to A+. (This applies to both the existing debt and the new bonds.)

The spreads for yields on corporate bonds above the US Treasuries yield curve are as follows:

| Credit rating | Spreads (basis points) |  |
| :--- | :---: | :---: |
| 4-year bonds | 10-year bonds |  |
| AA | 30 | 50 |
| AA - | 40 | 60 |
| A + | 45 | 70 |

The rate of taxation on company profits is $30 \%$

## Required

Calculate the weighted average cost of capital in the company if the project goes ahead and is financed entirely by ten-year bonds. Assume that there will be no change in the company's business risk.

## Answer

## Total value of the company

In the previous example, it was possible to re-calculate the total value of the company using the Modigliani and Miller formula $\mathrm{V}_{\mathrm{G}}=\mathrm{V}_{\mathrm{U}}+\mathrm{Dt}$.

In this example, this is not possible, because the company is raising additional debt capital. The MM formula applies only when the size of the company is unchanged, and equity replaces debt or debt replaces equity. The formula cannot be used when additional debt capital is raised.

An assumption will therefore be made (even though it is unsatisfactory) that when the company obtains its additional debt capital of $\$ 3$ billion, the value of the equity will be unchanged at $\$ 10$ billion.

## Preliminary workings

We need to calculate the current cost of equity in the company, in order to use the MM formula for Proposition 2. We know the current WACC which is $6.5 \%$. We also know that the current credit rating for the four-year debt capital is AA, which means that the pre-tax cost of debt is the risk-free yield of $4.00 \%$ plus a spread of 30 basis points. The pre-tax cost of debt is therefore $4.30 \%$ and the after-tax cost of debt is $4.30 \%(1-0.30)=3.01 \%$.

We now have enough data to calculate the current cost of equity.

The WACC, which is $6.5 \%$, is calculated as follows:

| Source of finance | Market <br> value | Cost | Market value <br> $\times$ Cost |
| :--- | ---: | ---: | ---: |
|  | $\$$ billion | $\%$ | $\mathrm{MV} \times \mathrm{r}$ |
| Equity | 10.0 | r | 10 r |
| Debt | 2.0 | 3.01 | 6.02 |
|  | 12.0 |  | $10 \mathrm{r}+6.02$ |
|  |  |  |  |

$\mathrm{WACC}=6.5 \%=\frac{10 \mathrm{r}+6.02}{12} \%$
$6.5(12)=10 r+6.02$
$10 \mathrm{r}=71.98$
$r=7.198 \%$

The current cost of equity is $7.198 \%$.

We also need to calculate the cost of debt in the company after the new bond issue.

|  | If AA - |  | If A + |  |
| :---: | :---: | :---: | :---: | :---: |
|  | 4 year | 10 year | 4 year | 10 year |
| Risk-free | 4.00 | 4.25 | 4.00 | 4.25 |
| Spread | 0.40 | 0.60 | 0.45 | 0.70 |
| Pre-tax cost | 4.40 | 4.85 | 4.45 | 4.95 |

We can calculate a weighted average pre-tax cost of the debt capital, as follows:

|  | Market value | AA |  | A + |  |
| :--- | :---: | ---: | ---: | ---: | ---: |
|  | \$bn | Cost | Cost $\times \mathbf{M V}$ | Cost | Cost $\times \mathbf{M V}$ |
| 4 year | 2 | 4.40 | 8.80 | 4.45 | 8.90 |
| 10 year | $\underline{3}$ | 4.85 | $\underline{14.55}$ | 4.95 | $\underline{14.85}$ |
|  | $\underline{5}$ |  | $\underline{23.35}$ |  | $\underline{23.75}$ |

Weighted average before-tax cost of debt if credit rating is $\mathrm{AA}-=23.35 / 5=4.67 \%$. Weighted average before-tax cost of debt if credit rating is $\mathrm{A}+=23.75 / 5=4.75 \%$.

Expected weighted average before tax cost of debt $=(0.75 \times 4.67)+(0.25 \times 4.75) \%$ $=4.69 \%$.

Expected weighted average after-tax cost of debt $=4.69 \%(1-0.30)=3.283 \%$.

## Calculate the cost of equity in the company after the new bond issue

Step 1. Calculate the cost of equity in an ungeared company at the current level of gearing. We know that the current cost of equity at the current level of gearing is 7.198\%.

$$
\begin{aligned}
& \mathrm{K}_{\mathrm{EG}}=\mathrm{K}_{\mathrm{EU}}+(1-t)\left(\mathrm{K}_{\mathrm{EU}}-\mathrm{K}_{\mathrm{D}}\right) \frac{\mathrm{D}}{\mathrm{E}} \\
& 7.198=\mathrm{K}_{\mathrm{EU}}+\left[(1-0.30)\left(\mathrm{K}_{\mathrm{EU}}-4.30\right) 2 / 10\right] \\
& 7.198=\mathrm{K}_{\mathrm{EU}}+0.14 \mathrm{~K}_{\mathrm{EU}}-0.602 \\
& 1.14 \mathrm{~K}_{\mathrm{EU}}=7.8 \\
& \mathrm{~K}_{\mathrm{EU}}=6.842 .
\end{aligned}
$$

Step 2. Calculate the cost of equity in the company at the new level of gearing, making the assumption about market value discussed above, but assuming no change in the cost of debt. (Changing the cost of debt in this calculation would produce an incorrect valuation for the cost of equity.)

$$
\begin{aligned}
& \mathrm{K}_{\mathrm{EG}}=\mathrm{K}_{\mathrm{EU}}+(1-t)\left(\mathrm{K}_{\mathrm{EU}}-\mathrm{K}_{\mathrm{D}}\right) \frac{\mathrm{D}}{\mathrm{E}} \\
& \mathrm{~K}_{\mathrm{EG}}=6.842+0.890 \\
& \mathrm{~K}_{\mathrm{EG}}=7.732 .
\end{aligned}
$$

## Calculate the WACC at the new level of gearing

We can now calculate the WACC after the bond issue.

| Source of finance | Market <br> value | Cost | Market value <br> $\times$ Cost |
| :--- | ---: | ---: | ---: |
|  | $\$$ billion | $\%$ | MV $\times \mathrm{r}$ |
| Equity | 10.0 | 7.732 | 77.320 |
| Debt (weighted average) | 5.0 | 3.283 | 16.415 |
|  | 15.0 |  | 93.735 |
|  |  |  |  |

$W A C C=93.735 / 15=6.249$, say $6.25 \%$.

A discount rate of $6.25 \%$ should be used to calculate the NPV of the proposed new capital investment.

## Asset beta

- Changes in gearing, the cost of equity and equity beta factors
- Definition of asset beta
- The asset beta formula
- Using the asset beta formula
- The asset beta formula: changing from one level of gearing to another
- Calculating a project beta factor
- Estimating an equity beta factor for a private company


## $7 \quad$ Asset beta

### 7.1 Changes in gearing, the cost of equity and equity beta factors

Modigliani and Miller's theories are used to demonstrate that as the level of gearing in a company increases, assuming no change in business risk, the cost of equity also increases. This also means that as gearing increases, the beta factor of the company's equity must also increase, since there is no change in the risk-free rate of return or the returns on the market portfolio.

Companies that are otherwise identical except for their level of gearing will therefore have different costs of equity and different equity beta factors.

Similarly, if a company changes its gearing level without changing its business risk profile, there will be a change in its cost of equity and equity beta factor.

### 7.2 Definition of asset beta

An asset beta is the beta factor that would be applied to a company's assets if financing risk is removed from the calculation. It is therefore a beta factor that applies to entire companies or individual projects. It provides a measure of the underlying business risk faced by the entire firm regardless of its financing structure.

An asset beta is sometimes called unlevered beta or ungeared beta, and could be described as the beta of equity in a company that is totally equity-financed.

### 7.3 The asset beta formula

Modigliani and Miller argued that there is a difference between the beta factor in an ungeared (all equity) company and the beta factor in a similar geared company. The beta factor in a totally ungeared company (the ungeared beta) is the asset beta for the firm.

The formula, adjusted for taxation, is:
$\beta_{\mathrm{A}}=\beta_{\mathrm{EG}} \times \frac{\mathrm{E}}{\mathrm{E}+\mathrm{D}(1-\mathrm{t})}+\beta_{\mathrm{D}} \times \frac{\mathrm{D}(1-\mathrm{t})}{\mathrm{E}+\mathrm{D}(1-\mathrm{t})}$
where
$\beta_{\mathrm{A}}=$ the asset beta or ungeared beta
$\beta_{\mathrm{EG}}=$ the beta of the equity in a geared company
$\beta_{\mathrm{D}}=$ the beta of the debt capital in the geared company
$\mathrm{E}=$ the market value of the equity in the geared company
$\mathrm{D}=$ the market value of the debt capital in the geared company
$t=$ the rate of taxation on profits (the rate of tax relief on interest payments on debt capital).

This formula is included in the formula sheet for your examination. You do not need to learn it, but you must be able to apply it.

For the purpose of examination questions, it is often assumed that debt capital in a company is risk-free. This is a simplifying assumption which makes the calculations much easier!

If it is assumed that debt is risk-free, the asset beta formula simplifies to:
$\beta_{\mathrm{A}}=\beta_{\mathrm{EG}} \times \frac{\mathrm{E}}{\mathrm{E}+\mathrm{D}(1-\mathrm{t})}$
Modigliani and Miller therefore argued that the beta factor in an all-equity company is lower than the beta factor in an identical but geared company, by a factor of $\frac{E}{[E+D(1-t)]}$

## Example

A company is all-equity financed, and its equity beta factor is 1.05 . Its total market value is $\$ 240$ million. This includes debt capital with a market value of $\$ 80$ million.

It is assumed that the debt capital is risk-free. The rate of tax is $30 \%$.

## Required

Calculate the asset beta for the company.

## Answer

Using the asset beta formula and assuming a risk-free cost of debt. (meaning that $\beta_{\mathrm{D}}=0$ ):
$\beta_{\mathrm{A}}=1.05 \times \frac{160}{160+80(1-0.30)}$
$\beta_{\mathrm{A}}=1.05 \times 0.74074=0.778$

### 7.4 Using the asset beta formula

The asset beta formula might be used in the following circumstances:

- when a company uses the CAPM to estimate a cost of equity and WACC, in a situation where a capital expenditure project will change the gearing of the company
- to estimate the beta factor for a capital investment project (a project beta), where the gearing will be different from the company's current capital structure
- in order to estimate a cost of equity in a private company by comparing it with a similar public company.


### 7.5 The asset beta formula: changing from one level of gearing to another

A company might be planning a capital investment that will alter its level of gearing. In these cases, to work out the equity beta factor and cost of equity at the new level of gearing, it is necessary to calculate the asset beta first (the beta for a similar ungeared company).

To calculate a cost of equity using the CAPM, where the level of gearing changes, we can calculate the asset beta and then convert this asset beta to an equity beta on the basis of the new expected level of gearing.

The approach should be as follows:

- Start with the company at its original level of gearing. You should be given the value of the company (the value of its equity and the value of its debt capital) and the cost of its equity and debt capital, and the equity beta factor.
- You may assume, unless instructed otherwise, that the debt capital in the company is risk-free, and the simplified version of the asset beta formula can be used.
- Having calculated the asset beta, you can calculate a new equity beta for the company at its new level of gearing, using the same formula.


## Example

A company has a total current value of $\$ 80$ million, consisting of $\$ 60$ million equity and $\$ 20$ million of debt capital. The cost of equity is $12 \%$ and the pre-tax cost of the debt capital is $7 \%$. The rate of tax on company profits is $30 \%$. The equity beta factor is 0.925 . The debt capital is risk-free. The return on the market portfolio is $13.67 \%$.

## Required

What will be the equity beta factor and the cost of equity in the company, if the company issues new equity to raise $\$ 10$ million and uses this money to repay $\$ 10$ million of debt?

## a

Answer

The equity beta of the company at its current level of gearing is 0.925 .

The asset beta of the company (the equity beta of a similar ungeared company) will be:
$\beta_{\mathrm{A}}=\beta_{\mathrm{EG}} \times \frac{\mathrm{E}}{\mathrm{E}+\mathrm{D}(1-\mathrm{t})}$
$\beta_{\mathrm{A}}=0.925 \times 60 /[60+20(1-0.30)]$
$\beta_{\mathrm{A}}=0.925 \times 60 / 74$
$\beta_{\mathrm{A}}=0.75$

We can calculate the total value of the company if it is ungeared, using the data for the company at its current gearing level.

Total value of an all-equity company:
$\mathrm{V}_{\mathrm{G}}=\mathrm{V}_{\mathrm{U}}+\mathrm{Dt}$
$\$ 80$ million $=\mathrm{V}_{\mathrm{U}}+(\$ 20$ million $\times 0.30)$
$\mathrm{V}_{\mathrm{U}}=\$ 74$ million.

At the new level of gearing, the market value of the debt capital will be $\$ 10$ million ( $\$ 20$ million minus $\$ 10$ million redeemed). We can calculate the total value of the company at the new gearing level, using the value of the company if it is ungeared.
$\mathrm{V}_{\mathrm{G}}=\mathrm{V}_{\mathrm{U}}+\mathrm{Dt}$
$\mathrm{V}_{\mathrm{G}}=\$ 74$ million $+(\$ 10$ million $\times 0.30)$
$\mathrm{V}_{\mathrm{G}}=\$ 77$ million.

Of this total value, $\$ 10$ million is debt capital; therefore the value of the equity is $\$ 67$ million.

We now have the information to calculate a beta factor for the equity at the new level of gearing. The asset beta was calculated earlier, and is 0.75 .

$$
\begin{aligned}
& \beta_{\mathrm{A}}=\beta_{\mathrm{EG}} \times \frac{\mathrm{E}}{\mathrm{E}+\mathrm{D}(1-\mathrm{t})} \\
& 0.75=\beta_{\mathrm{EG}} \times \frac{67}{67+10(1-0.30)} \\
& 0.9054 \beta_{\mathrm{EG}}=0.75 \\
& \beta_{\mathrm{EU}}=0.828, \text { say } 0.83 .
\end{aligned}
$$

Using the CAPM, we can calculate the cost of equity at the new gearing level as:
$7 \%+0.83$ (13.67-7.0)
$=12.5361$, say $12.5 \%$.

### 7.6 Calculating a project beta factor

The CAPM can be applied to capital expenditure appraisal. The cost of capital for a project can be determined according to the risk of the project. If the beta factor for a project can be determined, the CAPM can then be used to decide what the cost of capital for the project should be, allowing for its beta and systematic risk.

In addition, adjustments can be made for differences in gearing.

## Example

An all-equity company operates in an industry where its beta factor is 0.90 . It is considering whether to invest in a completely different industry. In this other industry, the average debt/equity ratio is $40 \%$ and the average beta factor is 1.25 . The risk-free rate of return is $4 \%$ and the average market return is $7 \%$. If the company does invest in this other industry, it will remain all-equity financed. The rate of taxation is $30 \%$. Assume that debt is risk-free.

## Required

What cost of capital should be used to evaluate the proposed investment?

## Answer

The appropriate discount rate should be one that applies to the industry in which the investment will be made. We know that the geared beta in this industry is 1.25, with a debt: equity ratio of 0.40 . We can calculate the asset beta (ungeared equity beta) for the industry as:
$\beta_{\mathrm{A}}=1.25 \times \frac{60}{[60+40(1-0.30)]}=1.25 \times \frac{60}{88}=0.85$
Since the company will be all-equity financed, the cost of equity to apply to the project is therefore:
$4 \%+0.85(7-4) \%=6.55 \%$.

Note: If the company has proposed to finance the project with some debt, the asset beta could be used to calculate a new geared beta for the appropriate level of gearing, and a new cost of equity and new WACC could be calculated.

### 7.7 Estimating an equity beta factor for a private company

The asset beta formula can also be used to estimate the equity beta for a private company (and its cost of equity) where the private company is comparable in size, business mix and business risk to a public company whose shares are traded on a stock market.

Data for the public company can be used to estimate an asset beta, and the asset beta can then be applied to the private company to obtain an equity beta for the private company.

Since there is no market data for the private company, an assumption has to be made that accounting book $\mathrm{b}=$ values in the balance sheet are a reasonable approximation to market values, and the calculations use book values for equity and debt rather than market values.

## Example

An investor is interested in acquiring Greenco, which is a private company with net assets of $\$ 240$ million and debt capital of $\$ 160$ million in its balance sheet.

Greenco can be compared to a public company Redco, which operates in the same industry and has a similar business mix and business risk. The following information is available about Redco:

| Market value of equity | $\$ 450$ million |
| :--- | :--- |
| Equity beta | 1.60 |
| Debt capital | Risk-free (debt capital beta $=0$ ) |
| Price to book value of equity | 1.50 times |
| Gearing (total debt to equity value) | 1.25 |

The rate of tax on profits is $30 \%$. The risk-free rate of interest is $5 \%$ and the equity risk premium is $3 \%$.

## Required

Use this information to estimate a cost of equity in Greenco.

## Answer

Greenco is a private company and estimating a cost of equity has to be based on a comparison with the similar public company, Redco.
The asset beta formula uses market values for debt and equity. Although we have these market values for Redco, we do not have them for Greenco. An assumption
should therefore be that book values are a reasonable approximation for market values, and book values will be used in the calculations.

It is also assumed that debt in Greenco is risk-free.
Taking data about Redco and book values rather than market values, we can estimate an asset beta for Redco and then assume that the same asset beta applies to Greenco.

Market value of Redco equity = \$450 million
Price to book value of equity $=1.5$
Book value of equity $=\$ 450$ million $/ 1.5=\$ 300$ million
Debt: equity ratio $=1.25$ times
Book value of debt $=\$ 300$ million $\times 1.25=\$ 450$ million
$\beta_{\mathrm{A}}=1.60 \times \frac{300}{300+450(1-0.30)}$
$\beta_{\mathrm{A}}=1.60 \times 0.4878=0.78$

This asset beta can be used to estimate an equity beta for Greenco:
$0.78=\beta_{\mathrm{EG}} \times \frac{240}{240+160(1-0.30)}$
$0.6818 \beta_{\mathrm{EG}}=0.78$
$\beta_{\text {EG }}=1.14$

The CAPM can now be used to estimate a cost of equity in Greenco:
$\mathrm{K}_{\mathrm{E}}=5 \%+1.14$ (3\%)
$\mathrm{K}_{\mathrm{E}}=8.42 \%$

## Weaknesses with this method

Using the asset beta formula to estimate a cost of equity in a private company, using data from a comparable public company, has some weaknesses and limitations.

- It is assumed that book values are a reasonable approximation for market values, but this assumption is questionable.
- It is assumed that the business risk faced by both companies is the same, which might not be true.
- It is assumed that the cost of debt and equity in the private company is not affected by factors such as greater default risk, its private company status, and its smaller size. This assumption is also questionable.


## Other aspects of capital investment appraisal

|  | Contents |
| :--- | :--- |
| 1 | Choosing the method of financing an investment |
| 2 | Adjusted present value method (APV method) of <br> project appraisal |
| 3 | Capital rationing |
| 4 | Real options |

## Choosing the method of financing an investment

- Theories about selecting the financing method for an investment
- Static trade-off theory
- Pecking order theory
- Market timing theory
- Agency effects on capital structure


## 1 Choosing the method of financing an investment

### 1.1 Theories about selecting the financing method for an investment

Capital investments have to be financed, and management must choose which method of financing they will use. There are different theories about how the method of financing is decided. These include:

- static trade-off theory
- pecking order theory
- market timing theory.

There is also a view that the choice of financing method is affected by agency costs.
These differing theories are considered in this section.

### 1.2 Static trade-off theory

Static trade-off theory argues that for each company there is an optimal capital structure, with an optimal level of gearing.

There is a trade-off between the benefits of taking on more debt and the costs of higher indebtedness.

- The benefits of taking on debt (rather than equity) are mainly in the tax relief that is obtained on debt interest. Modigliani and Miller have argued that although the cost of equity rises as gearing increases, the tax relief on debt means that the company's weighted average cost of capital falls as gearing rises. It is therefore beneficial to take in more debt and increase gearing up to the point where the marginal costs of extra debt start to exceed the marginal benefits of extra debt.
- The marginal costs of extra debt are related to the greater risks from 'financial distress.' If lenders perceive that a company with high levels of debt could be in financial distress (and in danger of failing to make payments of interest and repayments of loan capital on schedule), it becomes much more difficult to raise extra debt finance. The cost of debt might therefore increase substantially to compensate a lender for the high credit risk.

The optimal gearing level for a company is reached at a point where:

- the marginal benefits of taking on additional debt capital
- equals the marginal costs of taking on the extra debt.

The optimal gearing level varies between companies, depending on their profitability. A very profitable company can take on higher gearing because the marginal costs of financial distress will not become significant until the gearing level is very high.

For a company with low profitability, the situation is different. These companies provide low returns to their shareholders, and increasing the gearing level by borrowing more would increase the risks of bankruptcy and the cost of borrowing. Companies with low profits will therefore try to avoid additional borrowing, and they will also be reluctant to incur the costs of making new equity issues. To finance an investment, they will therefore rely on retained profits. They might even decide against investing in a capital project with a positive NPV unless they can finance it with funds from retained profits.

## Static trade-off theory summarised

Static trade-off theory therefore states that:

- Companies have an optimal level of gearing.
- In choosing the method of financing for a new investment, they will try to maintain or achieve the optimal gearing level.
- The optimal gearing level is higher for companies with high profits than companies with low profits.
- This means that there is a positive correlation between profitability and gearing level.


### 1.3 Pecking order theory

Pecking order theory takes a different view of gearing and methods of financing new investments. It was put forward by Myers in 1984 as a challenge to static tradeoff theory.

This theory states that companies show preferences for the source of finance that they use. There is an order of preference or 'pecking order'.

- 1st. The source of finance that is preferred most is retained earnings.
- 2nd. Debt capital is the source of finance second in the order of preference.
- 3rd. New equity capital (an issue of new shares) is the least preferred source of finance for investment.

This means that if a company has an opportunity to invest in a capital project with a positive NPV, it will prefer to fund the project from retained profits. If it is unable to do this, it will look for debt capital to finance the investment. Only if retained profits and debt capital are unavailable (because cash flows are weak and profitability is low) will the company consider a new issue of shares.

Companies are likely to choose a long-term dividend policy that will allow them to finance future investments largely through retained earnings.

The reasons for the pecking order of preferences for sources of finance can be explained by practical considerations.

- Using retained earnings is convenient. If a company wants to finance a new investment with equity, it is much simpler and cheaper to use retained earnings than to arrange a new share issue. Retained earnings are also much more convenient than new borrowing.
- If a company cannot finance an investment with retained earnings, it will prefer new borrowing to a new issue of shares because borrowing is cheaper. It is cheaper to arrange a loan than to issue new shares. The cost of debt is also less because of the tax relief on interest payments.

Pecking order theory states that the gearing of a company is the result of a series of financing decisions based on these preferences for sources of finance. An optimal level of gearing does not exist, and companies do not try to achieve an optimal gearing level.

## Pecking order theory summarised

Pecking order theory therefore states that:

- Companies do not have an optimal level of gearing.
- In choosing the method of financing for a new investment, they have an order of preference: retained earnings followed by new debt capital followed by an issue of new shares.
- Companies with high profits can rely on retained profits as a source of finance more than companies with low profits.
- This means that there is a negative correlation between profitability and gearing level.


### 1.4 Market timing theory

Market timing theory states that the choice of financing method for companies can be driven by opportunities in the capital markets. These opportunities occur because of 'asymmetries of information'. These occur when the managers of a company have more information and better information about the company than shareholders and other investors.

Management should know when the future prospects for the company are better than investors are expecting, and when the prospects for the future are worse than investors are expecting. Company management might therefore recognise occasions when the company's shares are currently under-valued or over-valued.

- Companies will therefore wish to make a new issue of shares when they consider the share price to be over-valued.
- They will consider share repurchases when they consider the share price to be under-valued.
Taking advantage of opportunities in the market to issue new shares or buy back existing shares affects the gearing level. A company therefore does not have a target optimal gearing level. Its financing decisions are determined more by market opportunity and market timing.


### 1.5 Agency effects on capital structure

Agency theory can be used to explain the capital structure of a company and its choices of financing for new investment. Agency theory, which was developed by Jensen and Meckling (1976), states that the governance of a company is based on conflicts of interest between the company's owners (shareholders), its managers and major providers of debt finance.

Each of these groups has different interests and objectives.

- The shareholders want to increase their income and wealth. Their interest is with the returns that the company will provide in the form of dividends, and also in the value of their shares. The value of their shares depends on the longterm financial prospects for the company. Shareholders are therefore concerned about dividends, but they are even more concerned about long-term profitability and financial prospects, because these affect the value of their shares.
- The directors and managers are employed to run the company on behalf of the shareholders. However, if the managers do not own shares in the company, they have no direct interest in future returns for shareholders, or in the value of the shares. Unless they own shares, or unless their remuneration is linked to profits or share values, their main interests are likely to be the size of their remuneration package, and other benefits from their job and position such as their status as company managers.
- The major providers of debt have an interest in sound financial management by the company's managers, so that the company will be able to pay its debts in full and on time. Major lenders will often be concerned that a company will borrow more because the cost of borrowing is fairly low, and invest the money in highrisk ventures.

These conflicts of interest can have implications for capital gearing and preferences for financing method.

- Shareholders might prefer debt finance as a new source of funding. When managers own shares in the company, a new issue of shares might dilute their interest in the company's equity, and other shareholders should want to prevent this from happening. Borrowing to finance growth rather than relying on equity also reduces the amount of free cash for managers to spend on personal interests and benefits.
- Providers of debt capital might be worried by the fact that debt capital gives shareholders an incentive to invest in high-risk projects. They might therefore oppose new borrowing by a company when they think that this will put their interest (the security of their investment and returns) at risk.

Jensen and Meckling argued that the 'optimal' capital structure for a company is obtained by trading off not just the marginal benefits and marginal costs of extra debt (as suggested by static trade-off theory) but also by trading off the 'agency costs' of additional debt and the 'agency costs' of additional equity.

## Adjusted present value method (APV method) of project appraisal

- Introduction to the APV method
- The decision rule for the APV method
- Base case NPV
- PV of other costs
- Present value of the tax shield (PV of the tax relief on interest costs)
- Summary of the APV method
- Reasons for using the APV method
- Comparison of NPV and APV methods


## 2 Adjusted present value method (APV method) of project appraisal

### 2.1 Introduction to the APV method

The most common method for a company to evaluate a proposed capital expenditure project is the discount the expected future cash flows of the project at the WACC. Where appropriate, the cost of equity and WACC may be adjusted to allow for the different systematic risk characteristics of the project.

An alternative approach to capital investment appraisal is the adjusted present value (APV) method.

The APV method may be used when a business entity is considering an investment in a project that will have different business risks and different financial risks from its current operations. For example, a business entity might wish to evaluate an investment in a different industry or a different market, and raise new capital to finance the investment.

The APV method is an alternative to calculating a new cost of equity and a new WACC, for example using the Modigliani-Miller formulae or the asset beta formula.

### 2.2 The decision rule for the APV method

A project is financially viable and should be undertaken if its adjusted present value (APV) is positive. The APV of a project contains three elements, and is calculated as follows:

|  | Base case NPV |
| :--- | :--- |
| minus | PV of other costs |
| plus | PV of tax relief on interest |
|  | Adjusted present value (APV) |

To calculate the NPV of a project, we must therefore calculate three amounts: the base case NPV, the PV of other costs and the PV of tax relief on interest.

### 2.3 Base case NPV

The approach with the APV method is to calculate an NPV ignoring entirely the way in which the project will be financed. The cost of capital should therefore be calculated using an asset beta that is typical for the industry or type of business in which the investment will be made. Having estimated an appropriate asset beta, a cost of equity (cost of equity in an ungeared company) can be calculated using the CAPM, and this is the discount rate.

The base case NPV is therefore calculated assuming the project is financed entirely by equity, so that the method of financing is ignored.

It is therefore necessary to calculate the cost of equity in an all-equity company in the same industry or the market in which the capital investment will be made. To do this:

- convert a geared beta for the industry to an asset beta (ungeared beta) for the industry, and then
- use this asset beta (ungeared beta) and the CAPM to establish the cost of equity in an ungeared company.

Normal DCF techniques are used to establish the expected cash flows for the project. Having established a cost of equity for an ungeared company, the expected project cash flows are discounted at this cost of equity to obtain the base case NPV.

## Example

A company operating in the insurance industry is considering whether to diversify by investing in a project in the transport industry.

The company has a gearing ratio of $30 \%$ debt and $70 \%$ equity, and its equity beta is 0.940 . Its debt capital is risk-free.

The transport industry has an average equity beta of 1.362 , and firms in the transport industry on average have a gearing ratio of $40 \%$ debt to $60 \%$ equity.

The risk-free rate of return is $5.3 \%$ and the expected market return is $8.3 \%$.
The rate of taxation on profits is $23 \%$.
The cash flows of the project after tax will be:

| Year 0 | $\$(600,000)$ |
| :--- | ---: |
| Years 1-3 | $\$ 250,000$ |

## Required

Calculate the base case NPV.

## Answer

The all-equity beta for the transport industry is calculated by un-gearing the equity beta for the transport industry.
$\beta_{\mathrm{A}}=1.362 \times \frac{60}{60+40(1-0.23)}$
$\beta_{\mathrm{A}}=0.90$.
Using the CAPM, we can now calculate the cost of an all-equity financed company in the transport industry:
$5.3 \%+0.90(8.3 \%-5.3 \%)=8.0 \%$.
This cost of capital should be used to discount the project cash flows, to obtain the base case NPV.

| Year | Cash flow | Discount <br> factor at $\mathbf{8 \%}$ | PV |
| :--- | ---: | :---: | ---: |
|  | $\$$ |  | $\$$ |
| 0 | $(600,000)$ | 1.000 | $(600,000)$ |
| $1-3$ | 250,000 | 2.577 | 644,250 |
| Base NPV |  |  | $+\mathbf{+ 4 4 , 2 5 0}$ |

### 2.4 PV of other costs

Other costs are the costs arising a consequence of the project, but not directly related to the project cash flows. Typically, these costs include the costs of raising new equity to finance the project, or the costs of obtaining debt finance.

Issue costs might be an allowable expense for tax purposes. When they are taxallowable, the PV of issue costs must allow for the reduction in tax payments that will occur. The PV of the issue costs is therefore net of the present value of any tax relief on the costs.

Unless you are given other information, you should assume that the PV of other costs is calculated using the risk-free cost of capital as the discount rate.

## Examination hint

If a company plans to raise new capital to finance a project, it will want to raise enough capital to finance the project after paying the issue costs. For example, suppose that a company intends to issue new equity to raise $\$ 20$ million and issue costs are 5\% of the amount raised. The company needs $\$ 20$ million after paying the issue costs, and presumably the issue costs will be paid for out of the proceeds from raising the finance.
$\$ 20$ million is therefore $95 \%(100 \%-5 \%)$ of the capital to be raised.
The capital to be raised $=\$ 20$ million $/ 0.95=\$ 21,052,632$.

This means that the issue costs will be $\$ 1,052,632$.
Since the finance will be raised at the beginning of the project, the PV of the issue costs for the equity is $\$ 1,052,632$, minus the PV of the tax relief on the issue costs.

## Example

Assume that in the previous example, the investment of $\$ 600,000$ would be financed by $\$ 400,000$ of new equity and $\$ 200,000$ of new debt.

Issue costs are $5 \%$ of the funds raised for equity and $2 \%$ of the funds raised for debt capital.

The risk-free cost of capital is $5.3 \%$. The rate of taxation on profits is $23 \%$. Issue costs are allowable for tax purposes.

## Required

Calculate the PV of the issue costs for financing the project. Assume that tax is paid in the year following the year in which the taxable profit occurs.

## Answer

The finance required is:

- Equity: $\$ 400,000$ after issue costs and $\$ 400,000 / 0.95=\$ 421,053$ before issue costs.
- Debt: $\$ 200,000$ after issue costs and $\$ 200,000 / 0.98=\$ 204,082$ before issue costs

| Issue costs before tax: |  | $\$$ |
| :--- | ---: | ---: |
| Equity | $\$ 400,000 \times 5 / 95$ | 21,053 |
| Debt | $\$ 200,000 \times 2 / 98$ | 4,081 |
| Total issue costs |  | 25,134 |

The PV of issue costs is calculated using the risk-free rate of $5.3 \%$ as the discount rate. There is tax relief on the issue costs, which will be received one year in arrears.

|  |  | Discount factor at |  |  |
| :--- | :--- | ---: | :---: | ---: |
| Year | Item | Cash flow | $5.3 \%$ | PV |
|  |  | $\$$ |  | $\$$ |
| 0 | Issue costs | Tax saved at $23 \%$ | $5,134)$ | 1.000 |
| 1 | 5,781 | $1 /(1.053)$ | $(25,134)$ |  |
| PV of issue costs |  |  | 5,490 |  |

### 2.5 Present value of the tax shield (PV of the tax relief on interest costs)

When a new project is financed wholly or partly with new debt finance, there will be tax relief on the interest. The PV of these tax benefits should be included in the APV of the project.

The PV of the tax relief on interest is calculated by:

- calculating the interest costs in each year

■ calculating the savings in taxation arising as a consequence, for each year of the project

- discounting these savings in taxation to a present value, using the pre-tax (before-tax) rate of interest on the debt as the discount rate.


## Calculating interest payments

The amount of interest costs in each year will depend on the terms of repayment of the debt capital.

- If the debt finance is repaid in full at the end of the term of the borrowing, the interest cost is calculated simply as:
Amount borrowed $\times$ Interest rate on the loan or the bonds
- If the loan is an amortising loan, and the debt is repaid gradually over the term of the loan, you will need to work out:
- the annual payment on the loan, principal repayment + interest, and
- separate this total annual payment into the principal repayment and the interest payment.

This separation of interest charges and the loan principal repayment is necessary because the tax relief applies to the interest payments only and not to the repayment of the loan principal.

## Example

In the previous example, suppose that the company obtains borrows $\$ 204,081$ in the form of a three-year amortising loan at $5.3 \%$ interest.

The annuity factor for years $1-3$ at $5.3 \%$ is 2.708 .
The rate of taxation is $23 \%$ and tax is payable in the year following the profit to which the tax relates.

## Required

Calculate the PV of the interest tax shield.

## Answer

The annual payments on the amortising loan can be calculated using the annuity factor for Years $1-3$ at $5.3 \%$, which is the cost of borrowing:

Amount borrowed
Annuity factor at 5.3\%, Years 1-3 $=\frac{\$ 204,081}{2.708}=\$ 75,362$

The interest costs each year are calculated on the amount of the loan principal still unpaid at the beginning of the year. The amount of principal repaid each year is the difference between the total annual payment and the interest charge. In this example, the figures are calculated as follows:

| Year | Loan principal unpaid | Interest at 5.3\% | Loan princi | repaid |
| :---: | :---: | :---: | :---: | :---: |
|  | \$ | \$ |  | \$ |
| 1 | $\begin{aligned} & 204,081 \\ & (64,546) \end{aligned}$ | 10,816 | (75,362-10,816) | 64,546 |
| 2 | $\begin{aligned} & 139,535 \\ & (67,967) \end{aligned}$ | 7,395 | (75,362-7,395) | 67,967 |
| 3 | $\begin{array}{r} 71,568 \\ (71,569) \\ \hline \end{array}$ | 3,793 | (75,362-3,793) | 71,569 |
|  | (1) | (rounding error) |  |  |

Having calculated the interest payments on the debt finance, we can now calculate the tax relief on the interest payments, and the PV of this tax relief.

| Year | Item |  | Cash flow | Discount <br> factor at 5.3\% | PV |
| :--- | :--- | ---: | ---: | ---: | ---: |
|  |  | $\$$ |  | $\$$ |  |
| 2 | Tax relief on Year 1 interest | $(23 \% \times 10,816)$ | 2,488 | $1 /(1.053) 2$ | 2,244 |
| 3 | Tax relief on Year 2 interest | $(23 \% \times 7,395)$ | 1,701 | $1 /(1.053) 3$ | 1,456 |
| 4 | Tax relief on Year 3 interest | $(23 \% \times 3,793)$ | 872 | $1 /(1.053) 4$ | 709 |
| PV of tax shield |  |  |  | 4,409 |  |

### 2.6 Summary of the APV method

Taking the three previous examples together, the APV of the investment project would be calculated as follows:

|  | $\$$ |
| :--- | ---: |
|  | Base case NPV |
| $-\quad$ PV of other costs | 44,250 |
| + | PV of tax relief on interest |
| $=$ | APV |

The APV of the project is positive; therefore on financial considerations (but ignoring the project risk) the project is worthwhile and should be undertaken.

### 2.7 Reasons for using the APV method

The APV method might be used in preference to adjusting the weighted average cost of capital (WACC) of the company using the Modigliani-Miller formulas. This is for several reasons.

- The APV method does not rely on assumptions about the new WACC of the firm if the project is undertaken.
- The APV method allows for the specific tax relief on the borrowing to finance the project, and does not assume that the debt will be perpetual debt.
- The APV method allows for other costs, such as the costs of raising new finance (issue costs).

It might be argued that APV is therefore the best method of estimating the effect of a new investment on the value of the business entity, and the wealth of its shareholders.

### 2.8 Comparison of NPV and APV methods

The NPV method of project evaluation and the APV method provide different valuations for proposed capital investment projects. It is possible that one method might indicate that a project is acceptable financially, whereas the other method indicates that the project is not viable.

The NPV and the base case NPV are calculated using the same cash flows, except that the cash flows for the base case NPV should exclude 'other costs' such as financing costs, whereas these are included in the calculation of an NPV.

## Example

A company is considering whether to invest in setting up a new business. The cost of the investment would be $\$ 9$ million. The project would be financed by raising $\$ 4.5$ million of new equity finance and the same amount of new debt finance. The $\$ 9$ million would be used to purchase assets for $\$ 8$ million. There would be an investment of $\$ 700,000$ in working capital, and issue costs would be $\$ 200,000$ for the equity and $\$ 100,000$ for the debt finance.

The purchased assets would attract capital allowances in the form of a $25 \%$ annual writing down allowance on a reducing balance basis.

The pre-tax operating cash flows from the investment are expected to be as follows:

| Year | Pre-tax operating <br> cash flows |
| :---: | :---: |
|  | $\$ 000$ |
| 1 | 1,800 |
| 2 | 2,100 |
| 3 | 2,400 |
| 4 | 2,700 |

Tax on profits is at the rate of $30 \%$. Assume that tax payments occur in the same year as the profit or benefit to which they relate.

At the end of Year 4, the intention is to sell off the business as a continuing operation, and the expected receipts from the sale, net of tax, are $\$ 3.60$ million.

The following information is also relevant:
(1) The investment will alter the company's gearing to $80 \%$ equity and $20 \%$ debt capital.
(2) The equity beta for the project is 1.25 .
(3) The debt finance for the project will be the same as for the company's other debt, which is $7 \%$ before tax.
(4) The issue costs of raising the new equity and debt finance would not be allowable for tax purposes.
(5) The risk-free rate of return is $5 \%$ and the market return is $9 \%$.

## Required

(a) Calculate the NPV of the project.
(b) Calculate the APV of the project.

Answer

## Workings: Capital allowances and tax relief

| Year | Written down value | Capital allowance <br> $\mathbf{2 5 \%}$ | Tax relief <br> $\mathbf{3 0 \%}$ |
| :--- | :---: | :---: | :---: |
|  | $\$ 000$ | $\$ 000$ | $\$ 000$ |
| 1 | 8,000 |  |  |
|  | $(2,000)$ | 600 |  |
| 2 | $\frac{(1,000}{4,500}$ |  |  |
| 3 | $\frac{(1,125)}{3,375}$ | 1,500 | 450 |
| 3 | $(844)$ | 1,125 | 338 |
| 4 |  | 844 | 253 |

There is no balancing charge, because it is assumed that the disposal value at the end of year 4, net of tax, includes the estimated tax effect of the disposal of the business.

## Net present value

Cost of equity $\mathrm{K}_{\mathrm{E}}=5 \%+1.25(9-5) \%=10 \%$.
$W A C C=[80 \% \times 10 \%]+[20 \% \times 7 \% \times(1-0.30)]=8.98 \%$, say $9 \%$.

| Year | 0 | 1 | 2 | 3 | 4 |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | \$000 | \$000 | \$000 | \$000 | \$000 |
| Investment | $(9,000)$ |  |  |  |  |
| Sale value |  |  |  |  | 3,600 |
| Operating cash flow |  | 1,800 | 2,100 | 2,400 | 2,700 |
| Tax (30\%) |  | (540) | (630) | (720) | (810) |
| Tax relief (workings) |  | 600 | 450 | 338 | 253 |
| Net cash flow | $(9,000)$ | 1,860 | 1,920 | 2,018 | 5,743 |
| Discount factor 9\% | 1.000 | 0.917 | 0.842 | 0.772 | 0.708 |
| Present value | $(9,000)$ | 1,706 | 1,617 | 1,558 | 4,066 |
| NPV $=-\mathbf{5 3 , 0 0 0}$ |  |  |  |  |  |

## Adjusted present value

The project will be financed $50 \%$ with equity and $50 \%$ with debt capital.
Asset beta $=1.25 \times 4,500 /[4,500+(4,500 \times 0.70)]=0.7353$.
Cost of ungeared equity $\mathrm{K}_{\mathrm{EU}}=5 \%+0.7353(9-5) \%=7.94 \%$, say $8 \%$.

The cash flows for discounting are the same as for the NPV method, with the exception that issue costs should be removed. These are \$300,000 in Year 0.

| Year | 0 | 1 | 2 | 3 | 4 |
| :--- | ---: | ---: | ---: | ---: | ---: |
|  | $\$ 000$ | $\$ 000$ | $\$ 000$ | $\$ 000$ | $\$ 000$ |
| Net cash flow | $(8,700)$ | 1,860 | 1,920 | 2,018 | 5,743 |
| Discount factor $8 \%$ | 1.000 | 0.926 | 0.857 | 0.794 | 0.735 |
| Present value | $(8,700)$ | 1,722 | 1,645 | 1,602 | 4,221 |

Base case NPV = + \$490,000

## Other costs

There is no tax relief on the issue costs, so the PV of issue costs (all in Year 0) is \$300,000.

## PV of the tax shield

It is assumed that interest costs each year will be $\$ 4,500,000 \times 7 \%=\$ 315,000$.
Tax relief on these interest costs at $30 \%$ will be $\$ 94,500$ per year for 4 years.
This tax relief should be discounted at $7 \%$.
PV of tax relief on interest $=\$ 94,500 \times 3.387=\$ 320,000$.

## Summary

|  | $\$ 000$ |
| :--- | ---: |
| Base case NPV | 490 |
| - | PV of other costs |
| + | PV of tax relief on interest |
|  | APV |

The APV is $+\$ 510,000$, indicating that the project should be undertaken, whereas the NPV is negative, indicating that it should not be undertaken.

In this particular example, the APV and NPV methods give conflicting recommendations.

## Capital rationing

- The nature of capital rationing
- Single period capital rationing
- Multi-period capital rationing and the simplex method
- The simplex method and your examination


## 3 Capital rationing

### 3.1 The nature of capital rationing

Capital investment decisions might be affected by capital rationing. Capital rationing occurs when there is not enough capital available to invest in every project with a positive NPV that the company would like to undertake. Since capital is in short supply, a decision has to be made about which projects to invest in with the capital that is available.

A distinction is sometimes made between 'hard' and 'soft' capital rationing.

- Hard capital rationing occurs when there is a real shortage of capital for investment. For example, a company might be unable to raise new capital in the capital markets or borrow large amounts from a bank. Its capital for investment might therefore be limited to the amount of capital it adds to the business each year as cash flows from retained profits.
- Soft capital rationing occurs when there is sufficient capital to invest in every project, but management has taken a policy decision that spending on capital investment should be limited to a budgeted maximum amount. The policy decision therefore sets a limit on the amount of capital available.

Several methods have been devised for indicating which projects should be selected for investment when there is capital rationing. These methods are based on the following assumptions:

- The choice of projects or investments should have the objective of maximising total NPV.
- The projects that are available for investment are comparable in terms of risk.

The method used to identify which projects to select depends on how many years of capital rationing there will be. Capital rationing might be for one year only (single period capital rationing) or might be for several years, and the investment projects will require some additional investment in each of the years when there will be capital rationing (multi-period capital rationing).

### 3.2 Single period capital rationing

When there is capital rationing in one year only (usually Year 0), the selection of projects for investment is normally straightforward. The method used depends on which of two assumptions should be used:

- Alternative assumption 1: The projects available for selection are indivisible. The choice is between investing in $100 \%$ of a project or not investing in the project at all.
- Alternative assumption 2: The projects available for investment are divisible, although there is a maximum amount that can be invested in any project. This second assumption might seem unusual, although it is likely to feature in an examination question on capital rationing.

When alternative 1 is the assumption, the combination of projects should be selected:

- for which there is sufficient capital and
- which will maximise the total NPV.

When alternative 2 is the assumption, the combination of projects should be selected:

- which uses all the capital available for investment, and
- maximises the NPV per $\$ 1$ invested in the year of capital rationing.


## Example

A company has $\$ 20$ million to invest. There are four projects available for investment, all similar in terms of risk. The amount of investment required and the NPV of each project are as follows:

| Project | Capital required <br> in Year 0 <br> $\$ m$ | Capital required <br> in Year $\mathbf{1}$ <br> $\$ m$ | NPV |
| :--- | :---: | :---: | ---: |
| A | 8 | 0 | $\$ m$ |
| B | 5 | 1 | +3.0 |
| C | 6 | 0 | +2.0 |
| D | 10 | 2 | +2.5 |
|  |  |  | +4.2 |

## Required

(a) Which projects should be selected if they are not divisible?
(b) Which projects should be selected if they are divisible?

## Answer

## (a) Projects are not divisible

If the projects are not divisible, the first step is to identify the combinations of projects that may be selected with the capital available in Year 0. The total

NPV for each combination should be calculated and the combination with the highest total NPV should be selected.

| Combination <br> of projects | Total capital <br> required in Year <br> $\mathbf{0}$ |  | Total <br> NPV |
| :--- | :---: | :---: | ---: |
|  | $\$ \mathrm{~m}$ |  |  |
| A + B + C | 19 | $(3+2+2.5)$ | +7.5 |
| A + D | 18 | $(3+4.2)$ | +7.2 |
| B + D | 15 | $(2+4.2)$ | +6.2 |
| C + D | 16 | $(2.5+4.2)$ | +6.7 |

Here, the combinations are easy to identify, and total NPV will be maximised by investing in projects $\mathrm{A}, \mathrm{B}$ and C .
(b) Projects are divisible

If the projects are divisible and there is a maximum investment in each project, the aim should be to maximise the NPV per $\$ 1$ invested in Year 0 (the year of capital rationing: Year 1 is ignored because there is no capital rationing in Year 1).

| Project | NPV | Capital <br> required in <br> Year 0 | NPV per \$1 <br> invested in <br> Year 0 | Priority for <br> investment |
| :--- | :---: | :---: | :---: | :---: |
|  | $\$ m$ | $\$ m$ | $\$ m$ |  |
| A + B + C | 3.0 | 8 | 0.375 | $4^{\text {th }}$ |
| A + D | 2.0 | 5 | 0.400 | $3^{\text {rd }}$ |
| B + D | 2.5 | 6 | 0.417 | $2^{\text {nd }}$ |
| C + D | 4.2 | 10 | 0.420 | $1^{\text {st }}$ |

The choice of investments should be as follows:
$\left.\begin{array}{lcccc}\text { Project } & \text { Priority } & \begin{array}{c}\text { Capital } \\ \text { required in } \\ \text { Year 0 }\end{array} & \begin{array}{c}\text { NPV per \$1 } \\ \text { invested in } \\ \text { Year 0 }\end{array} & \begin{array}{c}\text { Total } \\ \text { NPV }\end{array} \\ & & \$ m & \$ \mathrm{~m}\end{array}\right)$

Total NPV is maximised by sending the $\$ 20$ million in Year 0 by investing in $100 \%$ of Projects D and C, and investing the remaining $\$ 4$ million in $80 \%$ of Project B. The total NPV will be (in \$ million) $4.2+2.5+(80 \% \times 2)=\$ 8.3$ million.

The total NPV should always be higher when projects are divisible than when they are not divisible.

### 3.3 Multi-period capital rationing and the simplex method

When there is capital rationing in more than one year, and some or all of the projects require additional finance in each year where there will be capital rationing, a different method is needed to identify the combination of projects that will maximise total NPV, where the projects are divisible.

The mathematical technique used to identify the NPV-maximising combination of projects is called linear programming, and the technique of linear programming you are most likely to encounter is the simplex method.

For your examination, you are not required to solve a linear programming problem. However, you should understand how to formulate a linear programming problem, and how the simplex method works.

The method will be explained using a simple example. (The example is simple because there is capital rationing in two years only, and there are only three projects. The same technique can be applied to problems when there is capital rationing in more than two years, and when there are many different divisible projects available for selection.)

## Example

A company has $\$ 9$ million to invest in Year 0 and $\$ 6$ million to invest in Year 1. There are three projects available for investment, all similar in terms of risk. The amount of investment required and the NPV of each project are as follows:

| Project | Capital required <br> in Year 0 <br> $\$ m$ | Capital required <br> in Year 1 | NPV |
| :--- | :---: | :---: | :---: |
| X | 6 | 3 | $\$ \mathrm{~m}$ |
| Y | 2 | 3 | +4.0 |
| Z | 5 | 1 | +2.0 |
|  | 5 | 2 | +2.0 |

## Required

Which projects should be selected if they are all divisible?

## Answer

There is capital rationing in both Year 0 and Year 1 because in each year the capital available for investment is less than the total needed to invest in all three projects.

## Formulating the linear programming problem

A linear programming problem is stated as an objective function that is subject to certain constraints or limitations. The objective function is to maximise or minimise something. With capital rationing, the objective function should be to maximise the total NPV, and the objective function can be stated as follows:

## Objective function:

Maximise $4 x+2 y+2 z$
where $\mathrm{x}, \mathrm{y}$ and z are the proportion of Projects $\mathrm{X}, \mathrm{Y}$ and Z respectively chosen for investment.

There are five constraints on investment. These are the amounts of capital available in Year 0 and Year 1, and the fact that it is impossible to invest in more than $100 \%$ of a project.

The five constraints can be stated as follows:

| Year 0 capital | $6 x+2 y+5 z$ | $\leq$ | 10.2 |
| :--- | ---: | :--- | :--- |
| Year 1 capital | $3 x+y+2 z$ | $\leq 6.0$ |  |
| Maximum investment in X | x | $\leq 1.0$ |  |
| Maximum investment in Y | y $\leq 1.0$ |  |  |
| Maximum investment in Z | z | $\leq 1.0$ |  |

(Note: The symbol $\leq$ means 'is less than or equal to'.)
There are also non-negativity constraints, which mean that the value of $x, y$ and $z$ cannot be a negative amount.

If there is a more complex problem, with capital rationing in more years, there will be a constraint for each year in which the capital rationing will occur.

## Formulating and interpreting the initial simplex tableau

The simplex method can be used to find the optimal solution to the linear programming problem. The method is to test feasible solutions to the linear programming, one feasible solution at a time, until the optimal solution is found that maximises (or minimises) the value of the objective function.

Each feasible solution is tested in a table or 'tableau'. A tableau should have:

- one column for each variable in the problem, one column for each constraint, and a total column
- one row for each variable in the solution and a row for the objective function. There will always be exactly as many variables in the solution as there are constraints in the linear programming problem. In our example, this is 5 .

In addition, we need to introduce some new variables into the problem. There should be one variable for each constraint.

- Let $\mathrm{Y}_{0}=$ the number of unused capital in Year 0
- Let $Y_{1}=$ the number of unused capital in Year 1 .
- Let $\mathrm{U}_{x}=$ the proportion of Project X not invested in.
- Let $U_{y}=$ the proportion of Project $X$ not invested in.
- Let $\mathrm{U}_{z}=$ the proportion of Project X not invested in.

The initial tableau for this problem is constructed as follows:

| Variable $\quad$ in the <br> solution | $\mathbf{x}$ | $\mathbf{y}$ | $\mathbf{z}$ | $\mathbf{Y}_{\mathbf{0}}$ | $\mathbf{Y}_{\mathbf{1}}$ | $\mathbf{U}_{\mathbf{x}}$ | $\mathbf{U}_{\mathbf{y}}$ | $\mathbf{U}_{\mathbf{z}}$ | Total <br> (million) |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| $\mathbf{Y}_{\mathbf{0}}$ |  | 6 | 2 | 5 | 1 | 0 | 0 | 0 | 0 |
| $\mathbf{Y}_{\mathbf{1}}$ | 3 | 1 | 2 | 0 | 1 | 0 | 0 | 0 | 6 |
| $\mathbf{U}_{\mathbf{x}}$ | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 1 |
| $\mathbf{U}_{\mathbf{y}}$ | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 1 |
| $\mathbf{U}_{\mathbf{z}}$ | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 1 |
| Objective function | -4 | -2 | -2 | 0 | 0 | 0 | 0 | 0 | 0 |

Notice that the figures in each row of the $x, y$ and $z$ columns are taken directly from the linear programme, and the objective function on the bottom row shows the NPV of each project with a minus sign.

The initial tableau tests the feasible solution that there is 9 million of unused capital in Year 0 and 6 million of unused capital in Year 1. The proportion of each project not invested in is $1.0(100 \%) Y_{0}, Y_{1}, U_{x}, U_{y}$, and $U_{z}$ are therefore the five variables in this feasible solution, and the values of $x, y$ and $z$ are 0 . The total NPV (bottom row, total column) is $\$ 0$ million.

## Testing other feasible solutions

This is clearly not the optimal solution, and the simplex method now produces another feasible solution where the value of the objective function is higher. The second tableau might be as follows:

| Variable $\quad$ in the <br> solution | $\mathbf{x}$ | $\mathbf{y}$ | $\mathbf{z}$ | $\mathbf{Y}_{\mathbf{0}}$ | $\mathbf{Y}_{1}$ | $\mathbf{U}_{\mathbf{x}}$ | $\mathbf{U}_{\mathbf{y}}$ | $\mathbf{U}_{\mathbf{z}}$ | Total <br> (million) |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| $\mathbf{Y}_{0}$ |  | 0 | 2 | 5 | 1 | 0 | -6 | 0 | 0 |

This tableau has introduced $x$ into the solution in place of $U_{x}$.

In this solution, there will be an investment in $100 \%$ of project $X$ (since $x=1.0$ in the tableau). This will leave unused capital of $\$ 3.0$ million in Year 0 and $\$ 3.0$ million in Year 2. The total NPV will be $\$ 4$ million.

This is not the optimal solution, because there are still some minus signs in the objective function row, and for every unit of y or z that is introduced into the solution, the total NPV can be increased by $\$ 2$ million.

## The final solution and its interpretation: dual prices or shadow prices

| Variable in the | $\mathbf{x}$ | $\mathbf{y}$ | $\mathbf{z}$ | $\mathbf{Y}_{0}$ | $\mathbf{Y}_{\mathbf{1}}$ | $\mathbf{U}_{\mathbf{x}}$ | $\mathbf{U}_{\mathbf{y}}$ | $\mathbf{U}_{\mathbf{z}}$ | Total <br> (million) |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | ---: | ---: |
| solution |  |  |  |  |  |  |  |  |  |
| Project Z (z) | 0 | 2 | 5 | 1 | 0 | -6 | 0 | 0 | 0.2 |
| $\mathbf{Y}_{1}$ | 0 | 1 | 2 | 0 | 1 | -3 | 0 | 0 | 1.6 |
| Project $\mathbf{X}(\mathbf{x})$ | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 1.0 |
| Project $\mathbf{Y}(\mathbf{y})$ | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 1.0 |
| $\mathbf{U}_{\mathbf{z}}$ | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 0.8 |
| Objective function | 0 | 0 | 0 | 0.6 | 0 | 1.6 | 0.8 | 0 | 6.4 |

The final tableau is shown here, with the solution that maximises total NPV. The problem in this example is fairly simple, so the solution is quite straightforward.

To maximise total NPV, the company should invest in $100 \%$ of Project $\mathrm{X}, 100 \%$ of Project Y and $20 \%$ of Project Z (since $\mathrm{x}=1.0, \mathrm{y}=1.0$ and $\mathrm{z}=0.2$ ). This will leave unused capital of $\$ 1.6$ million in Year 1 (since $Y_{1}=1.6$ ). The proportion of Project $Z$ not invested in is 0.8 .

Total NPV will be $\$ 6.4$ million.
The solution also shows the dual prices or shadow prices of the variables that are not in the solution. These are $Y_{0}, U_{x}$ and $U_{y}$.

- All the available capital in Year 0 is used up by the solution. The dual price indicates that if $\$ 1$ of extra capital could be made available in Year 0, the total NPV could be increased by $\$ 0.4$.
- The maximum investment is made in project $X$, so $U_{x}=0$ in the final solution. The dual price for $U_{x}$ indicates that if the maximum investment in project $X$ could exceed $100 \%$, the total NPV could be increased by $\$ 1.6$ million for every additional project X that is available.
- The maximum investment is made in project Y , so $\mathrm{U}_{\mathrm{y}}=0$ in the final solution. The dual price for $\mathrm{U}_{\mathrm{y}}$ indicates that if the maximum investment in project Y could exceed $100 \%$, the total NPV could be increased by $\$ 0.8$ million for every additional project Y that is available.

Dual prices for projects are not particularly significant. However, the dual price or shadow price for capital is significant. It shows by how much total NPV could be increased if more capital could be made available in that year (given no change in any other constraint in the problem).

For example, this solution indicates that if the available capital in Year 0 could be increased by, say, $\$ 1$ million, from $\$ 9$ million to $\$ 10$ million, there would be a different optimal solution and the total NPV for this solution would be higher by $\$ 400,000$ ( $\$ 1$ million $\times 0.4$ ).

### 3.4 The simplex method and your examination

The study guide for the examination syllabus indicates that you need to know about 'multi-period capital rationing to include the formulation of programming methods and the interpretation of their output'. This indicates that some knowledge is required for the examination of linear programming and the use of the simplex method for solving capital rationing problems.

You should therefore try to familiarise yourself with the nature of a simplex tableau, and what the solution tells you (including the meaning of dual prices or shadow prices).

## Real options

- Definition of a real option
- Real options theory
- Real options: the theoretical framework
- Different types of real option
- The valuation of real options


## 4 Real options

### 4.1 Definition of a real option

Options are normally associated with financial market derivative instruments and share options of companies. A different type of option is a real option, sometimes called an 'embedded real option'.

A real option is an alternative or choice that exists with an investment project or investment opportunity. They are real choices that management will be able to make at some stage during the life of the project. For example, if a company undertakes a particular project, there might be a real option (a choice, and not an obligation) to take one or more of the following courses of action at some stage during the project's life:

- Make a further incremental investment, and increase the total amount invested in the project.
- Abandon the project early because it is losing money.
- Defer the investment in the project, instead of having to invest immediately.

For example, a mining company might be negotiating a ten-year lease on a mine, and it might want to include a 'break clause' (an option) in the lease contract that will enable it to cancel the lease after five years. The option to cancel the lease and abandon the investment would be a real option. The company would have to consider how much it would be prepared to pay for the break clause to be included in the terms of the lease.

They are called 'real' options because they are generally associated with choices in relation to 'real' tangible assets.

Real options provide flexibility by giving choice - the flexibility for example to avoid losses and to take opportunities that unexpectedly emerge.

### 4.2 Real options theory

Real options theory can be used for investment appraisal, as an alternative to simple net present value analysis.

- Like NPV analysis, investment appraisal with real options analysis calculates a present value for a project by discounting future cash flows.
- Unlike NPV analysis, a real options approach also puts a financial value to the real options that are embedded in the investment opportunity, and which have some value. The value of the real options is taken into consideration in deciding whether to invest, or which of two or more alternative projects to invest in.

There are two aspects to real options theory:

- providing a theoretical framework for the valuation of real options in investment appraisal
- developing methods of measuring the value of real options for the purpose of investment appraisal.


### 4.3 Real options: the theoretical framework

When a company decides to invest in a capital project, it faces the risk that actual results will be different from what was expected when the investment decision was made. The risk in a project comes from a combination of:

- the uncertainty about the future, and the different situations that might arise, and
- the means that management have at their disposal to deal with any of these situations if they arise.

There are many different sources of uncertainty in a project. They include the risks from:

- changes in the condition of the overall economy
- technological change
- a change in regulations
- unforeseen actions by a competitor.

Any of these events or changes in situation could affect the value of the investment.
Management deals with uncertainty in two stages.

- Stage 1. At the project selection stage. When the investment decision is made, management considers the different possible outcomes and assesses the risk in the project. Different investment options are considered, and management decides to invest in the projects that appear to offer the best prospects for a good return.
- Stage 2. Project management. After the decision has been made to invest in a project, the project is managed. Management takes action to reduce the risks and uncertainty in the project, and seeks to enhance the positive effects of unexpected events.

Using the NPV method of project selection, the financial analysis focuses entirely on the project selection stage. All the different possible outcomes for the project are considered, and these are aggregated into a single expected value of future cash flows. The NPV analysis does not consider all the future decisions that might be made during the project management stage that will be contingent or dependent on how the project develops.

## Example

A simple example is a management decision about whether to invest in Project A or Project B, which are mutually exclusive projects. Both projects would last for ten years. Estimated cash flows have been prepared and evaluated for each project and Project A has a higher NPV.

However, with Project B there would be an opportunity to withdraw from the project after five years if the returns are poor and the project is value-destroying. This option does not exist with Project A.

Simple NPV analysis would not consider the real option with Project B to withdraw after five years, and Project A would be selected. With a real options analysis, the comparison between Project A and Project B would put a value on the real option with Project B, and take this option into consideration when making the choice between the two projects.

## Real options: analysing project management risk and flexibility

The NPV approach to project appraisal helps management with the project selection process but is not concerned with active risk management. In making choices between alternative projects for investment, it does not allow for the different amounts of flexibility (real options) that each project provides. For example, one project involving the use of more expensive but more flexible technology might have a lower NPV than an alternative investment involving the use of cheaper but more rigid technology. The flexible use of the more expensive technology provides a real option with value, but with simple NPV analysis this factor would be ignored.

An approach to project appraisal using real options considers all the different possibilities that might arise during the project management stage of a capital investment project. Instead of starting with an 'aggregate' or 'expected value' scenario of what might happen in the future, real options analysis starts by trying to identify all the future events or developments that might occur that would affect the project's value. The differing 'volatilities' of alternative investments are critical, and some projects will involve more uncertainty than others.

Once the risk profile of each investment alternative has been understood, the next step in the analysis is to identify the major project management decisions that could be made (the real options available with each investment alternative). Real options provide flexibility for project management, and these real options should be evaluated and taken into consideration (both at the project selection stage and during project management).

### 4.4 Different types of real option

There are different types of real option. The examination syllabus refers to real options to 'delay, expand, re-deploy and withdraw'. A more detailed analysis of the types of real option is set out below.

| Invest/grow options | Scale up | Make sequential, incremental investments as the markets grow. This type of real option includes start-up options. |
| :---: | :---: | :---: |
|  | Switch up | A speedy commitment to a first generation of a technology migh give a company an advantage if the opportunity eventually occurs to switch to a next-generation of the technology. This might be called a 'market power option'. |
|  | Scope up | Real option that provides an opportunity to increase the scope or range of activities. For example, investing in one industry might give a company the opportunity to invest in another industry at the same time and at a low cost. |
| Defer/delay/learn options | Sundry start | Option to delay an investment until new information is available or new skills are acquired. A deferral option. |
|  | Scale down | Option to reduce the size of a project part-way through its life, or to disinvest entirely and withdraw from the investment. This option might be used if new information changes the expected payback from the project. |
| Divest/withdraw/reduce options | Switch down | Where the company has invested in flexible assets, there might be an option to switch to a more costeffective use of the assets |
|  | Scope down | There might be an option for a company to reduce the scope of its operations at a low cost if it is discovered that there is no potential for further business development. |

### 4.5 The valuation of real options

To estimate the value of a project using a real options approach, it is necessary to identify when and under what conditions each real option would be exercised, and a value has to be calculated for each option. For each real option that is identified for a particular project, the following data should be obtained:

- When the option will be available during the project life, and for how long.
- How the option might have a positive effect on the value of the project, and what that effect might be.
- What additional investment would be needed to exercise the option.
- Whether any additional investment is needed now to make the option available in the future.
- Whether the exercise of the option will then make other follow-up options available at a later time.

Each investment project should be assessed both for their NPV and the value of the future opportunities they might provide during the project management stage, such as the opportunity to expand in the same or related markets, or the flexibility to change the level or scale of operations, or for the ease of halting further expenditure or withdrawing entirely from an investment at a low cost.

Real options analysis puts a financial value to flexibility in the management of projects, and the value of real options is therefore higher when the project risk and uncertainty is high.

However, the real options approach has an important weakness. A considerable amount of academic research has been carried out into the valuation of real options, but a simple and satisfactory approach to valuation has not yet been established. Some academics believe that real options can be valued using a model based on the Black-Scholes option pricing model. This is a pricing model for financial options, which is described in a later chapter.

Some companies have also used a real options approach to project evaluation, particularly where real options can have a significant impact on capital investment project selection - for example in the pharmaceuticals industry and other industries where expenditure on research and development is high, and also in mining (where mining leases might include important real options for the mining company).

# International investment and financing decisions 

| Contents |  |
| :--- | :--- |
| 1 | Factors affecting foreign investment decisions |
| 2 | Estimating exchange rates and the impact of <br> exchange rate movements on investment <br> decisions |
| 3 | DCF appraisal: investment in a developing <br> country |
| 4 | International sources of finance |

## Factors affecting foreign investment decisions

- Factors restricting foreign investment
- Exchange rate risk (currency risk)
- Political risk
- Exchange controls

■ Taxation and international investment
■ Fiscal risk

## 1 Factors affecting foreign investment decisions

### 1.1 Factors restricting foreign investment

International companies invest in other countries because they hope to make a good financial return from their investment, and attractive opportunities might exist in other countries. Developing countries might be particularly attractive for foreign investment because these countries might be expected to achieve considerable economic growth in the future.

However, investments by private companies in developing countries might be restricted for several reasons.

- The country might not have an infrastructure of transport and communication systems to support commerce and trade. International companies might be reluctant to invest in capital projects in companies where the road network is inadequate for transporting heavy goods or large quantities of goods.
- The general standard of education of the local population might be low. International companies might need its employees to have certain basic skills, and if standards of education are low, these skills might not exist (and might be difficult to create with training programmes).
■ There is always a political risk, that the government of the country will take measures against foreign investors. For example, there have been examples in the past of governments forcing foreign companies to sell their assets to domestic companies, or taking a business into state ownership. A foreign government might also impose measures such as restrictions on foreign currency dealings or the imposition of withholding tax on payments of dividends and interest to foreign companies.
- There is a general economic risk that the economy of a developing country will not develop as well as expected. It might be difficult for a foreign company to make a good return on an investment in a country with a weak economy.

Companies considering a major capital investment in another country also need to consider:

- the exchange rate risk
- the risk of exchange controls and similar cash flow restrictions
- taxation on remittances to the parent company's country.


### 1.2 Exchange rate risk (currency risk)

Exchange rate risk, also called FX risk and currency risk, is the financial risk from the possibility (or probability) that foreign currency exchange rates will change. The risk is greater when a foreign exchange rate is volatile, and moves by fairly large amounts over time, often both up and down. Two aspects to exchange rate risk are:

- transaction risk
- translation risk.


## Transaction risk

Transaction risk affects any company that receives income or makes payments in a foreign currency. It is the risk that when a quantity of foreign currency will be received or paid at a time in the future, the exchange rate might move between 'now' and the time that receipt or payment of the currency will occur. As a consequence of the exchange rate movement the amount of money received or paid in the company's own currency (domestic currency) will be less or more than originally expected.

Exchange rates can move favourably as well as adversely, but the main concern for risk management is with the possibility and consequences of an adverse exchange rate movement.

## Example

A UK company has bought goods from a foreign supplier and must pay $\$ 700,000$ for them in three months' time. The current exchange rate is $£ 1=\$ 2$, and the company therefore expects to pay $£ 350,000$ in three months.

If the exchange rate changes so that after three months it is $£ 1=\$ 1.75$, and the company has done nothing to hedge its currency exposure (hedging is described in a later chapter), it will have to pay $£ 400,000$ to acquire the dollars to pay the supplier. Due to the exposure to currency transaction risk and the adverse movement in the exchange rate, the UK company has to pay $£ 50,000$ more than originally expected.

Transaction risks can affect international companies investing in another country in two ways:

- the foreign investment might purchase goods from other countries, or sell goods to other countries, and so will be exposed continually to transaction risks on its trading
- more significant for the purpose of the examination, the multinational parent company will require its foreign subsidiary to remit the profits from the venture to the parent, in the form of interest or dividends. Transaction exposures exist over the entire life on the capital investment project, because the interest payments or dividend payments from the foreign investment must be converted into the parent company's own currency. Changes in the exchange rate will affect the amount of dividends that the parent company receives in its own currency.


## Translation risk

Translation risk is a financial reporting risk for companies with foreign investments. For the purpose of preparing consolidated financial accounts, the financial statements of foreign subsidiaries must be translated into the reporting currency of the parent multinational. Changes in the exchange rate create gains or losses on translation, which affect the reported results of the group.

For example, suppose that a foreign subsidiary has net assets of 20 million francs at the beginning of a financial year for its parent company, whose domestic currency is dollars. The exchange rate is $\$ 1=4$ francs at the beginning of the year and $\$ 1=5$ francs at the end of the year.

Due to the adverse movement in the exchange rate during the year, the net investment at the beginning of the year, which had a value on translation of $\$ 5$ million, will only be included in the consolidated balance sheet at a value of $\$ 4$ million at the end of the year, a fall in value on translation of $\$ 1$ million.

International companies may therefore wish to consider the consequences of translation risk for their reported consolidated statements.

### 1.3 Political risk

Political risk is the risk for an international company that the government of a foreign country might take action that affects the operations or profitability of its investment in its country, or places restrictions on the ability of the foreign subsidiary to remit interest or dividends to the parent country.

Political action against an international company might include the nationalisation of some or all of the assets of its foreign subsidiary, punitive additional tax charges or exchange controls. The government might simply pursue economic policies and other policies that are 'unfriendly' to businesses and to foreign businesses in particular.

### 1.4 Exchange controls

Exchange controls are actions by a government that:

- restrict or prevent the ability of its own nationals to buy foreign currency in order to make payments to foreign suppliers
- restrict or prohibit the payment if interest or dividends to foreign investors, including payments from subsidiary companies to their foreign parent
- restrict the flow (payments) of capital out of the country.


## Example

An international company is considering whether to make an investment in Francland. The estimated after-tax cash flows from the project (all in francs) have been estimated as follows:

| Year | 000 francs |
| :--- | :---: |
| 0 | $(2,000)$ |
| 1 | 800 |
| 2 | 1,000 |
| 3 | 1,200 |
| 4 | 600 |

Taxation is ignored for the purpose of simplifying this example.
The cost of capital for evaluating all aspects of this project is $16 \%$.
The government of Francland has announced that there will be a total ban on the payment of any dividends overseas to foreign investors or parent companies. In the second and third years companies in Francland will be able to remit up to $70 \%$ of their annual profits for that year. All exchange controls will be removed entirely from the end of Year 4.

The exchange rate is expected to be as follows, at the end of each year:

| Year | Forecast rate (francs to \$1) |
| :--- | :---: |
| 0 | 5.0 |
| 1 | 5.2 |
| 2 | 5.4 |
| 3 | 5.7 |
| 4 | 6.0 |

## Required

(a) Calculate the NPV of the project if no exchange controls were in place and the foreign subsidiary remitted all available profits as dividends to the parent company.
(b) Calculate the NPV of the project allowing for the exchange controls, assuming that the foreign subsidiary remits the maximum permitted dividends each year to the parent company.

Answer
(a) Ignoring exchange controls

| Year | Franc cash <br> flow | Exchange <br> rate | \$ cash flow | Discount <br> rate $16 \%$ | PV |
| :--- | ---: | :---: | ---: | :---: | ---: |
|  | FR 000 |  | $\$ 000$ |  | $\$ 000$ |
| 0 | $(2,000)$ | 5.0 | $(400.0)$ | 1.000 | $(400.0)$ |
| 1 | 800 | 5.2 | 153.8 | 0.862 | 132.6 |
| 2 | 1,000 | 5.4 | 185.2 | 0.743 | 137.6 |
| 3 | 1,200 | 5.7 | 210.5 | 0.641 | 134.9 |
| 4 | 600 | 6.0 | 100.0 | 0.552 | 55.2 |
|  |  |  |  | NPV | +60.3 |

(b) Allowing for the exchange controls

| Year | Franc cash <br> flow: | Remitted <br> to the <br> parent | Exchange <br> rate | \$ cash <br> flow | Discount <br> rate $\mathbf{1 6 \%}$ | PV |
| :--- | ---: | :---: | :---: | :---: | :---: | :---: |
|  | FR 000 | FR 000 |  | $\$ 000$ |  | $\$ 000$ |
| 0 | $(2,000)$ | $(2,000)$ | 5.0 | $(400.0)$ | 1.000 | $(400.0)$ |
| 1 | 800 | 0 | 5.2 | 0.0 | 0.862 | 0.0 |
| 2 | 1,000 | 700 | 5.4 | 129.6 | 0.743 | 96.3 |
| 3 | 1,200 | 840 | 5.7 | 147.4 | 0.641 | 94.5 |
| 4 | 600 | 2,060 | 6.0 | 343.3 | 0.552 | 189.5 |
|  |  |  |  |  | $\mathbf{N P V}$ | $(19.7)$ |

Mainly because of the exchange controls, a project that would otherwise be a worthwhile investment should be avoided.

### 1.5 Taxation and international investment

Taxation affects investment decisions, and it is necessary to estimate the tax payments for a proposed capital investment project in another country. Taxation is considered in more detail in a later chapter, with respect to international or multinational companies. Briefly, however, there are two important aspects to tax on the profits and distributions of foreign subsidiaries.

- Withholding tax. Some countries levy a withholding tax on interest or dividends paid by companies to foreign investors, including foreign parent companies. Withholding tax is additional tax, reducing the net cash flows for the parent company from its foreign subsidiary.
- Double taxation agreements. Many countries have double taxation agreements with each other. The purpose of a double taxation agreement is to prevent punitive taxation by taxing profits twice, once in each country. A double taxation agreement allows an international company to set off the tax payable in its own country on the profits of or income received from a foreign subsidiary, against the tax already paid by the subsidiary in its own country. The effect of double taxation agreements is to help to make international investments more attractive by avoiding excessive and punitive tax on the pre-tax returns that the investments make.


## Example

An international company pays tax on its worldwide profits in its own country at the rate of $25 \%$. It has a subsidiary in a foreign country where the rate of tax on profits is only $12 \%$, but dividends paid to a foreign investor are subject to additional withholding tax of $10 \%$. This is $10 \%$ of the cash paid as dividends.

The parent company intends to remit as much as possible from its foreign subsidiary in the form of dividends. The foreign subsidiary makes a profit before tax of $1,000,000$ dinars. (The exchange rate is $\$ 1=1$ dinar and has been fixed at this rate for a long time.) There is a double taxation agreement between the two countries.

The subsidiary will pay tax on profits in its own country at $12 \%$, or 120,000 dinars.

This leaves 880,000 dinars after tax, but if it wants to remit as much as possible to the parent company and pays withholding tax of $10 \%$, the parent company will only receive $\times 100 / 110$ of the available profits after tax (gross distribution). The parent will receive 800,000 dinars and withholding tax will be (10\%) 80,000 dinars.

The parent will exchange the dinars into $\$ 800,000$ and it has paid tax at the effective rate of $20 \%$ on its foreign subsidiary's profits. The tax rate in the parent company's country is $25 \%$ on world-wide profits, so tax on $\$ 1,000,000$ will be $\$ 250,000$. However, since tax of 250,000 dinars $(\$ 250,000)$ has already been paid in the subsidiary's country, and a double taxation agreement exists, the parent company must pay $\$ 50,000$ in extra tax to its domestic tax authorities.

Note. If the rate of tax had been lower in the parent company's country, say $18 \%$ rather than $25 \%$, the subsidiary would pay profits on tax of 120,000 dinars and withholding tax of 80,000 dinars, but no further additional tax would be payable by the company to its domestic tax authorities.

The net effect is for the total amount of tax payable to be equivalent to tax at the higher rate of the two countries concerned.

### 1.6 Fiscal risk

Fiscal risk is the risk that after a capital investment project has been implemented, the government might increase the rate of tax payable on the profits or cash flows from the project. Higher tax payments could significantly affect the returns from a project.

Fiscal risk varies between countries. Some countries have a reputation for fiscal stability, so that any changes in tax are fairly insignificant. In other countries there is a much higher risk of tax changes. Obviously, companies considering an investment in a country, even their domestic country, should take fiscal risk into consideration when deciding whether or not to invest in a project.

## Example

The oil and natural gas industry is an example of an industry where fiscal risk has had a significant impact on capital expenditure decisions.

Oil companies purchase concessions to explore for oil or gas, and extract any that they find. If they fail to find any oil or gas, they will lose their investment. However, they face a risk that if they are successful in finding and extracting oil, the government will decide to tax their profits. One way of doing this is to charge a windfall tax on the profits of firms in the industry that operate in the country.

In the past the UK government has charged a windfall tax on the profits of companies in the North Sea oil business. The matter was debated in Parliament in 2002, where it was claimed that if the government raised the rate of tax on the profits of offshore oil and gas extraction to $40 \%$ or $50 \%$, further exploration by the oil companies would cease to be viable.

## Reducing fiscal risk

Fiscal risk is unavoidable for companies that pay tax on their profits. However, the risk might sometimes be reduced for large companies. A large company can try to negotiate with the government of a country before deciding whether to invest in the country. They might argue that they will not invest unless the government gives an assurance of fiscal stability.

The risk of an unexpected additional tax (windfall tax) cannot be removed simply by a government's promise, because promises might be broken. Another approach that might be used by some companies is to offer to pay a royalty tax on their output. For example, an oil company might undertake to pay a royalty on every barrel of oil that it extracts under a concession agreement. If the company pays tax in this way, so that the royalty payments increase with the volume of business, the government might be less inclined to charge additional tax on what it sees as excessive profits.

Estimating exchange rates and the impact of exchange rate movements on investment decisions

- Estimating future 'spot' exchange rates
- Purchasing power parity theory
- International Fisher effect
- Interest rate parity theory


## 2 Estimating exchange rates and the impact of exchange rate movements on investment decisions

### 2.1 Estimating future 'spot' exchange rates

A company might wish to estimate future 'spot' exchange rates in order to carry out a capital investment appraisal of a proposed foreign investment, where:

- the cash flows will be in a foreign currency, but
- at some stage in the life of the project the cash will be converted into cash flows in the currency of the investing parent company.

An estimate of a future spot rate is a forecast of what the exchange rate might be at a future date, sometimes several years ahead.
(Confusion can arise because an estimate of a future spot rate is not the same as a forward exchange rate that can be obtained in the foreign exchange markets. Forward exchange rates for forward exchange contracts are described in a later chapter.)

There are two related methods of estimating future spot exchange rates:

- purchasing power parity theory
- interest rate parity theory.

Remember, however, that spot exchange rates move up and down continually, and any forecast of what the spot rate will be can only be an approximate estimate of what the average exchange rate for a future period will be.

### 2.2 Purchasing power parity theory

Purchasing power parity (PPP) theory states that the spot rates between two currencies will change over time in relation to the rate of inflation in the countries from which the currencies originate. When the rate of inflation is higher in Country A than in Country B, the currency of Country A will fall in value against the currency of Country B.

Estimated spot rate in Year $\mathrm{n}=$ Current spot rate $\times \frac{\left(1+\mathrm{i}_{\text {VBLE }}\right)^{\mathrm{n}}}{\left(1+\mathrm{i}_{\text {BASE }}\right)^{\mathrm{n}}}$
where:
$i_{\text {VBLE }}$ is the forecast annual rate of inflation for the variable currency $i_{\text {BASE }}$ is the forecast annual rate of inflation for the base currency $n$ is the number of years in the future

The base currency is the currency for which there is 1 unit in the quoted exchange rate, and the variable currency is the currency whose value is expressed as a number of units in value per 1 unit of the base currency. For example in the exchange rate $£ 1$ $=\$ 1.90$, the pound sterling is the base currency and the dollar is the variable currency.
(This formula is included in the formula sheet for your examination, using different symbols. You can use the formula provided, but you need to understand what the various symbols in the formula mean.)

Note that this formula calculates the expected exchange rate as at the end of the year. However in DCF analysis it is assumed that cash flows occur at the year end. The formula therefore provides the exchange rate required for DCF analysis of international investments.

## Example

The current exchange rate for euro/British pound ( $£ / € 1$ ) is 0.6750 . A UK company wishes to evaluate a proposed capital investment in Germany. The investment will be for five years, and a large proportion of the project cash flows will be in euros.

The company needs to forecast what the euro/British pound spot rate will be each year for the full project period. It has been estimated that inflation in the UK will be $3 \%$ each year UK for years 1 to 4 and $5 \%$ in year 5 . It has also been estimated that inflation in the euro zone will be $1.5 \%$ each year for three years, and $2 \%$ in each of years 4 and 5 .

The spot exchange rates at the end of each year can be estimated using PPP theory, as follows. (The variable currency in this example is the British pound, since $€ 1=$ £0.6750).

| Year |  | Forecast rate |
| :--- | :--- | ---: |
| 0 | Actual rate | 0.6750 |
| 1 | $0.6750 \times(1.03) /(1.015)$ | 0.6850 |
| 2 | $0.6750 \times(1.03)^{2} /(1.015)^{2}$ | 0.6951 |
| 3 | $0.6750 \times(1.03)^{3} /(1.015)^{3}$ | 0.7054 |
| 4 | $\left.0.6750 \times(1.03)^{4} /\left[(1.015)^{3}\right)(1.02)\right]$ | 0.7123 |
| 5 | $0.6750 \times\left[(1.03)^{4}(1.05)\right]\left[(1.015)^{3}(1.02)^{2}\right]$ | 0.7332 |

### 2.3 International Fisher effect

The economist Irving Fisher argued that investors in all countries expect the same real rate of return, after allowing for inflation, and the difference in interest rates between two countries could be explained by differences in the rates of inflation in those countries. From this argument and PPP theory, it is possible to derive interest rate parity theory.

### 2.4 Interest rate parity theory

This states that it is possible to predict future spot exchange rates from differences in interest rates between the currencies.
Estimated spot rate in Year $n=$ Current spot rate $\times \frac{\left(1+r_{\text {VBLE }}\right)^{n}}{\left(1+r_{\text {BASE }}\right)^{n}}$ where:
$r_{\text {VBLe }}$ is the forecast annual rate of interest for the variable currency $r_{\text {BASE }}$ is the forecast annual rate of interest for the base currency.

This formula is similar to the PPP theory formula, except that the forecast annual interest rate is used instead of the annual forecast rate of inflation. The formula is also given in the formula sheet for the examination.
(This formula is also used to obtain a forward exchange rate for forward exchange contracts. This is explained in a later chapter on currency risk and currency risk management.)

## Example

The current exchange rate for British pound against the euro ( $€ / £ 1$ ) is 1.4815 . The forecast annual interest rate for the British pound is $5 \%$ in years 1 and 2 and $6 \%$ in years 3 and 4 .

The forecast annual interest rate for the euro is $2.5 \%$ in year 1 and $3 \%$ in each of years 2-4.

## Required

Use this data to estimate a spot rate $(€ / £ 1)$ at the end of each year for years $1-4$.

## Answer

The current exchange rate for British pound against the euro ( $£ / £ 1$ ) is 1.4815 . The forecast annual interest rate for the British pound is $5 \%$ in years 1 and 2 and $6 \%$ in years 3 and 4 .

The forecast annual interest rate for the euro is $2.5 \%$ in year 1 and $3 \%$ in each of years 2-4.

| Year |  | Forecast rate |
| :--- | :--- | ---: |
| 0 | Actual rate | 1.4815 |
| 1 | $1.4815 \times(1.025) /(1.05)$ | 1.4741 |
| 2 | $1.4815 \times[(1.025)(1.03)] /(1.05)^{2}$ | 1.4462 |
| 3 | $\left.1.4815 \times\left[(1.025)(1.03)^{2}{ }^{2}\right] /(1.05)^{2}(1.06)\right]$ | 1.4053 |
| 4 | $\left.1.4815 \times\left[(1.025)(1.03)^{3}\right]\left[(1.05)^{2}\right)(1.06)^{2}\right]$ | 1.3655 |

## Example

Expected changes in an exchange rate can have a significant effect on the financial viability of an international investment project, as this example demonstrates.

A company is considering a four-year investment in another country, where it will establish a subsidiary. The subsidiary will remit all its profits each year to the parent company, and no additional tax will be paid by the parent company in its own country on the profits and dividends of its foreign subsidiary.

The investment will cost $\$ 230,000$ in Year 0, and expected after-tax profits of the subsidiary (all distributable) are as follows:

| Year | 000 pesetas |
| :--- | :---: |
| 1 | 200 |
| 2 | 400 |
| 3 | 500 |
| 4 | 300 |

The current exchange rate is $\$ 1=4$ pesetas. The cost of capital for evaluating all aspects of the project is $15 \%$.

The expected rate of inflation in the parent company's country and Peseta-land is as follows:

| Year | Forecast annual inflation |  |
| :--- | :---: | :---: |
| Dollar-land | Peseta-land |  |
| 1 | $2 \%$ | $5 \%$ |
| 2 | $3 \%$ | $6 \%$ |
| 3 | $3 \%$ | $8 \%$ |
| 4 | $2 \%$ | $5 \%$ |

## Required

(a) Calculate the expected NPV of the project if changes in the exchange rate are ignored.
(b) Calculate the NPV of the project allowing for expected changes in the exchange rate.

Answer
(a) Ignoring changes in the exchange rate

| Year | Pesetas cash <br> flow | Exchange <br> rate | \$ cash flow | Discount <br> rate $15 \%$ | PV |
| :--- | :---: | :---: | :---: | :---: | :---: |
| 000 pesetas |  | $\$ 000$ | $(225)$ | 1.000 | $\$ 000$ |
| 0 |  |  | 50 | 0.870 | $43.0)$ |
| 1 | 200 | 4.0 | 100 | 0.756 | 75.6 |
| 2 | 400 | 4.0 | 125 | 0.658 | 82.3 |
| 3 | 500 | 4.0 | 75 | 0.572 | 42.9 |
| 4 | 300 | 4.0 |  | NPV | +14.3 |
|  |  |  |  |  |  |

(b) Allowing for changes in the exchange rate

Year
$1 \quad 4.0 \times(1.05) /(1.02)$
$24.0 \times(1.05)(1.06) /(1.02)(1.03)$
$3 \quad 4.0 \times(1.05)(1.06)(1.08) /(1.02)(1.03)^{2}$
$4 \quad 4.0 \times(1.05)^{2}(1.06)(1.08) /(1.02)^{2}(1.03)^{2}$

Estimated exchange
rate
4.12
4.24
4.44
4.57

| Year | Pesetas cash <br> flow | Exchange <br> rate | \$ cash flow | Discount <br> rate 15\% | PV |
| :--- | :---: | :---: | ---: | :---: | :---: |
|  |  | 000 pesetas | $\$ 000$ | $(225)$ | 1.000 |
| 0 |  |  | 48.5 | 0.870 | $(23000)$ |
| 1 | 200 | 4.12 | 94.3 | 0.756 | 71.2 |
| 2 | 400 | 4.24 | 112.6 | 0.658 | 74.1 |
| 3 | 500 | 4.44 | 65.6 | 0.572 | 37.5 |
| 4 | 300 | 4.57 |  | NPV | $(4.9)$ |
|  |  |  |  |  |  |

In this example, the project will not achieve a return of $15 \%$ and its NPV is negative, because of the expected movement in the exchange rate during the project period.

## DCF appraisal: investment in a developing country

- Features of a foreign country investment appraisal
- Method of making the DCF appraisal


## 3 DCF appraisal: investment in a developing country

An examination question might ask you to evaluate a proposed capital investment in a foreign country, possibly a developing country where the risks of investment are comparatively high due to a weak local currency and the risk of exchange controls.

### 3.1 Features of a foreign country investment appraisal

The features of investing in a foreign country include the following:

- The investment could be a very high-risk investment, and you might be required to establish a special cost of capital for evaluating the project, possibly using the CAPM and a beta factor for the project.
- Most of the cash flows for the foreign investment will be in the currency of the foreign country, although some cash flows might be in the currency of the parent company.
- If the foreign country is a developing country, there will probably be expectations of high rates of inflation in future years. If so, estimated cash flows should be calculated allowing for the expected inflation rates. (These cash flows including an allowance for inflation should be discounted at the money cost of capital.)
- If the foreign country is a developing country, there might be restrictions on the amount of payments that can be made from the foreign country, due to exchange control restrictions. This means that the cash profits from the project might not be payable immediately in full as dividends to the investing company.


### 3.2 Method of making the DCF appraisal

## Two stages in the appraisal

When a company is considering an investment in another country, a DCF analysis should be carried out in two stages, and two net present values should be calculated.

Stage 1. Ignore the fact that the proposed investment is an investment in another country, and consider the investment as a capital project for the subsidiary company in the foreign country. Calculate an NPV for the project on the basis of cash flows for the subsidiary in the foreign country. This should be an NPV based on foreign currency cash flows.

If the NPV is positive and the risk seems acceptable, you should then go on to Stage 2 of the DCF analysis.

Stage 2. Consider the project from the viewpoint of the parent company, and estimate the cash payments and receipts for the parent company in its own currency. These might include costs incurred in the parent company's own country to set up the project. They will also include the dividend or interest payments received from the foreign subsidiary, in the currency of the parent company. These cash flows should be discounted at an appropriate cost of capital, which might be different from the cost of capital used in Stage 1.

The Stage 2 analysis uses different cash flows from the Stage 1 analysis.

- Stage 1 evaluates the cash flows and cash profits in the foreign country. Stage 2 evaluates the actual returns received by the parent company.
- Stage 1 is an evaluation of the foreign currency cash flows. The Stage 2 evaluation is an evaluation of the cash flows for the parent company in its own currency.

This approach to evaluating the NPV of a foreign investment therefore involves two separate NPV calculations:

- Calculating the NPV of the cash flows in the foreign country, at an appropriate cost of capital.
- If the NPV calculated in this way is positive, calculating a different NPV for the estimated cash flows for the project in the company's domestic currency, probably using the WACC as the discount rate. The cash flows in the company's domestic currency will be different from the cash flows in the currency of the foreign country for several reasons:
- There may be some costs incurred in the company's domestic currency and outside the country where the investment is made. For example, the company's head office may incur costs in its own currency to establish the project in the foreign country.
- There may be restrictions on dividend payments and other cash transfers out of the country where the investment is made.
- The amount paid as dividends from the foreign country will also vary over time with changes in the foreign exchange rate between the currency of the investment country and the currency of the investing company.


## The project is financially viable only if both NPVs are positive.

## Stage 1

The Stage 1 NPV calculation may be summarised as follows:

## Comments

## Establish the cash flows of the project for the foreign subsidiary, in the currency of the foreign subsidiary.

Establish a discount rate for the foreign project.


Where appropriate, allow for estimated rates of inflation. Where you are given the tax regulations, calculate the after-tax cash flows.

If you are given a beta factor for the project, you can use this to establish the cost of capital.

The project might be viable if it has a positive NPV. A positive NPV would show that the capital investment would be viable if the company were based in that country. However, you should assess the project risk as well as the return.

## Stage 2

The Stage 2 NPV calculation may be summarised as follows:

## Comments

Estimate the actual dividends that will be paid from the foreign country out of the project cash flows, allowing for foreign exchange controls in that country.


Where you are given inflation rates or interest rates for the company's own country and the foreign country, you should estimate what the future exchange rates are likely to be, and convert the dividends in each year at the appropriate estimated exchange rate for that year.

Include any other cash flows for the project that the company will expect to incur in its own currency, as part of the project.

Use the company's WACC as the discount rate.

The project is financially viable, subject to an assessment of the project risk, if the NPV at Stage 2 is positive.

## Example

A company is considering an investment in another country, by setting up a foreign subsidiary. The investment would cost 20 million foreign currency units (CUs).

At current prices, the cash flows from the project in the other country are expected to be as follows, before allowing for tax:

| Year | CUs |
| :--- | ---: |
| 0 | $(20.0)$ million |
| 1 | 8.0 million |
| 2 | 10.0 million |
| 3 | 8.0 million |

Taxation in the foreign country is at $50 \%$, and is payable in the year following the year in which the taxable profits occur.

In addition, in Year 1 of the project, the foreign government will refund $50 \%$ of the cost of the investment as a tax allowance. This will be refunded in the form of a cash payment to the company's subsidiary.

Other information:

- The rate of inflation in the foreign country is expected to be $10 \%$ each year.
- The equity beta for the new project will be 1.80 .
- The company is all-equity financed, and would finance the new project with equity.
- The risk-free rate of return is $10 \%$ and the market return is $15 \%$, in the foreign country.
- The company's own cost of capital is $11 \%$.
- Due to foreign exchange control regulations, the company will not be permitted to withdraw its surplus cash from the country in dividends until the end of Year 3. At the end of Year 3, the dividends paid from the foreign subsidiary to the parent company will be the total cash profits from the project, less the net tax payments.
- The company will also have to spend $\$ 200,000$ in its own country at the start of the project on legal fees and set-up costs. The tax rate in the company's own country is $30 \%$.
- The expected rate of inflation in the company's own country over the next three years is expected to be $6 \%$ each year.
- The current rate of exchange is $\$ 1=4$ Currency Units.


## Required

Calculate the NPV of the investment, and recommend whether it should be undertaken.

## Answer

Step 1: Calculate the NPV of the cash flows in the foreign country
The project is all-equity financed. The cost of capital for the project can therefore be calculated directly from the CAPM, using a beta factor of 1.80.

Cost of equity $=10 \%+1.80(15 \%-10 \%)=19 \%$.
The cash flows for the project in the other country must now be estimated. The cash flow estimates should allow for both taxation at $50 \%$ and annual inflation at $10 \%$. The tax cash flows are one year in arrears of the profits or losses that give rise to them.

The following cash flow estimates are all in millions of CUs:

| Year | $\mathbf{0}$ | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ |
| :--- | ---: | ---: | ---: | ---: | ---: |
| Cash flows before tax and inflation | $(20.00)$ | 8.00 | 10.00 | 8.00 |  |
| Cash flows allowing for inflation at $10 \%$ | $(20.00)$ | 8.80 | 12.10 | 10.65 |  |
| Tax at 50\% |  | $\frac{10.00}{}$ | $\frac{(4.40)}{7.70}$ | $\frac{(6.05)}{4.60}$ | $\frac{(5.33)}{(5.33)}$ |
| Net cash flows | $(20.00)$ | 18.80 |  |  |  |

The NPV of the project within the foreign country can now be calculated, using a discount rate of $19 \%$.

| Year | Cash flow | Discount <br> factor at 19\% | PV |
| :--- | ---: | :---: | ---: |
|  | CUs (millions) |  | CUs (millions) |
| 0 | $(20.00)$ | 1.000 | $(20.00)$ |
| 1 | 18.80 | 0.840 | 15.79 |
| 2 | 7.70 | 0.706 | 5.44 |
| 3 | 4.60 | 0.593 | 2.73 |
| 4 | $(5.33)$ | 0.499 | $(2.66)$ |
| NPV |  |  | +1.30 |

The investment would have a positive NPV on the basis of the cash flows in the foreign country.

## Step 2

Calculate the NPV of the investment for the parent company, by calculating the cash flows to the parent from the investment and discounting these at the parent's (WACC).

To do this, an estimate is needed of the future exchange rate in the years when the cash flows from the foreign investment will be received.

| Year | Cash flow in CUs | Estimated exchange rate (see working) | Cash flow in \$ |
| :---: | :---: | :---: | :---: |
|  | CUs |  | \$ |
| 0 Capital expenditure | $(20,000,000)$ | 4.0 | $(5,000,000)$ |
| 0 Other expenditure: legal fees, set-up costs |  |  | $(200,000)$ |
| 1 Tax relief on Year 0 spending at $30 \%$ |  |  | 60,000 |
| 3 Dividends from foreign subsidiary: |  |  |  |
| Cash profits ( $8.8+12.1+10.65$ ) | 31,550,000 |  |  |
| Net tax ((10.0) + $4.4+6.05+5.33)$ | (5,780,000) |  |  |
| Dividend | 25,770,000 | 4.47 | 5,770,000 |

## Working

Using the PPP theory formula shown earlier, the estimated exchange rate at the end of Year $3=$
$\$ 1=4 \mathrm{CUs} \times(1.10)^{3} /(1.06)^{3}$
$\$ 1=4 \times 1.331 / 1.191$
$\$ 1=4.47$.

At this exchange rate, the dividend of 25.77 million CUs will be exchanged into: 25.77 million $/ 4.47=\$ 5.77$ million.

These cash flows should be discounted at the parent company's WACC to arrive at an NPV for the project.

The parent company's WACC is $11 \%$.

| Year | Cash flow | Discount <br> factor at 11\% | PV |
| :--- | ---: | ---: | ---: |
|  | \$ million |  | \$ million |
| 0 | $(5.20)$ | 1.000 | $(5.20)$ |
| 1 | 0.06 | 0.901 | 0.05 |
| 3 | 5.77 | 0.731 | 4.22 |
|  |  |  | $(0.93)$ |
|  |  |  |  |

The NPV is negative; therefore the project should not be undertaken. The negative NPV has been caused by:
■ the delay until Year 3 in being able to repatriate the investment profits in the form of dividend payments

- the falling value of the foreign Currency Units between Year 0 and Year 3.


## International sources of finance

- Currency borrowing to reduce the currency risk in capital investments
- Local capital markets for borrowing


## 4 International sources of finance

### 4.1 Currency borrowing to reduce the currency risk in capital investments

International capital investment is similar in many respects to capital investment in a company's own country, although there are some extra risks. As explained earlier, a major additional risk is the currency risk. Changes in the value of a foreign currency relative to the investing company's currency could have a significant effect on a project NPV.

One way of reducing currency risk is to borrow in the currency of the investment to finance some or all of a proposed international project. Borrowing in the currency of the investment reduces risk in two ways.

- It reduces the translation risk. By borrowing in the currency of the investment, the parent company is able to match assets in a foreign currency with liabilities in the same currency. To the extent that the assets and liabilities are matched in value, any movement in the exchange rate will have an equal effect on the translated value of the assets and liabilities. For example, if a company invests in a project costing 20 million foreign currency units, and borrows 20 million foreign currency units to finance the project, assets and liabilities in the foreign currency are matched, and translation risk is eliminated.
- It also reduces the transaction risk. This is because when a company borrows in a foreign currency, it must pay interest and repay the loan in the same currency. It will use cash flows from the project to make the payments. In doing so, it is able to match some of its receipts in the foreign currency with payments in the same currency. To the extent that receipts and payments in the same foreign currency can be matched, transaction risk is eliminated


### 4.2 Local capital markets for borrowing

The ability of a company to borrow in the currency of an international investment depends on:

- whether there is a large and active local capital market, and
- if there is a large capital market for the foreign currency, whether the company is able to borrow in that market.

An international company might be able to borrow in the local currency from a bank, possibly through its foreign subsidiary. It is much less likely that the foreign country will have a well-developed corporate bond market for issuing bonds in the local currency. The international bond markets are also inappropriate for borrowing in any currency except the major international currencies (mostly the US dollar and the euro).

## Mergers and acquisitions

## Contents

1 Mergers and takeovers: strategic and regulatory issues
2 Valuation for mergers and acquisitions: book value plus and market relative models
3 Valuation for mergers and acquisitions: cash flow models
4 Shareholder value added (SVA) method of valuation

5 Economic value added (EVA) and market value added (MVA) valuation models
6 Other aspects of valuation
$7 \quad$ Financing acquisitions and mergers

## Mergers and takeovers: strategic and regulatory issues

- The distinction between acquisitions and mergers
- The objective of a merger or takeover
- Criteria for choosing an acquisition target
- Synergy
- The high failure rate of acquisitions
- The regulatory framework for acquisitions
- Defences against a hostile takeover bid
- Will shareholders in the target company accept an offer for their shares?


## 1 Mergers and takeovers: strategic and regulatory issues

### 1.1 The distinction between acquisitions and mergers

A company with a growth strategy might seek to grow by means of a merger or acquisition. Mergers and acquisitions involve the amalgamation of two companies.

- Typically, one company acquires the other and becomes the parent company. The second company becomes a subsidiary within the group.
- Occasionally, a new parent company might be created, and the two merging companies become subsidiaries of the new parent company.

A merger is an amalgamation of two companies of approximately equal size. In a takeover, a larger company usually acquires a majority of the share capital in a smaller 'target' company. In practice, business combinations are usually an acquisition of one company by another.

However the financial management issues are broadly the same for mergers as for acquisitions.

### 1.2 The objective of a merger or takeover

The objective of a takeover should be consistent with the overall objectives of the company making the takeover, which should be to increase the wealth of its shareholders. Similarly, the objective of a merger should be to satisfy the objectives of both companies - to increase the wealth of the shareholders of both companies. It is not always obvious, however, how a merger or takeover might add value and create wealth for shareholders.

More specifically, the purpose of an acquisition or a merger is to grow the company. In an acquisition, the acquiring company takes over the business of the target company - its net assets, its other resources, its sales and markets and (hopefully) its profits.

## Alternatives to acquisitions

A company that is pursuing an expansion strategy might consider alternative expansion strategies. The main alternatives are:

- growth through internal development of the business
- growth by means of a joint venture.


## A comparison of growth by acquisition and internal growth

Compared to growth through internal development, growth by acquisition has several advantages.

- By far the most important advantage of an acquisition is that growth is achieved much more quickly. By making an acquisition, a company immediately gets bigger. If the target for acquisition has been selected well, the company should be able to move towards its strategic goals more quickly than if it tried to grow internally.
- It is often argued that when a target company is acquired, it should be possible to achieve 'synergies' and add value by increasing the combined profits of the two companies. Synergy is explained in more detail later.
- When a company is trying to grow its business in another country, acquisition might be better than internal growth, because the company will acquire skilled employees who already understand the business, the country, its laws and culture and its language.
- Unless a company acquires available target businesses, its competitors might acquire them instead and the strategic threat from the enlarged competitor might increase.

There are several reasons, however, why growth through internal development might be preferred to growth by acquisition.

- There is a high risk that the price paid for an acquisition will be too high, and the financial return from buying an over-valued company will be low.
- Many acquisitions are failures. This means that an acquisition strategy is a highrisk strategy. The reasons why acquisitions often fail are explained later.
- With a strategy of internal growth, the company's management should be able to plan and control the development of the business more effectively, because the practical problems associated with acquisitions do not arise. The practical problems include difficulties with employees in the acquired company and the management time needed to combine the systems of the two companies after the acquisition.
- An acquisition is not usually 'ideal' and there will be some features of the target company that the acquirer might not want to buy. After the acquisition, the acquiring company might want to sell off unwanted parts of the business. This can be a time-consuming process, and the prices obtained from selling off these operations and assets might be low.


## A comparison of growth by acquisition and growth by joint venture

An opportunity might arise to grow the business by forming a joint venture with one or more other entities. Growth through joint ventures has some advantages over growth through acquisition.

- When a company wants to enter the market in a foreign country, there might be a risk that an acquisition will create a hostile response from the government of the country concerned and from the customers of the target company. There might even be legal restrictions on the maximum percentage of shares in a domestic company that a foreign investor is allowed to acquire. These difficulties might be avoided by setting up a joint venture with a local company.
- Joint ventures should involve a lower capital investment, because the setup costs will be shared with the joint venture partner. The business risks and (in a foreign country) political risks are also shared.
- A joint venture partner might bring skills and expertise to the venture that the company does not have. The companies in a joint venture might therefore complement each other, and improve the prospects for success of the venture.

Joint ventures also have disadvantages, compared with an acquisition.

- The joint venture partners might have different strategic objectives, and might want different end-results from the business venture. This might eventually lead to a conflict of interests and disagreement between the partners.
- With an acquisition, the acquirer obtains management control of the target business. In a joint venture, control is shared with the other partner or partners.
- At some stage in the future, it is likely that one joint venture partner will want to buy-out the other, if the venture is a success. For the company selling its share of the business to the other partner, the joint venture will not have been successful in achieving long-term growth in the business, only a short-term financial return.


### 1.3 Criteria for choosing an acquisition target

The choice of acquisition targets might be based on any of the following criteria:

- Strategic aims and objectives. An acquisition target is usually selected because acquiring the target would help the acquiring company to achieve its strategic targets. For example, a company might be seeking to grow the business by expanding its product range for its existing markets, or moving into new geographical markets. Acquiring a suitable business would enable a company to expand its product range or move into new geographical markets. Some companies have pursued a strategy of buying up a large number of small companies in a fragmented market, with the intention of becoming the largest company and market leader.
- Cost and relative size. Although there are occasional examples of small companies acquiring much larger ones in a 'reverse takeover', target companies are usually selected because they are affordable.
- Opportunity and availability. In many cases, targets for acquisition are selected because of circumstances. An opportunity to acquire a particular company
might arise, and the acquiring company might decide to take the opportunity whilst it is available.
- Potential synergy. Acquisition targets might possibly be selected because they provide an opportunity to increase total profits through improvements in efficiency. One reason for the success of private equity funds in acquiring target companies has been their ability to achieve additional efficiencies and economies that the previous company management had been unable to do. A strategy of private equity funds might be to look for target companies that they consider under-valued, with the intention of improving their operations and creating extra value.


### 1.4 Synergy

Synergy is sometimes called the ' $2+2=5$ ' effect. It is the concept that the combined sum of two separate entities after a merger or acquisition will be worth more than their sum as two separate entities. When two separate entities come together into a single entity, opportunities might arise for increasing profits.

Synergies might be divided into three categories:

- Revenue synergies
- Cost synergies
- Financial synergies.


## Revenue synergies

Revenue synergies are increases in total sales revenue following a merger or acquisition, by increasing total combined market share. For example, if Company A has annual sales revenue of $\$ 500$ million acquires Company B which has annual sales revenue of $\$ 200$ million, the combined revenue of the two companies after the merger might be, say, $\$ 750$ million.

It is unusual for revenue synergies to occur, but they might occur in the following circumstances:

- The acquisition or merger creates an enlarged company that is able to promote its brand more effectively, and market share increases because customers are attracted by the new brand image.
- The acquisition or merger creates an enlarged company that is able to bid for large contracts, such as contracts to supply the government, which the two companies were unable to do before they combined due to their smaller size.


## Cost synergies

Cost synergies are reductions in costs as a consequence of a merger or takeover. They might arise because it is possible to improve efficiency. For example, it might be possible to reduce the size and cost of administrative departments by combining the administrative functions of the two companies. It is not unusual for takeovers to result in staff redundancies, partly for this reason.

Cost synergies might also be possible by combining other activities, such as combining warehouse facilities.

Experience has shown, however, that companies often have difficulty in achieving planned cost synergies after a takeover, because combining the activities of the two companies after the takeover is often a long and complex process.

## Financial synergies

A larger (combined) company or group might be able to raise finance in a cheaper way. The enlarged company might have access to financial markets, such as the bond market, that the two individuals companies could not access before the takeover, due to their smaller size.

The larger company might also be seen as a lower credit risk, so that it is able to borrow from banks at a lower rate of interest.

### 1.5 The high failure rate of acquisitions

Many acquisitions fail, and do not provide the value for shareholders that was expected when the acquisition was made. There are several reasons for failure.

- The purchase price paid for an acquisition is often too high.
- The expected synergies do not occur.
- There are serious problems with integrating the acquired company into the new group.
- Employees in the acquired company might find it difficult to accept the different culture of the acquiring company, and a new set of policies and procedures. The loss of staff might be high, and valuable knowledge and expertise might be lost.
- $\quad$ There might be problems with establishing effective management control in the acquired company. Control systems might have to be reviewed and changed.
- Senior management in the acquiring company might not give the acquired company sufficient time and attention to make the acquisition operationally and financially successful.
- Competitors might react to an acquisition with a new competitive strategy of their own. Increased competition might drive down the profits for all participants in the market.


### 1.6 The regulatory framework for acquisitions

The regulatory framework for mergers and acquisitions varies between countries.
In the UK, listed companies are required to obtain shareholder approval for major acquisitions or a merger, and subject to the requirements of competition law mergers or acquisitions do not need government approval and are not prohibited by law. This means for example that UK regulations do not restrict the ability of nonUK companies to acquire UK companies.

In other countries, there are protectionist measures against the acquisition of domestic companies by foreign acquirers, especially companies in 'strategic industries' (however these are defined by government policy or national law).

## Takeover rules

The law or other regulations might impose regulations on the conduct of a takeover bid. In the UK, when the target company is a listed company, a 'Takeover Code' applies. The Takeover Code sets out rules that the bidding company must follow. For example, the bidding company is allowed to buy shares in the target company in the stock market, but only up to a certain proportion of the total number of shares in issue. To acquire more shares, it must then make a formal offer for all the shares in the target company. This rule prevents a buying company from acquiring a target company by stealth, through the purchase of a controlling interest in the shares by means of stock market purchases. It also ensures that the shareholders in the target company are treated equally by the bidder.

## Competition law

Most countries have some form of competition law to protect the interests of the consumer or general public. The purpose of competition law is to prevent any company from acquiring such a large share of the market by means of a merger or acquisition that it will be able to exert a strong influence over:

- prices
- output or
- other market conditions.

An ability to 'control' the market could be against the public interest and unfair to consumers.

In the UK, a proposed acquisition that might give the acquirer undue influence over the market can be referred to a government-appointed commission (the Competition Commission) for investigation. The Commission has the power to prohibit an acquisition, or to allow an acquisition to go ahead only on condition that the acquirer complies with certain conditions that would reduce its market influence. For example, if one company that operates a chain of supermarkets proposes to acquire a rival company, the acquisition might be referred to the Competition Commission. The Commission might allow the acquisition to go ahead only on condition that the acquirer agrees to dispose of certain stores operated by the target company.

### 1.7 Defences against a hostile takeover bid

In a friendly takeover bid, the company wanting to make the acquisition makes a bid that the board of the target company will recommend to their shareholders. In this situation, the target company will allow the acquirer access to its accounts, records and management information, so that the acquirer is able to carry out 'due diligence' and satisfy themselves that they are offering a fair price for the business.

A hostile bid occurs when the board of the target company rejects a takeover offer and refuses to recommend it to the shareholders.

A hostile takeover bid is risky, because the bidder has to make the offer for the target company's shares without the benefit of management information about the company (for example, management accounting information about current financial performance). If the takeover bid is 'friendly' and welcomed by the board of directors of the target company, confidential information will probably be made available to the bidder.

The directors of a target company have several defences against a takeover bid. These vary between countries. In the UK, the terms of the offer for the company's shares must be given to the company's shareholders, who are then free to decide whether or not to accept the offer for their shares. If enough shareholders accept the offer, the bid will be successful.

The most important defence against a hostile takeover (in the UK) is to persuade the shareholders to reject the bid. There are several ways in which this might be done. In each case, the directors of the company must deal honestly with the shareholders, and should not give them information that is incorrect or misleading.

Methods of persuading the shareholders to reject a bid include the following:

- Announcing that the company expects to increase its profits substantially in the future; therefore the offer price from the bidder is too low.
- Announcing an intention to return a large amount of equity to the shareholders in the near future, in the form of an increased dividend or a share buy-back; therefore the offer price from the bidder is too low.
- Explaining the future strategy of the board of directors for increasing the company's profits in the future.

When it is uncertain whether shareholders will be persuaded to reject the offer, other methods of defence in the UK are:

- To try to get the takeover referred to the Competition Commission, which has the power to prevent a takeover if it is likely to affect competition adversely in the industry, for example by creating a monopoly company in the industry.
- To find a 'white knight'. A white knight is another company that is prepared to make its own takeover bid for the company, at a higher price or on better terms.


### 1.8 Will shareholders in the target company accept an offer for their shares?

A company making a takeover bid wants to win sufficient acceptances from shareholders in the target company that it acquires full control of the company. The willingness of shareholders to accept a bid for their shares will depend on several factors:

- the offer price: share price valuations are explained in the following sections of this chapter
- the purchase consideration - cash, debt capital or shares in the bidding company: the significance of the purchase consideration is explained later
- job security: when the directors, managers and employees of the target company are major shareholders, job security might be relevant. These shareholders might reject an offer if they are concerned about their job security after a takeover.
- alternative options: there might be the possibility of a rival and more attractive offer.


## Valuation for mergers and acquisitions: book value plus and market relative models

- The need for a valuation
- Valuation methods
- Valuation based on asset values
- Valuation based on earnings and $\mathrm{P} / \mathrm{E}$ ratio
- Dividend valuation model: constant annual dividends
- Dividend valuation method: constant rate of growth in annual dividends
- Advantages and disadvantages of the dividend valuation models


## 2 Valuation for mergers and acquisitions: book value plus and market relative models

### 2.1 The need for a valuation

Valuation and the offer price are key issues in a merger or acquisition. When a merger is negotiated, the two companies need to reach agreement on the valuation of shares in each company for the purpose of deciding the terms of the merger. In a takeover:

- the acquiring company needs to decide what price it is prepared to offer for the target company
- the directors of the target company need to decide whether the offer is acceptable and whether it should be recommended to the shareholder, and
- the shareholders in the target company need to decide whether they are willing to accept the offer made for their shares.

The target company might be a public company whose shares are already quoted on a stock market. In such cases the current share price is a useful guide to valuation, but a bidder will not succeed unless the offer price is higher than the current market price. Without a higher offer, the shareholders in the target company have no reason to accept and agree to sell their shares to the bidder.

When the target company is a private company, there is no market price that can act as a benchmark or guide to an offer price.

In bidding for both public and private companies, the acquiring company therefore needs to make estimates of the value of the target company, in order to decide what price it might be prepared to offer. The price should be sufficiently high to win acceptance from the target company shareholders, but not so high that the bidding company pays too much. When an acquisition is over-priced, the return on the investment will be low and the acquiring company will lose value for its shareholders.

It is not usual to make a bid to buy the debt capital or the preference shares of a target company. A valuation is required for the equity shares in the target company, and not a valuation for the company as a whole.

## Valuation is an art, not a science

It is important to remember that the valuation of companies for the purpose of a merger or acquisition is not a science. Although there are valuation methods, these are all based on estimates and assumptions.

Valuation methods can be used to decide on an offer price, or may be used to justify an offer price. However, the final price is often agreed through negotiation, and the management of the bidding company must use judgement in deciding how high a price they might be willing to pay. The only 'correct' valuation is the price that the bidder makes and the shareholders in the target company accept.

### 2.2 Valuation methods

You might be required in your examination to use several methods for valuation of a target company. Each method will produce a different valuation, but each valuation can provide useful information and help with deciding what the offer price should be. You should be prepared to use each of the different valuation methods, and then discuss the assumptions and estimates on which the valuation is based. You might also be required to compare the different valuations produced by each method, and then recommend (with reasons) a valuation that you consider appropriate as a basis for making an offer to the target company shareholders.

There are several approaches to making a valuation of the shares in a company:

- A valuation based on the net asset value of the target company.
- Market-based valuations, using estimates of future earnings or dividends
- Cash-flow based valuations, using discounted cash flow of expected future returns from the acquisition
- A valuation based on an expected value added (EVA) model: this is another form of cash flow-based model.


### 2.3 Valuation based on asset values

## Disposal value

The minimum value of a target company is the value of its net assets. (Net assets are the value of the company's total assets minus its liabilities.) If net assets can be valued according to the disposal value of the assets, this would indicate the amount that could be obtained for the shareholders of the company in the event that the company is liquidated and its assets sold off.

However the net disposal value (or 'break-up' value) of the target company's assets is usually irrelevant, unless asset value is higher than the value of the business as a going concern. A target company is usually acquired with the intention of
continuing its business operations, and the value of assets in a going concern should be higher than their break-up value.

In a takeover bid, it is also impractical to estimate the disposal value of the target company's assets, except perhaps as a very approximate estimate.

## Book value

In practice, a minimum valuation for a target company might be based on the book value of its assets. The minimum value of the equity would then be the book value of the assets minus the book value of the liabilities.

Some assets might be under-valued, but where non-current assets are re-valued regularly book value might be sufficient for an asset-based valuation without the need for further adjustments.

If you are required to make an asset-based valuation in your examination, you should be prepared to consider some adjustments to the valuation of certain assets, where information in the question indicates that the valuation of certain assets (particularly intangible assets) might be too high or too low.

A valuation based on the book value of net assets should be considered a minimum valuation, and not one that the target company shareholders are likely to accept. An offer price would have to be in excess of book value ('book value plus') for the bid to have any chance of success.

## Example

The following information is available about a private company, Company Z.

|  | $\$ 000$ |
| :--- | ---: |
| Tangible non-current assets | 250 |
| Intangible no-current assets | 75 |
| Current assets | 60 |
|  | 385 |
| Ordinary shares of $\$ 1$ | 50 |
| Revaluation reserve | 80 |
| Retained profits | 145 |
|  | 275 |
| Bank loans | 90 |
| Current liabilities | 20 |
|  | 385 |

## Required

Provide an asset-based valuation of the shares in Company Z.

## Answer

The book value of the net assets is $\$ 275,000$ or $\$ 5.50$ per share. However, this valuation is based on the assumption that the tangible non-current assets are suitably valued, and that $\$ 75,000$ represent a realistic value for the intangible noncurrent assets.

It is therefore unlikely that the target company shareholders will accept an offer below $\$ 5.50$ per share, and the offer will almost certainly need to be higher than $\$ 5.50$ if the takeover is to succeed.

Valuations based on other valuation methods should be compared with the assetbased valuation. There should be some concern (for the bidding company) if a valuation based on expected earnings, dividends or cash flows is lower than the asset-based valuation.

### 2.4 Valuation based on earnings and P/E ratio

A simple method of estimating a value for a company in the absence of a stock market value is to use earnings per share and a price-earning ratio:

## Value $=$ EPS $\times$ Estimated P/E ratio.

The problem with this valuation method for a private company is that a suitable estimate must be obtained for both EPS and the P/E ratio.

- The EPS might be the EPS of the target company in the previous year, an average EPS for a number of recent years or a forecast of EPS in a future year. Any of these estimates for EPS could be used.
- Since a private company does not have a market value for its shares, the shares do not have a P/E ratio. A P/E ratio must therefore be selected by looking at the P/E ratio for similar companies whose shares are traded on a stock market. The P/E ratio selected might be based on the average P/E ratio of a number of similar companies whose shares are traded on a stock market, for which a current $\mathrm{P} / \mathrm{E}$ ratio is therefore available.
However, shares in public companies should be worth more than shares in an identical private company, because they are more 'liquid'. They can be sold on the stock market at any time, whereas shares in a private company cannot be sold at all unless a willing private buyer can be found.

Since shares in a quoted company ought to have a higher price than an identical private company, the $\mathrm{P} / \mathrm{E}$ ratio of a quoted company should also be higher than the $\mathrm{P} / \mathrm{E}$ ratio applied to a valuation of the private company's shares.

For example, if a quoted company has a P/E ratio of 11.5 times, the $\mathrm{P} / \mathrm{E}$ ratio used to value shares in a similar private company in the same industry should be less than 11.5. Unfortunately, there is no reliable method of deciding how much lower than 11.5 a suitable P/E ratio should be.

Another problem with selecting a suitable $\mathrm{P} / \mathrm{E}$ ratio is that it might not be possible to identify quoted companies that are very similar to a private company. Some
quoted companies might operate in the same industry, and might even be competitors of the private company in some markets. However, a comparable quoted company is likely to be larger than the private company, and might well operate in a wider range of markets and in different geographical markets.

## Example

The EPS of a private company, ABC Company, was $\$ 2.50$ last year and is expected to rise to $\$ 2.80$ next year. Similar companies whose shares are quoted on the stock market have P/E ratios ranging from 10.0 to 15.6 . The average $\mathrm{P} / \mathrm{E}$ ratio of these companies is 12.5 .

A valuation of the company might be to take the prospective EPS and apply the average $\mathrm{P} / \mathrm{E}$ ratio for similar companies:

Valuation $=\$ 2.80 \times 12.5=\$ 35.00$ per share.
An alternative evaluation might be to take the actual EPS last year and apply the lowest P/E ratio of any other similar stock market company, reduced by, say, $10 \%$ to allow for the fact that ABC Company is a private company and does not have a stock market quotation.

Valuation $=\$ 2.80 \times(90 \% \times 10)=\$ 25.20$.
Here, a P/E ratio of $9(90 \% \times 10)$ has been used in the valuation.
Another valuation might be to use the EPS for last year and a P/E ratio of 9. This would give a share value of $\$ 2.50 \times 9=\$ 22.50$.

By making different assumptions, the earnings-based valuation method has provided valuations in a wide range of $\$ 22.50$ to $\$ 35$. From this example, it might be apparent that the $\mathrm{P} / \mathrm{E}$ ratio valuation method has a number of weaknesses:

- It is based on subjective opinions about what EPS figure and what P/E ratio figure to use.
- It is not an objective or scientific valuation method.
- It is based on accounting measures (EPS) and not cash flows. However, the value of an investment such as an investment in shares is derived from the cash that the investment is expected to provide to the investor (shareholder).

The valuation can be compared with valuations using other valuation methods.

### 2.5 Dividend valuation model: constant annual dividends

The dividend valuation model is a more objective and cash-based approach to the valuation of shares. It is based on the view that the value of a share to shareholders is the value of all the future dividends that they expect to receive from the shares in the future.

If the fair value of a share represents the value of all expected future dividends, this value can be estimated by discounting expected future dividends to a present value.

All expected future dividends 'in perpetuity' may be discounted to a present value at the shareholders' cost of capital. The shareholders' cost of capital is the return on investment that shareholders expect to receive from their shares in the company.

Without going into the mathematics to prove the valuation model, it can be shown that if it is assumed that the company will pay a constant annual dividend every year into the foreseeable future, the present value of those dividends, and so the value of the shares, is:
$P_{0}=\frac{D}{K_{e}}$
where:
$P_{0}$ is the current value of the share ex dividend
D is the amount of the annual cash dividend.
$\mathrm{K}_{\mathrm{e}}$ is the shareholders' cost of capital expressed as a proportion (so $9 \%=0.09$, etc).
This valuation model assumes that the dividend is paid annually. This is the assumption that is commonly used in examination questions.

A share price ex dividend is a price that excludes the value of the annual dividend in the current year. For an 'exact' valuation using this model, it should be assumed the next dividend is payable in one year's time.

If the annual dividend in the current year has not yet been paid, but will soon be paid, the value of the share is its value 'cum dividend'.

Value cum dividend
$=$ Value ex dividend + Amount of the dividend soon to be paid.

## Example

A company is expected to pay an annual dividend of $\$ 0.48$ per share into the foreseeable future and the shareholders' cost of capital is $12 \%$. The most recent annual dividend has just been paid.

Using the dividend valuation model, the value of the share (ex dividend) ought to be $\$ 0.48 / 0.12=\$ 4.00$.

If expectations about future annual dividends change from $\$ 0.48$ per share to $\$ 0.54$ per share, the valuation of the share will also change, to $\$ 4.50$ ( $=\$ 0.54 / 0.12$ ).

If future annual dividends are expected to be $\$ 0.48$ per share, but the shareholders' cost of capital changes to $12.5 \%$, the valuation of the share will fall to $\$ 3.84$ (= $\$ 0.48 / 0.125)$.

The dividend valuation model therefore provides an explanation of how the value of shares will rise or fall when there are changes in either:

- the expected annual dividend, or
- the shareholders' required rate of return (the equity cost of capital).


## Note on the cost of shareholder equity

To use the dividend valuation model (or the dividend growth model for valuation, which is described next) we need an estimate of the cost of equity capital. When the bidding company is a quoted company, it could use its own cost of equity to make the valuation. Estimating the cost of a company's equity is described in the chapter on the cost of capital.

If the bidding company is also a private company, an estimate of the cost of equity is required. This might be obtained from estimates of the cost of equity of similar quoted companies.

### 2.6 Dividend valuation method: constant rate of growth in annual dividends

An alternative assumption in the dividend valuation model is that the annual dividend will grow in the future. A valuation for shares in a company can therefore be obtained by making a forecast of what the future rate of dividend growth each year will be.

A simplifying assumption is that the dividends will grow at a constant annual percentage rate.

If it is assumed that the company will pay an annual dividend that grows by a constant percentage amount every year into the foreseeable future, the present value of those dividends, and so the value of the shares, is:
$P_{0}=\frac{D_{1}}{K_{e}-g}$
where:
$P_{0}$ is the current value of the share ex dividend
$D_{1}$ is the expected amount of the annual cash dividend in one year's time (next year)
$\mathrm{K}_{\mathrm{e}}$ is the shareholders' cost of capital expressed as a proportion (so $9 \%=0.09$, etc)
g is the expected annual growth rate in dividends expressed as a proportion ( $3 \%=$ 0.03 etc )

This is the valuation of the share ex dividend. Note that this valuation formula is based on the assumptions that:

- the dividend is paid annually, and
- the dividend for the current year has just been paid.


## Example

A company has just paid an annual dividend of $\$ 0.48$. Dividends are expected to grow by $4 \%$ each year into the foreseeable future. The shareholders' cost of capital is 12\%.

Using the dividend valuation model, the expected value of the share (ex dividend) is:
$P_{0}=\frac{0.48(1.04)}{0.12-0.04}=\$ 6.24$

This provides a useful guide to a potential buyer of the shares, to suggest what the valuation of the shares should be.

If there is no expected growth in annual dividends, and the company is expected to pay a constant annual dividend in the future, the share valuation would have been $\$ 4.00$ (see the earlier example). Because the annual dividend is expected to increase every year, the valuation is much higher.

Using the dividend growth model, the fair value of shares changes with:

- changes in expected future dividends (for example, changes in the expected annual growth rate in dividends), or
- changes in the shareholders' required rate of return (the equity cost of capital).


## Example

In the previous example, if the expected growth rate in annual dividends falls from $4 \%$ to $3 \%$, the valuation of the shares will fall to:

$$
P_{0}=\frac{\$ 0.48(1.03)}{(0.12-0.03)}=\$ 5.49
$$

## Change in the annual rate of dividend growth

The dividend growth model can also be used to value shares when the expected growth rate in dividends is expected to change.

## Example

A company has just paid an annual dividend of $\$ 0.63$. This dividend is expected to remain constant for two more years, but from Year 3 it is expected to grow by $3 \%$ each year into the foreseeable future. The cost of shareholders' funds is $10 \%$.

The expected value of the shares (ex dividend) at the end of Year 2 can be calculated using the dividend growth model:
$P_{2}=\frac{0.63(1.03)}{0.10-0.03}=\$ 9.27$

The expected current value of the share is:

- this valuation at the end of Year 2 discounted to a Year 0 value,
- plus the present value of the expected dividends at the end of Year 1 and Year 2.

| Year |  | Discount <br> factor at 10\% |  |  |
| :--- | :--- | ---: | ---: | ---: |
|  |  | $\$$ |  | $\$$ |
| 1 | Dividend | 0.63 | 0.909 | 0.57 |
| 2 | Dividend | 0.63 | 0.826 | 0.52 |
| 2 | End of Year 2 value | 9.27 | 0.826 | 7.66 |
|  | Share value |  |  | 8.75 |
|  |  |  |  |  |

## Note on retained earnings: Gordon's growth model

Gordon's growth model was explained in the earlier chapter on cost of capital. Dividend growth can be achieved by retaining some profits (retained earnings) for reinvestment in the business. Reinvested earnings should provide extra profits in the future, so that higher dividends can be paid. When a company retains a proportion of its earnings each year, the expected annual future growth rate in dividends can be estimated using the formula:
$\mathrm{g}=\mathrm{br}$
where:
$\mathrm{g}=$ annual growth rate in dividends in perpetuity
$b=$ proportion of earnings retained (for reinvestment in the business)
$r=$ the shareholders' required return on capital, which is also assumed to be the rate of return that the company will make on its investments of retained earnings.

## Example

A company has just achieved annual earnings per share of $\$ 0.50$, of which $60 \%$ has been paid in dividends and $40 \%$ has been reinvested as retained earnings. The company is expected to retain $60 \%$ of its earnings every year and pay out the rest as dividends. The return on investment achieved by the company is expected to be $10 \%$ per annum.

The anticipated annual growth in dividends $=40 \% \times 10 \%=4 \%$ or 0.04 .

Using the dividend growth model, the expected value per share is:
$P_{0}=\frac{0.30(1.04)}{0.10-0.04}=\$ 5.20$

### 2.7 Advantages and disadvantages of the dividend valuation models

The dividend valuation models provide a present value of expected future dividends that is based on future cash flows (dividends) for a shareholder. It is therefore a form of cash flow based model as well as a market-based model of valuation.

The main weakness of the dividend growth model for the valuation of shares in an acquisition is that the bidding company hopes to win control of the target company. When it obtains control, it will decide the dividend policy of the acquired company. It might decide to increase or reduce the dividend payout ratio (as a percentage of earnings). This would affect the expected dividend growth rate.

It can also be argued that a more appropriate method of valuation should be one based on the total cash flows of the acquired company, not just the dividends that will be remitted to the parent company.

Cash flow-based company valuations are described in the next three sections of this chapter.

Valuation for mergers and acquisitions: cash flow models

- The advantages of a cash flow basis for business valuation
- Net present value method


## 3 Valuation for mergers and acquisitions: cash flow models

### 3.1 The advantages of a cash flow basis for business valuation

Cash flow models of valuation are based on obtaining a valuation for a company or business operation by putting a value to future expected cash flows. There are several strong arguments in favour of a cash flow approach to valuation.

- Cash flows earned by a business are much more closely correlated to value and shareholder wealth than accounting profits. Valuations should therefore be based on expectations of future cash flows rather than expectations of profit or accounting return on investment.
- Returns on an investment, such as the acquisition of another company, must be sufficient to cover the costs of the finance used to make the acquisition. This includes the cost of equity finance as well as the cost of any debt finance. It is therefore appropriate to assess the value future cash flows from an acquisition in terms of whether they provide a return in excess of the total cost of financing.

A key point to note is that if the purchase price for an acquisition exceeds the value of the returns that will be obtained from the acquisition, allowing for the acquirer's cost of capital, the acquisition will destroy value. The value of the acquisition will be less than the acquirer pays for it.

This is a strong argument for suggesting that businesses should be valued on the basis of expected future cash flows and the cost of capital.

Three cash flow models will be described:

- The net present value method
- The shareholder value added (SVA) method of valuation
- Economic value added (EVA) and market value added (MVA)


### 3.2 Net present value method

The net present value method is straightforward. The valuation of a company or a business should be calculated as the net present value of expected future cash flows from the business, discounted at an appropriate cost of capital.

The following steps are required to apply the NPV valuation method.

- Estimate the relevant incremental cash flows from the business to be acquired.
- Discount these cash flows at an appropriate cost of capital that reflects the risk of the investment.
- The maximum price that should be paid for the business is the present value (or net present value) of these future expected cash flows.

The NPV method might be appropriate where the future cash flows from an acquired business will be affected by the acquisition. For example, the acquirer might have plans to invest extra capital in the acquired business after the acquisition has taken place. This additional investment will affect future cash flows, and so should affect the valuation.

## Example

QX Company is considering the acquisition of an operating division from a multinational company. If it acquires the division, it will invest an additional \$12 million immediately to modernise equipment.

It has been estimated that the annual after-tax cash flows after the acquisition and additional investment will be as follows:

| Year | \$ million |
| :--- | ---: |
| 1 | 6 |
| 2 | 8 |
| 3 | 10 |
| 4 | 8 |
| 5 onwards (per year) | 4 |

An appropriate after-tax cost of capital for valuing the acquisition is $8 \%$.

## Required

Estimate the maximum amount that QX Company should be prepared to pay to acquire the operating division from the multinational company.

## Answer

| Year | Cash flow | Discount <br> factor | PV |
| :--- | ---: | ---: | ---: |
|  | \$ million | $8 \%$ | \$ million |
| 0 | $(12)$ | 1.000 | $(12.000)$ |
| 1 | 6 | 0.926 | 5.556 |
| 2 | 8 | 0.857 | 6.856 |
| 3 | 10 | 0.794 | 7.940 |
| 4 | 8 | 0.735 | 5.880 |
| 5 onwards (per year) | 4 | 9.188 | 36.752 |
|  |  |  | 50.984 |

## Working:

Discount factor at $8 \%$
In perpetuity $(1 / 0.08)$
Years 1-4
Year 5 in perpetuity

| 12.500 |
| ---: |
| 3.312 |
| 9.188 |

The maximum price to pay for the operating division is about $\$ 51$ million.

## Example

Company WY is considering the acquisition of another company, Company BN. Company BN has 4 million shares in issue.

After the acquisition Company WY intends to invest a further $\$ 6$ million for the purchase of new operating assets in Company B. It is expected that annual cash flows for equity will be $\$ 1.0$ million in the first year following the acquisition, and that these will increase by $8 \%$ each year for the next four years, after which the rate of growth in annual cash flows will fall to $3 \%$.

The cost of capital is $10 \%$.

## Required

Calculate a valuation of the equity capital of Company BN.

## Answer

| Year | Cash flow | Discount factor <br> at $\mathbf{1 0 \%}$ | Present <br> value |
| :--- | ---: | :---: | ---: |
|  | $\$ 000$ |  | $\$ 000$ |
| 0 | $(6,000)$ | 1.000 | $(6,000)$ |
| 1 | 1,000 | 0.909 | 909 |
| 2 | 1,080 | 0.826 | 892 |
| 3 | 1,166 | 0.751 | 876 |
| 4 | 1,260 | 0.683 | 861 |
| 5 | 1,360 | 0.621 | 845 |
| 6 onwards | 20,615 | 0.621 | 12,802 |
| Valuation |  |  | $+11,185$ |

A valuation based on the projected cash flows is $\$ 11.185$ million or about $\$ 4.10$ per share for the 4 million shares.

## Workings

After Year 5 the annual rate of growth in cash flows falls to 3\%, so the cash flow in Year 6 (in $\$ 000$ ) will be 1,401.
The valuation of $\$ 1,401,000$ in Year 6 , rising by $3 \%$ in perpetuity, can be calculated using the dividend growth model formula. The end of Year 5 present value of these cash flows is:

$$
\text { PV in } \$ 000=\frac{1,401(1.03)}{(0.10-0.03)}
$$

$$
=(\text { in \$000) 20,615. }
$$

This is an end-of-Year 5 discounted value, which needs to be converted to a present value using the Year 5 discount rate.

## Note on the method of calculation used in this example

Make sure that you understand the technique for valuation of annual cash flows from a year in the future, where the cash flows are expected to grow by a constant percentage amount each year in perpetuity. This use of the Gordon's growth model for an NPV valuation or shareholder value added valuation could well feature $n$ an examination question.

Shareholder value added (SVA) method of valuation

- Seven value drivers
- The steps in the SVA valuation method
- Valuation of the operational assets
- Definition of free cash flow


## 4 Shareholder value added (SVA) method of valuation

### 4.1 Seven value drivers

The shareholder value added method of valuation of a business is also based on estimated cash flows and present values. The cash flows used for the valuation are the 'free cash flows' of the business. The definition of free cash flows for the purpose of SVA is explained later.

The SVA method also identifies seven factors or value drivers in the valuation. These are as follows:

- Sales growth. Sales growth is expressed as a percentage growth in sales revenue each year.
- Operating profit margin. This is the expected operating profit as a percentage of sales revenue.
- Tax rate. This is the rate of tax on operating profit. It is a 'cash' tax rate, and ignores deferred taxation.
- IWCI. This is the incremental working capital investment each year. An additional working capital investment reduces free cash flow.
- RFCI and IFCI. RFCI is replacement fixed capital investment, which is the amount of investment needed to replace non-current assets that reach the end of their economic life. Ignoring inflation, it is often assumed that RFCI is equal to the annual depreciation charge for non-current assets. IFCI is incremental fixed capital investment, which is investment each year in non-current assets in excess of the amount needed for the replacement of existing non-current assets. IFCI reduces free cash flow.
- The cost of capital. This is used to discount future free cash flows to a present value, for the purpose of valuation of the business.
- The 'competitive advantage period' or 'value growth duration'. This is the period of time during which the business is expected to achieve growth in its current form, and obtain a return in excess of its cost of capital. After the competitive advantage period, it is often assumed that annual free cash flows will be a constant annual amount.


### 4.2 The steps in the SVA valuation method

There are three steps in the SVA method.

- Step 1. Calculate the value of the operational assets of the business.
- Step 2. Add the value of the non-operational assets, such as cash and investments.
- The sum of the value of operational assets and non-operational assets is the total value of the business assets.
- Step 3. Subtract from this total the value of the company's debt. This gives a valuation for the business equity.


### 4.3 Valuation of the operational assets

The most complex part of the valuation is the valuation of the operational assets of the business. This is calculated as follows:

- For each year of the competitive advantage period, calculate the expected free cash flow.
- Discount these annual cash flows to a present value, using the cost of capital. Add these PVs to obtain the value of all expected future free cash flows during the competitive advantage period.
- Calculate a present value for all the expected annual free cash flows after the competitive advantage period. This is an annual free cash flow in perpetuity, usually assuming no further growth.
- Add the value of free cash flows during the competitive advantage period and those after the competitive advantage period to obtain a total valuation for the operational assets of the business.


### 4.4 Definition of free cash flow

For the purpose of the SVA method, the following definition of annual free cash flow is used:

| Operating profit | X |
| :--- | :---: |
| Less: Tax | $(\mathrm{X})$ |
| Equals: Profit after tax | X |
| Add back: annual depreciation charge | X |
| Equals: Operating cash flow | X |
| Adjustments for |  |
| RFCI | $(\mathrm{X})$ |
| ICFI | $(\mathrm{X})$ |
| IWCI | $(\mathrm{X})$ |
| Equals: Free cash flow | X |

## Notes:

1. RFCI, IFCI and ICWI are all additional capital investments when the business is expected to grow. They are all therefore reductions in free cash flow.
2. It is often assumed that the annual depreciation charge and RFCI are the same amount each year.

## Example

You are required to use the SVA method to obtain a valuation for the equity of a company that is the target for a takeover bid. The following information is available:

Current annual sales: \$10 million
Competitive advantage period: 5 years
Cost of capital: 10\%
Other value driver information:

| Year | 1 | 2 | 3 | 4 | 5 | After year <br> 5 (annual) |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: |
|  | $\$ \mathrm{~m}$ | $\$ \mathrm{~m}$ | $\$ \mathrm{~m}$ | $\$ \mathrm{~m}$ | $\$ \mathrm{~m}$ | $\$ \mathrm{~m}$ |

Annual depreciation charges are $\$ 4$ million.
The company has investments with a market value of $\$ 2$ million and the total value of its debt liabilities is $\$ 4.5$ million.

## a <br> Answer

\(\left.$$
\begin{array}{lrrrrrr}\text { Year } & \mathbf{1} & \mathbf{2} & \mathbf{3} & \mathbf{4} & \mathbf{5} & \begin{array}{r}\mathbf{6}\end{array}
$$ <br>
\& \& \& \& \& <br>

onwards\end{array}\right]\)| $\$ \mathrm{~m}$ |
| :--- |
|  |
| Sales |


| RFCI | $(4.00)$ | $(4.00)$ | $(4.00)$ | $(4.00)$ | $(4.00)$ | $(4.00)$ |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: |
| IFCI | $(0.04)$ | $(0.04)$ | $(0.03)$ | $(0.02)$ | $(0.01)$ | 0 |
| IWCI | $(0.16)$ | $(0.15)$ | $(0.10)$ | $(0.09)$ | $(0.05)$ | 0 |
| Free cash flow | 1.31 | 1.43 | 1.42 | 1.24 | 1.35 | 0.94 |
| Discount factor at $10 \%$ | 0.909 | 0.826 | 0.751 | 0.683 | 0.621 | 6.209 |
| PV of free cash flow | 1.19 | 1.18 | 1.07 | 0.85 | 0.84 | 5.84 |
| Total PV of free cash flows: $\$ \mathbf{1 0 . 9 7 m}$ |  |  |  |  |  |  |

The value of the operational assets is $\$ 10.97$ million, say $\$ 11.0$ million.

|  | $\$ \mathrm{~m}$ |
| :--- | ---: |
| Valuation of operational assets | 11.0 |
| Value of non-operational assets | 2.0 |
|  | 13.0 |
| Less: Value of debt | $(4.5)$ |
| Equals: Value of equity | 8.5 |

## Workings and assumptions:

1 Replacement fixed capital investment is equal to the annual depreciation charge.

2
Discount factor at $10 \%$
In perpetuity $(1 / 0.10) \quad 10.000$
Years 1-5
Year 6 in perpetuity
3.791 6.209

## Economic value added (EVA) and market value added (MVA)

- The origins of the EVA method
- The concept of economic value added
- Calculating EVA
- Market value added: MVA
- The link between EVA and MVA
- EVA and MVA as a valuation method for companies
- Advantages and disadvantages of EVA and MVA for valuation


## 5 Economic value added (EVA) and market value added (MVA)

Another approach to the valuation of companies is to calculate how much value is added to the company each year, and use this annual increase in value as a basis for making a valuation for the company. One approach to calculating the increase in value during a period is to measure economic value added or EVA®. (EVA is a registered trade mark of the consultancy firm Stern Stewart.)

### 5.1 The origins of the EVA method

Economic value added or EVA is a measure of performance that provides a useful assessment of how much shareholder value has been added during a period.

- One method of measuring the creation of shareholder value is to take the change in the share price during the period and add dividend payments. However, this measure of the increase in shareholder value is unreliable and so unsatisfactory, because short-term movements in share prices are dependent on factors other than financial performance.
- It is usual to assess management performance by the profit earned during a period or the return on investment ROI (return on net assets RONA). However, accounting profit is an unsatisfactory measurement of shareholder value added. ROI is also criticised as a measure of performance because it can encourage management decision-making that is not in the best interests of the company such as deferring capital investments and choosing not to invest in research and development, because these would reduce ROI in the short-term.

EVA was developed as a measure of performance that is closely correlated to shareholder wealth. In addition, when EVA is used as a basis for a management incentive scheme, it can encourage management to make decisions that are in the best interests of the company, such as investing for the longer term, in capital expenditure, research and development (innovation) and brand-building.

### 5.2 The concept of economic value added

Measuring EVA is a complex calculation, but the basic concept is very simple. (It is similar to the concept of residual income.)

- Managers should be charged for the capital that they use. This should include equity capital as well as debt capital.
- A company, and each operating division within a company, should make enough profits after tax to provide the returns that are expected by the providers of capital (equity and debt).
- A charge for the use of capital should therefore be deducted from operating profit after tax.
- If operating profit exceeds the capital charge, economic value has been added and shareholder value has been created.
- If operating profit is not sufficient to cover the capital charge, value has been lost. Providers of capital would be justified in wanting to move their capital to an investment yielding higher returns, or in wanting to replace the current managers with new management who are better at creating value.

The economic value added during a period is calculated as follows:

## Net operating profit after tax - (Capital employed $\times$ Cost of capital \%)

## EVA $=$ NOPAT $-($ Capital employed $\times$ WACC $)$

Where:
(1) WACC is the weighted average cost of capital.
(2) Capital employed is the capital employed at the beginning of the financial year, with some adjustments (as described later).
(3) The net operating profit figure is after deduction of a charge for tax (unlike normal measurements of ROI and residual income). The cost of debt interest is included in the capital charge; therefore interest costs are not deducted in arriving at the figure for net operating profit. NOPAT is operating profit before deducting interest charges but after deducting an amount for tax.

### 5.3 Calculating EVA

The concept of EVA was developed by the consultancy firm Stern Stewart.
It is argued that accounting measurements of operating profit and capital employed are unreliable and unrealistic in various respects. The figures that should be used are:

- the economic value of capital employed, and
- an estimate of economic profit.

These figures can be estimated by making adjustments to the accounting figures for capital employed and profit. Stern Stewart have identified a large number of
adjustments that should be made to the accounting figures for profit and capital employed. Originally they identified 164 adjustments that should be made. However making all these adjustments would be time-consuming and many of them would not have a material impact on the resulting figure for EVA.

In practice, companies using EVA therefore make about 5 to 15 adjustments, which affect EVA by a material amount. Some of these are explained here.

## Adjustments to capital employed

Several adjustments should be made to the accounting (balance sheet) value of capital employed to arrive at an estimate for the economic value of capital employed.

- Expenditure on some 'intangible items' is treated as an expense in the income statement, and written off in the year that it is incurred. However, some spending on intangibles adds to the economic value of a company's assets. The expenditure should therefore be capitalised and amortised over a number of years. An important example is spending on research and development. It could also be argued that some spending on training and on advertising to develop a brand name are items that should be capitalised and amortised over several years.
- Provisions or allowances in the accounting balance sheet are not 'real' reductions in capital, and should be added back. In particular, any deferred tax reserve and allowance for doubtful debts should be added back to capital.
- On the other hand, long-term leases should not be capitalised. If they have been capitalised, they should be deducted from capital. The rent paid on a lease is a measure of its economic cost. Instead of including depreciation of a leased asset and a finance charge in the income statement, these should be replaced by the actual lease rental for the period.

Capital employed can be measured in either of two ways:

```
Assets method
Adjustments to assets/equity as described
above
Equals: Capital employed
```

Liabilities and equity method

```
Liabilities and equity method
Interest-bearing current liabilities (for example, a
Interest-bearing current liabilities (for example, a
bank overdraft)
bank overdraft)
Plus
Plus
Interest-bearing non-current liabilities (for example,
Interest-bearing non-current liabilities (for example,
bonds and long-term loans)
bonds and long-term loans)
Plus (or minus)
Plus (or minus)
Adjustments to assets/equity as described above
Adjustments to assets/equity as described above
Equals: Capital employed
```

```
Equals: Capital employed
```

```

\section*{Adjustments to NOPAT}

Some adjustments must also be made to the accounting figure for NOPAT in order to reach an estimate of the economic profit before capital charge.
- Amounts charged as an expense in the income statement that should be capitalised (such as research and development expenditure) should be added
back to profit, but an amortisation charge should be made to reflect the loss of economic value in the intangible asset during the period.
- Depreciation and finance charges for leased assets (finance leases) should be added back to profit, but the actual lease rental should be deducted.
- Any addition to the deferred taxation reserve should be added back to NOPAT (or any reduction in deferred tax should be subtracted from NOPAT). The tax charge in calculating NOPAT should be the actual amount of tax that the company expects to pay on its profits for the year.
- A calculation of NOPAT might also allow for the fact that the 'cash' tax charge includes tax relief on debt interest. The tax relief on debt interest is allowed for in the weighted average cost of capital; including tax relief in the tax charge would be 'double-counting'. The tax charge should therefore be increased by the amount of tax relief on debt interest.

\section*{Capital charge}

The capital charge for the year is the economic value of capital employed multiplied by the company's weighted average cost of capital. Usually, the WACC is applied to the capital employed as at the beginning of the year.

\section*{EVA and depreciation and impairment of non-current assets}

You might notice that the adjustments to capital employed and NOPAT do not include any adjustment for the depreciation and impairment of non-current assets.

Assets lose economic value each year, and it is appropriate to make a charge for the loss of economic value - economic depreciation or economic impairment. A figure for economic depreciation should be used in calculating EVA, and this should replace the accounting figure for depreciation.
- Economic depreciation must be recovered from the company's cash flows. It might help to think of this as cash that is needed for the essential replacement of worn-out assets. There must be a charge for economic depreciation in calculating EVA.
- It is often assumed that accounting depreciation is a reasonable approximation of economic depreciation of tangible non-current assets.
- For these reasons, there is no adjustment to EVA for accounting depreciation charges.

It is also appropriate to assume that any charge for impairment of assets (including goodwill) in the accounting balance sheet is an appropriate measure of the impairment of economic value.

As a result, no adjustments are made to the depreciation charge or the amounts written off for impairment of tangible non-current assets, goodwill or other intangible non-current assets in the accounting balance sheet.

\section*{Note}

EVA is NOT an estimate of cash flow profits and should not be confused with cash flows or 'free cash flow'.
- NOPAT is an estimate of economic profit, and it includes charges for depreciation and amortisation.
- The charge for the cost of capital includes a charge for the cost of equity, which is not a cash item of cost.

A simplified example is shown below to illustrate how EVA might be calculated.

\section*{Example}

At the beginning of a financial year, a company's balance sheet capital employed was \(\$ 150\) million (accounting measurements).

During the year, the company's profits were as follows in the financial accounts:
\begin{tabular}{|c|c|c|}
\hline & \$ million & \$ million \\
\hline Profit before interest and tax & & 24.0 \\
\hline \multirow[t]{2}{*}{Interest} & & 4.0 \\
\hline & & 20.0 \\
\hline \multicolumn{3}{|l|}{Taxation} \\
\hline Tax on current year profits & 6.0 & \\
\hline Deferred tax & 2.0 & \\
\hline & & 8.0 \\
\hline Profit after tax & & 12.0 \\
\hline Dividends & & 5.0 \\
\hline Retained profit for the year & & 7.0 \\
\hline
\end{tabular}

The following additional information is available:
(1) For the first time, the company has invested in research and development. Expenditure during the year was \(\$ 5\) million and the full cost has been written off as an expense in the income statement. To measure EVA, it should be assumed that the amortisation charge for \(R \& D\) for the year should be \(\$ 1\) million.
(2) There was an increase in the allowance for doubtful debts from \(\$ 1.5\) million to \(\$ 2\) million.
(3) Taxation is \(30 \%\) of profits.
(4) The company's weighted average cost of capital is \(10 \%\).

\section*{Required}
(a) Calculate an estimate of economic value added for the year. Assume for the purpose of this example that the capital charge is calculated using the yearend value of capital employed, before adding EVA for the year.
(b) Calculate the economic value of the company's capital employed as at the beginning of the following financial year.

\section*{Answer}
\begin{tabular}{lr} 
Capital employed & \$ million \\
At the beginning of the year (no adjustments) & 150.0 \\
Adjustments for: & 2.0 \\
Allowance for doubtful debts & 4.0 \\
Research and development & \\
(\$5 million minus \$1 million amortisation) & 156.0 \\
Year-end economic value of capital employed & -1
\end{tabular}

Note: The capital charge will be calculated in this example on the year-end value of the capital employed before adding EVA. In practice, the beginning-of-the-year value of capital employed is normally used.
\begin{tabular}{|c|c|c|c|}
\hline \multirow[t]{2}{*}{Method 1} & \multicolumn{3}{|c|}{Method 2} \\
\hline & \$m & & \$m \\
\hline Profit before interest and tax & 24.0 & Accounting profit after tax & 12.0 \\
\hline Tax charge on profit & (6.0) & Tax relief on interest & (1.2) \\
\hline Plus tax relief on interest & (1.2) & Add back & \\
\hline & 16.8 & Interest charge & 4.0 \\
\hline Adjustments & & Deferred tax & 2.0 \\
\hline Increase in allowance for doubtful debts & 0.5 & & 16.8 \\
\hline R\&D expenditure capitalised & 5.0 & Adjustments for & \\
\hline Amortisation of R\&D & (1.0) & Increase in allowance for doubtful debts & 0.5 \\
\hline NOPAT & 21.3 & R\&D capitalised & 5.0 \\
\hline & & Amortisation of R\&D & (1.0) \\
\hline & & NOPAT & 21.3 \\
\hline
\end{tabular}
\begin{tabular}{lr} 
Economic value added & \$ million \\
NOPAT & 21.3 \\
Capital charge (\$156 million \(\times 10 \%\) ) & 15.6 \\
\cline { 2 - 2 } EVA & 5.7 \\
& \\
Economic value of capital employed & \(\$\) million \\
Year-end value before adding EVA & 156.0 \\
EVA for the year & 5.7 \\
Dividends paid & 5.0 \\
At the beginning of the next financial year & 156.7 \\
\hline
\end{tabular}

\subsection*{5.4 Market value added: MVA}

Market value added or MVA is a measurement of value that builds on the Stern Stewart EVA model. EVA measures the economic value that has been added to a company during a specific period of time, such as a financial year. MVA provides a link between EVA and the market valuation of companies.

\section*{Measuring MVA}

MVA can be calculated as follows:
- Take the total amount of money that has been invested in the company, through share issues, borrowing and retained earnings. Make adjustments to retained earnings, to allow for differences between accounting profit and economic value. The resulting figure should be the economic book value of the capital employed in the company.
- Calculate the current total market value of the company's capital - equity and debt.
- Calculate the difference between these two amounts, to obtain MVA.

MVA \(=\) [Current market value of equity and debt] minus [Economic value of assets/capital employed]

MVA is therefore the premium over the economic book value of the company's assets that investors have given to the company's securities.
- If MVA increases, value has been created.
- If MVA falls, value has been lost or destroyed.
- Over time, if a company adds shareholder value each year, and has a positive EVA, MVA should increase.

\subsection*{5.5 The link between EVA and MVA}

Another way of defining MVA is as follows:
If MVA is positive, it represents a premium that investors place on the company's securities, in excess of the economic book value of its assets. It therefore represents a value that investors give to the expected future growth opportunities for the company. This value can be defined as the present value given by investors to the expected EVA that the company will create in future years.


\subsection*{5.6 EVA and MVA as a valuation method for companies}

EVA and MVA could be used as a method of putting a valuation to a company (public or private company) or to an operating division of a company. A valuation could be obtained as follows:
\begin{tabular}{llc} 
& Current economic value of assets & X \\
Add & Estimated MVA & Y \\
& Valuation of company/division & \(\mathrm{X}+\mathrm{Y}\) \\
\hline
\end{tabular}

The MVA can be calculated by estimating the expected EVA for the company or division over a selected number of years into the future. These EVAs should then be discounted to a present value using a suitable cost of capital, to obtain a valuation for MVA.

\section*{Example}

ABC, a public limited company, is considering a takeover bid for XYZ, a private company. The takeover will be 'friendly', but ABC needs to decide a price that it is prepared to offer for the acquisition of the shares in XYZ.

The accounts of XYZ for the past few years have been analysed, and adjustments have been made to obtain an estimated economic book value of the assets of XYZ , which is \(\$ 50\) million. It has also been estimated that for each of the past five years, XYZ has made economic value added of \(\$ 4\) million.

ABC is prepared to pay for the economic value of XYZ 's assets plus a premium based on MVA. It proposes to calculate MVA as the present value of expected annual EVAs for the next ten years. It is estimated that EVA will be \(\$ 4.5\) million per year in the future. A suitable cost of capital to apply to the valuation is \(9 \%\).

The price that ABC will offer is therefore calculated as follows:
Current economic value of assets
Estimated MVA (see working)
Valuation of company
Working: Calculation of MVA
Annual EVA \(=\$ 4.5\) million
Discount factor, years \(1-10\), at \(9 \%=6.418\)
MVA \(=\$ 4.5\) million \(\times 6.418=\$ 28.9\) million

\subsection*{5.7 Advantages and disadvantages of EVA and MVA for valuation}

There are several advantages with the EVA/MVA valuation method.
- The concept is quite simple, although the calculation of EVA might be complex.
- It can be applied to private companies and also to divisions of companies.
- It links the valuation of companies to the creation of shareholder value. (Research appears to show that over time, changes in the MVA of companies have been closely correlated to EVA.)

There are several disadvantages with the EVA/MVA valuation method.
- To use EVA for a company valuation, we use estimates of EVA in future years. EVA was actually devised as a historical measure of past performance.
- It could be argued that a valuation based on the PV of free cash flows from the acquired company is more appropriate than a valuation based on a conceptual value such as EVA.

\section*{Other aspects of valuation}
- Other factors to consider in pricing a takeover bid
- Valuation assumptions
- Choice of valuation method and cost of capital
- APV method of assessing a company valuation
- The iterative method of equity valuation
- Valuation of high-growth start-up companies

\section*{6 Other aspects of valuation}

\subsection*{6.1 Other factors to consider in pricing a takeover bid}

When a company is considering a takeover bid for another company, there a several important factors to consider before deciding what offer price might be appropriate. These include:
- Synergy
- Risk exposure. An acquisition or merger might alter the exposure to market risk, and either increase or reduce the asset beta of the enlarged company. Before masking a takeover bid, or deciding on a suitable offer price, the company making the acquisition should consider what changes might happen to its risk exposures, and how significant they might be.
- Real options. An acquisition might create real options to expand operations or re-deploy at some time in the future. Real options are described in another chapter. Briefly, a real option is an option to take a course of action at some time in the future, often depending on how the business develops. If an acquisition creates a real option, such as an option to expand into a new geographical market at some time in the future, this option has some value.
- Financing. An acquisition often requires substantial financing, and the acquiring company needs to consider how it might be able to raise the finance. It might be difficult to raise new debt finance to acquire a highly-geared target company (depending on conditions in the debt markets) when the acquiring company is also highly-geared. Using retained earnings to finance acquisitions might have implications for dividend policy.
- Valuation assumptions.

\subsection*{6.2 Valuation assumptions}

In a takeover process, separate valuations for the target company are made by the company making the takeover bid and the directors of the target company. Shareholders in both companies might also have their own views on valuation. Valuations are based on assumptions, but many assumptions are based on judgements rather than 'hard evidence'. Estimates might be unreliable, such as estimates of the growth rate in future profits or free cash flows of the target
company. To allow for negotiations on price, the directors of the company making the takeover bid should consider an initial offer price and a price that they might be willing to go up to. The directors of the target company should consider whether to reject an initial offer from the bidding company, but then consider the minimum price they would be willing to agree to and recommend to their shareholders.

\subsection*{6.3 Choice of valuation method and cost of capital}

Several valuation methods have been described in this chapter. A company preparing to make a takeover bid should consider several valuation methods and compare the different valuations they produce. Judgement should then be used to arrive at a price that should be offered for the target company shares.

Another factor to consider in the choice of valuation method is the risk profile of the acquiring company, and how this might change after the acquisition. There are two aspects to the risk profile:
- Financial risk: the financial risk of the acquiring company might change because of a change in financial gearing due to the method used to finance the acquisition. For example, if gearing increases because the acquisition is financed largely by debt capital, the financial risk of the company will increase as a consequence of the acquisition.
- Business risk. The target company might operate in an industry or market sector where the business risk is very different from the business risk profile of the acquiring company. When this happens, the business risk profile of the company will change as a result of the acquisition.

\section*{No change in the risk profile after the acquisition}

If there is no change in the financial risk profile or business risk profile as a consequence of the acquisition, the company's current WACC can be used for a cash flow-based valuation (NPV, SVA, EVA) and the current cost of equity can be used for a dividend-based valuation.

\section*{Change in the financial risk but not the business risk}

When there is a change in the financial risk of the acquiring company as a consequence of an acquisition, one approach to valuation is to re-calculate a cost of equity at the new gearing level and a new WACC (using the method described in the earlier chapter on cost of capital).

An estimated valuation for a proposed acquisition can also be assessed using the APV method.

\subsection*{6.4 APV method of assessing a company valuation}

The APV method was described in an earlier chapter in the context of capital expenditure appraisal. The same method can be applied to the valuation of a target company when the financial risk profile of the acquiring company changes as a result of the acquisition.
An example will be used to illustrate the technique.

\section*{Example}

Big Company is considering a takeover bid for Little Company. Big Company is currently financed by \(75 \%\) equity and \(25 \%\) debt capital. The equity beta of the company's shares is 1.10. The debt capital is considered risk-free. Tax on profits is at the rate of \(30 \%\). The market rate of return is \(9.5 \%\).

Big Company thinks it will be prepared to offer \(\$ 15\) million to acquire the share capital of Little Company, and would finance the acquisition entirely with debt capital at the risk-free rate of \(6 \%\). For simplicity, assume that the debt capital is borrowed in perpetuity.

The costs of the acquisition would be an additional \(\$ 2.5\) million in Year 0 , with tax relief on all these costs in Year 1. The issue costs would be financed from retained profits.

By acquiring the share capital of Little Company, Big Company will increase the post-tax operating profits by \(\$ 2\) million per year in perpetuity. Assume that the debt capital will be repaid in full at the end of ten years.

An APV valuation can be made as follows.
(1) The ungeared beta can be calculated as follows:
\(\beta_{\mathrm{EU}}=1.10 \times 75 /[75+25(1-0.30)]=0.81\)
(2) Cost of equity \(=6 \%+0.81(9.5-6.0) \%=8.8 \%\), say \(9 \%\).
(3) Base case NPV \(=\$ 2\) million \(/ 0.09=\$ 22.2\) million. The acquisition cost f \(\$ 15\) million must be deducted, giving a net base case NPV of \(\$ 7.2\) million.
(4) Tax relief on interest \(=(6 \% \times \$ 15\) million \() \times 30 \%=\$ 270,000\).

PV on tax relief (in perpetuity) \(=\$ 270,000 / 0.06=\$ 4.5\) million.
(5) PV of acquisition costs \(=\$ 2.5\) million \(+(\$ 2.5\) million \(\times 30 \% \times 0.943)=\$ 1.80\) million.
(6) APV:
\begin{tabular}{lr} 
& \(\$ \mathrm{~m}\) \\
Base case NPV & 7.2 \\
PV of other costs & \((1.8)\) \\
PV of tax relief on interest & 4.5 \\
\cline { 2 - 2 } & 9.9 \\
\hline
\end{tabular}

The APV is positive, indicating that the acquisition is worthwhile at a purchase cost of \(\$ 15\) million.

\subsection*{6.5 The iterative method of equity valuation}

Another approach to obtaining a valuation for the equity of a company can be used where the target company is in a different industry sector and it is not certain what the cost of equity and cost of capital should be used for the valuation. The valuation method is independent of how the acquisition will be financed.

The method is 'iterative', which means that the same method is used to make a large number of repeated valuations until a 'steady state' is reached, and each successive valuation produces the same total equity value.

Each repetition of the calculation process uses a cost of equity that is calculated from an ungeared equity beta for the industry.

\section*{Initial assumptions and estimates}

The starting point is to decide a value of debt for the company. This value does not change in any of the iterative calculations. Next, an arbitrary valuation for equity should be selected: any valuation is acceptable as a starting point. If in doubt, the book value of equity should be used, but any other reasonable starting point is just as good.

The expected cash flows from the acquisition should be estimated next. These cash flows should be the earnings before interest and tax, or a cash flow equivalent (which is even better). Tax should then be deducted to arrive at future estimates for earnings or cash flow before interest but after tax (EBIBAT).

\section*{The iterative process}

The iterative process starts with a calculation of the geared equity beta, using the ungeared equity beta for the industry and the initial assumptions about capital structure.

For example, suppose the valuation for the debt capital in the company is taken as \(\$ 1,000,000\) and the starting point for the valuation of equity is the book value of equity, which is \(\$ 800,000\). The rate of tax is \(30 \%\) and the ungeared industry beta is 0.85 . The re-geared beta for the company would then be calculated as follows:
\(0.85=\) Geared beta \(\times 800 /[800+(1,000 \times 0.7)]\)
Geared beta \(=1.5937\).
The re-geared beta can be used to calculate a cost of equity, using the CAPM. This cost of equity should then be used to obtain a present value of the estimated future EBIBAT cash flows from the acquisition.

This first iteration gives a new value for equity, different from the starting valuation. The same calculation process should now be repeated using the new equity valuation but the same debt value and the same ungeared industry beta as before. This gives a new re-geared equity beta, a new cost of equity and a new equity valuation.

The process should be repeated, often many times, before the valuation of equity is the same with each repeated calculation. This 'steady state' valuation provides a valuation for equity in the company, given a valuation of the debt.

\section*{Example}

Large Company is considering a bid for the equity in Small Company. Small Company is in a different industry where the ungeared equity beta is 0.90 .

The value of Small Company's debt is \(\$ 600,000\) and the book value of its equity is \(\$ 800,000\).

The following cash flows would be expected, before interest and taxation, if the takeover is successful:
\begin{tabular}{ll} 
Year & \(\$\) \\
1 & 200,000 \\
2 & 500,000 \\
3 & 550,000 \\
4 onwards & 600,000 per year
\end{tabular}

The rate of tax is \(30 \%\). The risk-free rate of interest is \(5 \%\) and the market risk premium is \(3 \%\).

Using the iterative method of valuation, it can be assumed that for the first iteration, the value of equity should be \(\$ 800,000\). The re-geared beta for the company would then be calculated as follows:
\(0.90=\) Geared beta \(\times 800 /[800+(600 \times 0.7)]\)
Geared beta \(=1.3725\).
The cost of equity \(=5 \%+1.3725 \times 3 \%=9.1 \%\), say \(9 \%\).
\begin{tabular}{|c|c|c|c|c|c|}
\hline Year & Cash before and tax & flow interest & After tax & Discount factor at 9\% & PV \\
\hline & \$ & & \$ & & \$ \\
\hline 1 & 200,000 & & 140,000 & 0.917 & 128,380 \\
\hline 2 & 500,000 & & 350,000 & 0.842 & 294,700 \\
\hline 3 & 550,000 & & 385,000 & 0.772 & 297,220 \\
\hline 4 onwards & 600,000 & & 420,000 & \((1 / 0.09) \times 0.772\) & 3,602,667 \\
\hline & & & & & 4,322,967 \\
\hline
\end{tabular}

This calculation process is repeated using an equity valuation of \(\$ 4,322,967\) rather than \(\$ 800,000\), to obtain another equity valuation.

The process is repeated until each successive repetition of the calculation process produces the same valuation for equity.

\subsection*{6.6 Valuation of high-growth start-up companies}

The techniques already described in this chapter can be used for any sort of company valuation. However, a particular problem arises in the case of highgrowth companies that are being set up, or have recently been set up. Valuations might be needed by venture capital organisations that have been asked to invest capital in the business.

The special features of high-growth start-up companies are as follows:
- The risk of failure of the business is high. Venture capital investors might allow for a high failure rate when they make their investment decisions.
- Estimates of cash flows will be subject to enormous uncertainty, and might be little better than a guess.
- The company's equity will have no value for a venture capitalist until an 'exit route' is available, and the venture capitalist is able to sell the shares, either in a stock market flotation or in a trade sale of the company to another more established company in the industry.

A venture capitalist might therefore be more interested in the exit valuation at the end of the investment period, rather than estimates of cash flows in the intervening years. An estimated exit valuation might be discounted to a present value, so that the venture capitalist can assess how much it might be willing to invest in the company.

The company's management need to prepare a robust business plan, to give a venture capitalist some reassurance that the business risks with the start-up business can be managed successfully.

Financing acquisitions and mergers
- Purchase consideration
- All-share consideration
- Cash or debt finance in the purchase consideration

\section*{\(7 \quad\) Financing acquisitions and mergers}

\subsection*{7.1 Purchase consideration}

In a merger, one company might acquire the other company by issuing new shares. The shareholders in the other company exchange their shares for the new shares in the first company. For example, if company A and company B merge, company A might issue new shares in exchange for the shares in company B. The company B shareholders therefore become shareholders in the enlarged company A.

Similarly, a takeover might be paid for by issuing new shares to the shareholders in the target company, who will then become shareholders in the buying company after the takeover.

\section*{Example}

Company X makes a successful bid to acquire the shares in company Y. Company Y consists of 500,000 shares of \(\$ 1\) each. Company \(X\) offers 3 new shares for every 2 shares in company Y. Company \(X\) therefore issues 750,000 new shares to purchase the shares in company \(Y\) and the company \(Y\) shareholders become shareholders of company \(X\). If the shares of company \(X\) are valued at \(\$ 4\) each before and after the takeover, the takeover price is \(\$ 3,000,000(750,000\) shares \(\times \$ 4)\).

Instead of making an acquisition by exchanging shares, a company might pay for some or all of the acquisition:
- in cash (which it might obtain by borrowing)
- by issuing debt securities in exchange for some of the shares in the target company

In the example above for example, company X might have acquired the shares in company Y for \(\$ 3\) million by paying \(\$ 2\) million in cash and \(\$ 2\) of redeemable loan stock for every 1 share in company Y.

Cash to finance a takeover bid might be obtained from existing cash resources, or by issuing new shares to other investors, or by borrowing.

The purchase consideration - both the total purchase price for a takeover and the nature of the purchase consideration (shares, cash or debt securities) - are usually subject to negotiation in a friendly takeover.
- In a friendly takeover, the directors of the target company negotiate terms that they then recommend to their shareholders for acceptance.
- In a hostile takeover, the bidding company makes an offer to the shareholders in the target company, without the co-operation of the board of directors of the target company. With a hostile takeover bid, the bid is much more likely to succeed if it is an all-cash offer, because the shareholders in the target company might not want to retain any investment in the acquiring company.

\subsection*{7.2 All-share consideration}

When a takeover is financed by issuing new shares as the purchase consideration, either the shareholders in the buying company or the shareholders in the target company will gain value at the expense of the shareholders in the other company.

\section*{Example}

Company A and company B are companies whose shares are traded on a stock market. Company A makes profits after tax of \(\$ 800,000\) each year and company B makes after-tax profits of \(\$ 400,000\). Both companies have 1 million shares in issue. Company A shares are valued on a price/earnings multiple of 10 and company B shares are valued on a P/E multiple of 9 ( \(\$ 3.60\) per share).

Suppose that company A makes an offer of \(\$ 4.40\) per share for company B, valuing company B at \(\$ 4.4\) million. This values the company on a P/E ratio of 11 .

Its own shares are valued at \(\$ 8\) each (EPS \(\$ 0.80 \times \mathrm{P} / \mathrm{E}\) ratio 10 ). If company A makes an all-share offer, it would therefore propose to issue 550,000 new shares to the shareholders in company B ( \(\$ 4.4\) million \(/ \$ 8\) per share). The terms of its offer would therefore be 11 new shares in company A for every 20 shares in company B.

If the offer is accepted by the shareholders of company B, company A would consist of \(1,550,000\) shares after the takeover. The combined after-tax profits of the company might be \(\$ 1.2\) million ( \(\$ 800,000+\$ 400,000\) ), which is the sum of the profits of the individual companies before the takeover.

Does the takeover create wealth for the shareholders of company A?
The answer depends on what happens to the share price of company \(X\) after the takeover. Unless there is an increase in the P/E ratio, or an increase in the profits of the combined companies, a takeover will destroy value for the shareholders of the company making the takeover when all of the following conditions apply:
- The purchase consideration is all shares.
- The purchase price is on a higher P/E multiple than the P/E ratio of the company making the takeover.
- There is no increase in the combined profits of the two companies after the takeover.

In this example, if the \(\mathrm{P} / \mathrm{E}\) ratio of the company after the takeover is 10 , the total value of the company will be \(\$ 12\) million ( \(\$ 1.2\) million \(\times 10\) ), which is \(\$ 7.74\) per share ( \(\$ 12\) million \(/ 1,550,000\) shares).
The former shareholders in the two separate companies will therefore lose or gain value in their investment, as follows:
\begin{tabular}{lrrrr} 
& \begin{tabular}{r} 
Number of \\
shares
\end{tabular} & \begin{tabular}{r} 
Value at \\
\(\$ 7.74\)
\end{tabular} & \begin{tabular}{r} 
Investment \\
value before \\
the takeover
\end{tabular} & \begin{tabular}{r} 
(Loss)/gain \\
in value
\end{tabular} \\
Shares owned by & & \(\$ \mathrm{~m}\) & \(\$ \mathrm{~m}\) & \begin{tabular}{r}
\(\$ \mathrm{~m}\) \\
\hline
\end{tabular} \\
Original shareholders in A & \(1,000,000\) & 7.74 & 8.00 & \((0.26)\) \\
Former shareholders of B & 550,000 & 4.26 & & 3.60 \\
& & 12.00 & & 0.66
\end{tabular}

There has been some increase in value in the combined company, because the earnings of company B are now valued on a higher \(\mathrm{P} / \mathrm{E}\) multiple. However, all the benefits are enjoyed by the former shareholders in company B. The original shareholders in company A have suffered a loss in value due to the takeover.

\section*{All-share consideration: the effect of synergy}

If there is synergy, and total annual earnings and cash profits increase, the total value of the businesses should be expected to increase, adding value for the shareholders.

In the example above, suppose that after the acquisition of company B by company A, there are benefits from economies of scale and total annual earnings increase by \(\$ 200,000\) to \(\$ 1,400,000\). Suppose also that the \(\mathrm{P} / \mathrm{E}\) ratio of the enlarged company A is 10. The total value of company A will now be \(\$ 1.4\) million \(\times 10=\$ 14\) million. The price per share will be \(\$ 9.03\).

The former shareholders in the two separate companies will therefore gain value in their investment, as follows:
\begin{tabular}{lrrrr} 
Shares owned by & \begin{tabular}{r} 
Number \\
of shares
\end{tabular} & \begin{tabular}{r} 
Value at \\
\(\$ 9.03\)
\end{tabular} & \begin{tabular}{r} 
Value \\
before \\
takeover
\end{tabular} & \begin{tabular}{r} 
Gain \\
in value
\end{tabular} \\
\hline & & \(\$ \mathrm{~m}\) & \(\$ \mathrm{~m}\) & \(\$ \mathrm{~m}\) \\
Original shareholders in A & \(1,000,000\) & 9.03 & 8.00 & 1.03 \\
Former shareholders of B & 550,000 & 4.97 & 3.60 & 1.37 \\
& & 14.00 & &
\end{tabular}

Because of the synergy and growth in combined earnings (or cash profits), shareholders in both companies benefit from the takeover.

\subsection*{7.2 Cash or debt finance in the purchase consideration}

Another way of providing extra value for shareholders in a takeover might be to pay for the acquisition in whole or in part with either cash (financed by borrowing) or debt securities. The benefit of paying in these ways is that there is tax relief on the interest cost.

The target company's shareholders might want to be paid in cash; therefore offering to buy the target company's shares for cash (financed by new debt) could be attractive to those shareholders, who will be cashing in their investment by accepting the takeover offer.

\section*{Example}

Returning to the previous example, suppose that a takeover price of \(\$ 4,400,000\) is agreed for company B, and that this purchase price is paid all in cash, with the money obtained from a bank loan at \(8 \%\) interest.

The rate of tax is \(25 \%\), and after the takeover there is no synergy. The company's shares continue to be valued on a P/E multiple of 10 .

The company's earnings and value after the takeover can be calculated as follows:
\begin{tabular}{|c|c|c|}
\hline & \$ & \$ \\
\hline Profits of 'old' company A & & 800,000 \\
\hline Profits of 'old' company B & & 400,000 \\
\hline & & 1,200,000 \\
\hline Interest on debt (\$4.4 million \(\times 8 \%\) ) & \((352,000)\) & \\
\hline Less tax relief at 25\% & 88,000 & \\
\hline Net cost of interest & & \((264,000)\) \\
\hline Annual earnings after the takeover & & 936,000 \\
\hline Value of company A shares & \$ & \\
\hline After the takeover (\$936,000 \(\times 10\) ) & 9,360,000 & \\
\hline Before the takeover & 8,000,000 & \\
\hline Gain to company A shareholders & 1,360,000 & \\
\hline
\end{tabular}

Company A will now be geared, and its cost of capital might therefore change. If so, the \(\mathrm{P} / \mathrm{E}\) ratio would no longer be 10 . Even so, financing a takeover with debt capital can add to the wealth of the shareholders because of the tax relief on debt interest.

\section*{10}

\section*{Corporate reconstruction and reorganisation}

\author{
Contents \\ 1 Predicting corporate failure \\ 2 Capital reconstructions \\ 3 Business reorganisation
}

\section*{Predicting corporate failure}
- Corporate failure prediction models
- Z scores (Zeta scores)
- Altman's Z score (Zeta score)
- Other Z score models
- KMV/Moody's failure prediction model
- Using Z score models
- Problems with Z score models

\section*{1 Predicting corporate failure}

\subsection*{1.1 Corporate failure prediction models}

Many user groups, including banks, suppliers, auditors and the government would like to be able to predict corporate failure. The collapse of a company occurs when it is unable to settle its liabilities when they fall due. Fierce competition in the market, high levels of debt and poor liquidity are often the main causes of failure.

It may be possible to assess a company's risk of failure and insolvency using financial statement analysis and the calculation and assessment of key financial ratios. Models have been developed that assess the likelihood that an entity may become insolvent.

Many failure prediction models are based on a small number of key financial ratios. These are ratios that have been identified as ratios that help to distinguish between companies that have a high probability of financial failure and those that do not, in the short-term or medium-term.

\subsection*{1.2 Z scores (Zeta scores)}

The ratios for a company are given weightings by the model, and the weighted ratios are then added up to obtain a 'score'. This score might be called a Z score or a Zeta score. The score provides a measure of the probability of corporate failure.

\section*{Multivariate discriminant analysis}

The ratios in corporate failure prediction models are based on factors or variables that are considered significant for the financial 'health' of a company. The probability of failure cannot be based on any single variable or ratio. Several variables and several different ratios need to be considered.

A corporate failure prediction model may therefore be constructed to carry out multivariate discriminant analysis, which is simply an analysis of several variables and ratios, for which weightings are given. The resulting total Z scores for different companies provide a probability distribution of survival or failure. The probability
distribution of Z scores is often assumed to be normally distributed, so that companies whose Z scores are at the extreme low end of the distribution have a high probability of failure.

\subsection*{1.3 Altman's Z score (Zeta score)}

The most well-known failure prediction model is Altman's Z score. His original Z score model was developed in the 1960s and had five key ratios, each with a different weighting:
\(Z=1.2 X_{1}+1.4 X_{2}+3.3 X_{3}+0.6 X_{4}+1.0 X_{5}\)
where:
\(\mathrm{X}_{1}=\) working capital/total assets
\(\mathrm{X}_{2}=\) retained earnings/total assets
\(\mathrm{X}_{3}=\) earnings before interest and tax/total assets
\(\mathrm{X}_{4}=\) market value of equity/book value of total debt
\(\mathrm{X}_{5}=\) sales/total assets.

The Z score is obtained by calculating the five ratios, and applying the weightings to each ratio. The sum of the five weighted ratios provides the Z score.

The original model was then used by comparing a company's Z score with the following indicators:
\begin{tabular}{ll} 
Z score & Prediction \\
\hline Less than 1.81 & High probability of corporate failure \\
\(1.81-2.67\) & No significance: in the 'grey area' \\
Greater than 2.67 & Very low probability of failure
\end{tabular}

\subsection*{1.4 Other Z score models}

Other Z score models have been produced, because it has been found that a single model with fixed weightings for specific key ratios does not apply with equal accuracy to different types of company and different industries.
- Different weightings are appropriate for companies in different industries.
- Similarly, the appropriate weightings for small companies may be different from the weightings that should be used for larger companies.

The terms 'Z score' and 'Zeta score' are often used as if they have exactly the same meaning. However, a Z score is a score based on a model that is applied generally to all companies, and a Zeta score is a 'customised' score based on the particular characteristics of the company and the industry or country in which it operates.

\subsection*{1.5 KMV/Moody's failure prediction model}

The KMV/Moody's model is a well-known example of another corporate failure prediction model. It is based on the Black-Scholes option pricing model. This model does not use key ratios from the financial statements. Instead, it uses data about share prices and the values of assets and debts of the company to predict failure.

The KMV / Moody's model is able to assess:
- default risk, and
- the value of a company's debt capital and the probability that some or all of the debt will be recovered, in the event of corporate failure.

The additional information provided by this model could be particularly valuable for bond investors and lending banks.

\subsection*{1.6 Using Z score models}

Corporate failure prediction models are used today by auditors and management consultants. They are also included in database systems used by banks for loan evaluation. A Z score can be used by a bank to decide whether to include a default risk premium in the interest rate it charges on a corporate loan, and if so what the size of the risk premium should be.

\subsection*{1.7 Problems with Z score models}

Z score models provide an objective statistical measure. However they have some problems.
- A Z score is only significant when it has an extremely low or high value. Z scores in the middle range are usually unreliable as predictors of failure. For example Altman was unable to draw any firm conclusions for a Z score between 1.81 and 2.67 .
- There is some possibility of statistical error in Z score predictions. A Z score model might indicate that a company is financially sound when it is in fact about to fail. Similarly, a Z score might indicate that a company is very likely to fail when in fact it is financially sound. Statistical errors should be unusual, but they cannot be avoided.
- Since a different model might be appropriate for different types of company and different industries, companies wanting to obtain Z score information must usually rely on external specialist consultants for data and advice.
- When a company collapses, its financial ratios might deteriorate very quickly, in the few months or weeks before the collapse occurs. However the most recent financial statements might be up to a year or more out of date. Z scores might therefore only indicate in retrospect, after the corporate failure has happened, that the company was at risk.
- A Z score model does not identify how quickly the company might go into liquidation.

\section*{Capital reconstructions}

■ Definition of a reconstruction
- The purpose of a reconstruction
- The problems with a reconstruction scheme
- Examination questions on reconstructions

\section*{2 Capital reconstructions}

\subsection*{2.1 Definition of a reconstruction}

A capital reconstruction is a major reorganisation of the capital structure of a company. When a reconstruction occurs, the reason is usually because the company is in financial difficulties. For example, its Z score might indicate that it is in danger of financial collapse. It is more probable however that cash flow projections, and possibly profit projections, indicate that the company cannot avoid financial collapse and liquidation unless urgent action is taken to 'reconstruct' the company's finances.

A reconstruction might involve:
- the conversion of debt capital into equity
- and possibly the conversion of equity shares from one form to another.

Existing equity shareholders might be required to relinquish all their shares and accept a much smaller quantity of new shares in the reconstructed company. The majority of the new shares on the reconstructed company might be held by lending banks or bond investors, who agree to exchange their loans or bonds for new shares in the reconstructed company. Reconstructions are likely to be subject to regulation by company law.

\subsection*{2.2 The purpose of a reconstruction}

The purpose of a capital reconstruction is to find a way of allowing the company to continue in business, and avoid insolvency and liquidation. A reconstruction is therefore only worth considering:
- if it is likely to be more beneficial to all the parties concerned than a liquidation of the company (and the sale of its assets), and
- the reconstructed company has a good chance of surviving and restoring itself to profitability.

A reconstruction will benefit all the parties if it is likely to result in them getting more cash or more value from the reconstruction than from an enforced liquidation of the company.

For example, the effect of a reconstruction scheme might be to:
- reduce the total debt of the company (because some debt capital might be exchanged for new equity), and
- defer the repayment obligations for other debt capital.

This reduction in total debt and deferral of interest payments and principal repayments might allow the company to make a profit and achieve positive cash flows. If the company is able to recover, it should be able to pay back its outstanding debts and also create value (in the share price) for the former lenders who are now equity shareholders.

The parties who must agree to a reconstruction therefore include all the creditors of the company (or a sufficient number of them, in accordance with the requirements of national company law) and the shareholders.

\subsection*{2.3 The problems with a reconstruction scheme}

There are some major problems with arranging a capital reconstruction. These are:
- finding a reconstruction arrangement that will benefit all the parties, and
- getting all the parties to agree to the proposal.

Each of the parties involved in a reconstruction (banks who are lending money, bondholders, unpaid trade creditors and the shareholders) need to believe that the reconstruction offers them the prospect of more cash or more value than a liquidation of the company. Each of the parties will therefore compare what they will probably receive:
- if the construction scheme is agreed, and
- if the scheme is rejected and liquidation occurs.

\section*{Example}

A company has 1 million shares of \(\$ 1\) in issue, \(\$ 5\) million in \(6 \%\) bonds and trade creditors of \(\$ 200,000\). The company is in severe financial difficulties and unless a reconstruction scheme is agreed, the company will be liquidated. The bondholders would expect to receive \(\$ 0.40\) in the \(\$ 1\) in a liquidation, and the trade creditors and equity shareholders would receive nothing.

The following reconstruction scheme might be proposed:
- The company cancels its existing equity shares and issues, say, 10 million new shares of \(\$ 0.05\) each.
- The bondholders are offered 9.9 million of these shares in exchange for their bonds.
■ The remaining 100,000 shares might be offered to the existing equity shareholders, in a 1 for 10 exchange. The shareholders will surrender their \(\$ 1\) shares in the company and receive one new share of \(\$ 0.05\) each for every 10 of their \(\$ 1\) shares.
- A bank might agree to provide a loan to the newly-reconstructed company.

■ The trade creditors might be offered \(\$ 0.30\) in the \(\$ 1\) in full settlement of the unpaid debts.

This scheme might be agreed, but only if the following conditions are met:
- The company will expect to be profitable and to have positive cash flows as a result of the new structure.
- The bondholders believe they will gain more from accepting equity in the company than forcing a liquidation to receive \(\$ 0.40\) in the \(\$ 1\).
- The trade creditors and equity shareholders also agree.
- The reconstruction complies with the requirements of company law.

\subsection*{2.4 Examination questions on reconstructions}

Company law on reconstructions varies between countries, and it is therefore unlikely that you will be expected to demonstrate a detailed understanding of the law. However, you should be aware that some legal process and approval for a proposed scheme of reconstruction might be required.

Capital reconstruction proposals can be fairly complex, so it is unlikely that in your examination you will be required to devise and recommend a detailed scheme of reconstruction for a particular company. However an examination question might give you some details about a company that is in financial difficulties, and give details of a reconstruction scheme that has been proposed. You might then be asked to comment on the proposed scheme.

To answer this type of question, you should consider three aspects to the proposal.
- The first step is to make sure that you understand what the capital of the reconstructed company will be. How many shares will be in issue and who will own them? How much debt capital will the company have, and what will be the rate of interest payable on the debt?
- Having established what the new capital structure will be, you should make a financial analysis to decide whether the company is likely to become profitable and have positive cash flows after the reconstruction has occurred. If the company is unlikely to survive in its reconstructed form, the reconstruction proposal should not be supported or recommended.
- If you consider that the company will return to profits and positive cash flows if the reconstruction occurs, you should then look at each of the investor groups in the company. If any investor group is unlikely to benefit from the reconstruction, it will not support the reconstruction proposals. For each investor group - ordinary shareholders, preference shareholders, bondholders and banks - you should consider what will happen to them:
- if there is no reconstruction
- if the company is reconstructed.

\section*{Business reorganisation}
- Definition of a business reorganisation
- Demergers
- Divestments
- Management buyouts and management buy-ins
- Share buy-back (share repurchase scheme)
- Going private

\section*{3 Business reorganisation}

\subsection*{3.1 Definition of a business reorganisation}

A capital reconstruction is a major change in the capital structure and ownership of a company in financial difficulties. A business reorganisation is similar, in the sense that it often involves a change in capital structure and a change in ownership. However, a reorganisation normally involves a company that is not in financial difficulties as the result of a business strategy decision, such as selling off a non-core part of the group's business operations, or de-merging two divisions of a company into two entirely separate and independent companies.

\subsection*{3.2 Demergers}

A demerger is the splitting up of a group of companies into two (or more) separate and independent companies. Often, the group in a demerger is a conglomerate, consisting of widely-different businesses. For example, some years ago in the UK, there was a demerger of chemicals company ICI from pharmaceuticals company Zeneca. In 2007 Cadbury Schweppes proposed either to demerge its confectionery division (Cadbury) and soft drinks division (Schweppes) into independent companies, or to sell off its soft drinks division as part of a strategic review.

The shareholders in the company before the demerger receive shares in both demerged companies. They can choose to hold all the shares, and be a shareholder in both companies. Alternatively, they can sell their shares in one of the companies and remain a shareholder in just one of the demerged companies. (They could also sell their shares in both companies.)

If the company before the demerger has debt capital, it will have to negotiate with its banks or representatives of its bondholders, to agree which of the demerged companies should take on the debt obligations.

\section*{Purpose of a demerger}

A demerger requires the consent of the company's shareholders, and the purpose of the demerger should be in the best interests of the shareholders. This means that a demerger is only justified if it is expected to increase the total wealth of the shareholders.

The reasons that may be put forward for a demerger are as follows:
- A conglomerate group of different businesses is often valued at less than the sum of the value of its separate businesses, if these were independent. This is because stock market investors often prefer 'specialised' businesses to conglomerates.
- 'Specialised' businesses can focus on their core competencies.
- Specialised businesses can develop their own strategies for the future and are more likely to be successful if they can decide their strategies independently.
- There might be some savings in head office costs, no longer incurred after the demerger.

\subsection*{3.3 Divestments}

A divestment is the sale of a business by a company to an external purchaser (or external purchasers). The business that is sold might be a subsidiary company or a sub-group of subsidiary companies within the overall group.

Examples of divestments would be:
- the sale of a business to another company
- the sale of a business to a management team, in a management buy-out (MBO): the management team, often supported by venture capital finance, become the shareholders of the divested company
- the sale of a business to external buyers who become the management team as well as the shareholders (a management buy-in).

The company making the divestment should try to ensure that it obtains a satisfactory price for the subsidiary or division. The buyers should try to ensure that they pay a reasonable price and that the purchase is financed in the best way possible.

In contrast to a demerger, where the shareholders of the original company become shareholders in both demerged companies, a divestment (or 'spin-off') involves the sale of the ownership of a business.

The purchase consideration for a divestment:
- normally consists mainly or entirely of cash
- although the selling company might retain an equity interest, and continue to hold a proportion of the shares.

\section*{Purpose of divestments}

A divestment can have any of the following purposes:
- to raise cash for the company, when it needs additional funding or has liquidity difficulties
- to get rid of a loss-making business
- because the company receives a very attractive offer price for selling the business
- for strategic reasons.

The strategic reason most commonly suggested for making a divestment is because the company is selling off a 'non-core' business. The company has a competitive advantage over rivals in businesses where it has 'core competencies', and it should therefore concentrate on these areas of business. In principle, the divestment should allow the company to invest all its resources in those parts of the business where it can make the greatest profits. This will benefit the shareholders.

\subsection*{3.4 Management buyouts and management buy-ins}

Management buyouts and management buy-ins are divestments for the company selling the business, but business ventures for the management team that buys it.

With a management buyout, the purchasers of the divested business include a team of managers who work for the subsidiary or division that is being sold off, and these managers will run the business (as entrepreneurs) after they have bought it.

A management buy-in involves the purchase of a divested business by an external management team, which run the business (as entrepreneurs) after they have bought it.

With both management buyouts and management buy-ins, it is usually the case that the management team do not have sufficient capital of their own to afford the business they are trying to buy, and they have to rely on the support of venture capital finance.

\section*{The structure of venture capital finance}

A venture capital firm might be willing to provide capital support to a management buyout or a management buy-in if it believes that:
- the management team have a robust and realistic business plan for the company
- the management team appear to have the capabilities to make the company a success
- the management team makes a significant capital contribution to the purchase of the business, so that they have a strong financial motive for making the business a success
- the financial returns appear satisfactory, allowing for the high business risk in the project.

Venture capital firms expect a high return on their investments, because a proportion of them are not successful and the investment risk is high.

The financing of a management buyout or management buy-in might be as follows:
- The individual managers in the management team agree to subscribe some of their own money to buy shares in the company they are buying. (If the MBO or MBI involves setting up a new company, the managers agree to buy a quantity of shares in the new company.)
- The venture capital firm also buys a quantity of equity shares, so that the proportions of shares owned by the management team and by the venture capital firm are in a satisfactory balance. The venture capital firm will want an equity stake in the company, but will not want its stake to be so large that there is no incentive or motivation for the management team to make the company a success.
- The venture capital firm will provide additional finance, possibly in the form of redeemable preference shares or unsecured lending. The intention should be that as the company grows and becomes more profitable and cash-rich, it will be able to redeem the preference shares or loan.
- The venture capital firm often negotiates additional loans, possibly secured bank loans, to bring the total capital available up to the amount needed to buy the company.
- The management team may be given an incentive scheme (such as a share option scheme) that will allow them to take more equity if the company is successful.
An MBO or MBI cannot go ahead unless enough capital is provided to purchase the business, and the expected profits from the business must be sufficient to meet the costs of debt interest and preference share dividends, and achieve a profit for the equity shareholders.

\subsection*{3.5 Share buy-back (share repurchase scheme)}

A share buy-back is the purchase by a company of some of its own equity shares, which are then cancelled.
- The shares might be purchased in the stock market at the current market price.
- Alternatively, the company might offer to buy back a quantity of shares at a fixed price from shareholders willing to sell them. Or the company might invite its shareholders to submit bids for the price at which they would be willing to sell a quantity of their shares to the company. When the company invites tenders from its shareholders in this way, it will buy back shares from the shareholders submitting the lowest prices for selling their shares.

Share repurchases are carried out as a part of a planned programme, which requires the prior consent of the shareholders, and which must treat all the equity shareholders fairly (for example, by giving them all an opportunity to sell back some of their shares).

After a share buy-back, the company will have fewer shares in issue.

\section*{Purpose of a share buy-back}

A share buy-back might be appropriate when the company has a large amount of surplus cash, and cannot find enough new investment opportunities for investing the money. Instead of holding the cash (and earning a low rate of interest), it would benefit shareholders if the cash is paid back to them.

An alternative to a share buy-back is a special dividend payment. A dividend would be paid to all the shareholders, and so all the shareholders would receive some cash. However, the company might wish to avoid giving the impression that higher dividends will continue to be paid in the future.

There might be a tax advantage for shareholders in selling back shares rather than receiving a higher dividend.

By reducing the number of shares in issue, the cash profits per share and earning per share after the buy-back should be higher, and the market value of the remaining shares should therefore increase. A share buy-back should therefore be expected to increase the wealth of the remaining shareholders.

\section*{Increasing the company's financial gearing}

Another reason for a share buy-back is to increase the company's gearing. The company may borrow debt capital and use the money to buy back and cancel some of its shares. Replacing equity with low-cost debt may reduce the WACC and increase the value per share of the remaining shares.

\subsection*{3.6 Going private}

A 'public company' might 'go private'. The public company must be a company whose shares are traded on a stock market. When it goes private, it abandons its stock market status and it shares are no longer traded on the market.

Going private usually involves a major change in the share ownership of the company. When its shares are traded on a stock market, the majority of the shares are probably held by institutional investors. When the company 'goes private', the shares are normally acquired by either:
- a small number of former managers in the company and venture capital organisations, or
- a private equity organisation.

Typically, the chairman or chief executive officer of the company, who might be a significant shareholder, decides to withdraw the company from the stock market. He seeks financial support from venture capitalists and banks, and makes an offer to the existing shareholders to buy their shares. The existing shareholders must be offered a sufficiently high price for their shares, to persuade them to sell.

\section*{Reasons for going private}

When a company goes private, the reasons may be as follows:
- The company is obtaining no benefit from its stock market status. For example:
- the company might not want to use the stock market to raise new equity capital
- investors might be unwilling to support a new share issue, so that the company is unable to obtain new funding from the stock market
- the company either does not want to issue new equity to finance takeovers, or is unable to persuade the shareholders of target companies to accept its shares as the purchase consideration for a takeover.
- The chairman or chief executive officer, who is also a major shareholder, might think that the shares are under-valued. By taking the company private and acquiring more of the shares, he will expect to increase his own wealth.

When a significant number of companies revert to private status and give up their stock market status, this would be an indication that the stock market is not acting efficiently in providing capital to companies and that having a stock market status does not help to increase share values.

\section*{Private equity}

Private equity organisations specialise in buying companies, often companies that are not as successful as they should be.
- The private equity firm has a fund for buying companies, and often borrows from banks to obtain large quantities of additional debt finance.
- A company acquired by a private equity firm may be a public company whose shares are traded on a stock market. If so, the private equity firm will buy the company and 'take it private'. This means that the company's shares are withdrawn from trading on the stock market, and the shares are 'de-listed'.

A new management team is appointed to run the company. These managers are usually offered large incentives to make the company successful. This is the key to a successful private equity venture: the acquired company must be changed from an under-performing company to a company with much more value.

In addition, the purchased company might have valuable assets such as land and buildings that the new owners use to raise more capital. For example, property might be sold and leased back or used as collateral for additional borrowing.

If the company is successful, the private equity firm may decide to 'cash in' its investment. One way of doing this is to re-list the shares and re-launch the company on the stock market. When the company is re-launched on the market, the private equity firm will sell its shares to other investors.

Private equity firms have been particularly successful in recent years. Some of this success has been due to the low cost of borrowing. A large proportion of the purchase price of target companies has been paid for with debt capital, on which the interest rate has been fairly low (and there is tax relief on the interest payments).

\section*{11}

\section*{The money markets. The treasury function}

\section*{Contents}

1 The money markets
2 Money market instruments
3 The treasury function
4 Risk management methods in banks
5 Finance of international trade

\section*{The money markets}
- Definition of the money markets
- Purpose of the money markets
- The inter-bank market
- The repo market
- Comparison of the repo market and inter-bank market

\section*{1 The money markets}

\subsection*{1.1 Definition of the money markets}

The money markets are wholesale markets for dealing in short-term lending and borrowing and for trading short-dated financial instruments.
- 'Wholesale' markets are markets for large-value transactions.
- Short-dated financial instruments are financial instruments that have one year or less to maturity when they are issued. They include Treasury bills, bills of exchange, commercial paper and certificates of deposit (CDs). The repo market, which is one of the money markets, also deals in short-dated bonds (bonds with a short time remaining to maturity) as well as other money market securities.

The main money markets are:
- the inter-bank market and
- the repo market.

There are international money markets for all the main currencies. For example, London is a major money market centre which operates large money markets in US dollars, euros, yen, Swiss francs and Canadian dollars as well as sterling.

\subsection*{1.2 Purpose of the money markets}

The main purpose of the money markets is to provide:
- short-term liquidity to entities needing money, and
- short-term investment opportunities for entities with surplus liquidity.

Banks are the most active participants in the money markets, but some large companies also have direct access to the markets. Smaller companies and individuals might be offered an opportunity by their bank to borrow or lend at money market interest rates, and their access to the money markets is made through the bank. This means that companies are able to borrow or deposit money shortterm in the money markets through their bank. When they participate through their bank, they can borrow or lend much smaller amounts of money than the normal size of money market transaction.

Money markets also provide ready access to short-term borrowing and lending opportunities in foreign currency. This is important for companies involved in international trade that need short-term finance for transactions involving a foreign currency.

\subsection*{1.3 The inter-bank market}

The inter-bank market is a market for borrowing and lending large amounts of money for a short-term, often 'overnight' (for one day) but possibly up to one year. As the name of the market indicates, it is mainly a market for borrowing or lending between banks, but as explained earlier, companies are able to participate directly or through their bank. (When they participate through their bank, they can borrow or lend much smaller quantities of money.)

Interest rates in the inter-bank market vary according to:
- the duration of the loan, and
- the credit rating or credit status of the borrowing bank.

Banks with a very high credit rating are able to borrow at lower rates than banks with a lower credit rating.

The interest rates charged to top-quality banks are monitored. In London for example, a number of banks report their borrowing rates to the British Bankers Association (BBA) each day, and the BBA uses these rates to determine the average borrowing rates for top-rated banks.

\section*{LIBOR}

The BBA publishes these rates as the BBA London Inter-Bank Offered Rate or LIBOR. LIBOR (also written as Libor) is an important rate in the London markets, because it provides a guideline on interest rates to banks and it is also used as a benchmark interest rate in some derivatives markets. (Derivatives are explained later.)

The financial press publishes LIBOR rates, which might be presented as follows:
\begin{tabular}{lccccc}
\begin{tabular}{lll} 
Market interest rates \\
Overnight
\end{tabular} & Month & \begin{tabular}{c} 
Three \\
months
\end{tabular} & \begin{tabular}{c} 
Six \\
months
\end{tabular} & One year \\
& & & 5.32000 & 5.36000 & 5.40906
\end{tabular}

LIBOR can be regarded as a risk-free money market rate. The money market yield curve is normally upward-sloping, but this is not always the case, and a yield curve may be downward sloping along all or a part of its length.

Only a few LIBOR rates are published in the financial press, and it is possible to borrow or lend for any money market period.

Money market rates are shown as annual rates of interest. The actual amount of interest payable on a money market loan, or receivable on a money market deposit, depends on the term of the loan or deposit as well as the interest rate.

There are specific rules for the calculation of money market interest, but for the purpose of your examination, you can assume that the interest payable or receivable is the annual interest rate multiplied by the number of months of the loan or deposit and divided by 12 . Here are some examples.
- For a three-month loan of \(\$ 1,000,000\) when three-month LIBOR is \(5.36 \%\), the interest rate is \((5.36 \% \times 3 / 12) 1.34 \%\) and the interest payable is \(\$ 13,400\).
- For a six-month deposit of \(€ 1,000,000\) when six month LIBOR is \(4.28488 \%\), the interest rate is \(2.14244 \%\) and interest receivable will be \(€ 21,424.40\).

Only top-rated banks are able to borrow at LIBOR. Other banks and companies borrow at a 'spread' above LIBOR or earn interest on deposits at a rate below LIBOR. For example a company might be able to borrow at LIBOR plus \(0.5 \%\) (plus 50 basis points), and if one-month LIBOR is \(6.125 \%\), it is able to borrow for one month at \(6.625 \%\).

\section*{Note on base rates}

Not all companies borrow from their bank at a rate of interest based on LIBOR. Many companies, especially smaller companies, might pay a rate of interest on an overdraft or variable rate loan at an interest rate linked to the bank's base rate. For example a company might borrow at base rate plus \(2 \%\). A base rate is an 'administered rate' which means that it is set by the lending bank and is not directly related to market rates of interest.

\section*{Changes in money market interest rates}

Money market interest rates can change rapidly, in response to market conditions and prospects for the economy. However, in the United States, the European Union and the UK, the biggest influence on LIBOR rates is the central bank. The central bank is able to raise or lower the money market interest rate for its dealings with commercial banks, and it uses interest rates to try to influence the state of the economy (and in particular the rate of inflation). If the central bank in the UK raises its interest rate by \(0.25 \%\) for example, banks will immediately increase their LIBOR rates by the same amount.

\subsection*{1.4 The repo market}

A repo is a sale and repurchase transaction. In a repo transaction, one party sells a quantity of short-dated bonds or money market securities to the other party and at the same time undertakes to buy them back at a specified future date at a higher price.

For example Bank A might arrange a repo transaction with Bank B, in which Bank A sells a quantity of short-dated government bonds and money market instruments to Bank B for \(\$ 20\) million and agrees to buy them back after 14 days for \(\$ 20.046\) million.

In effect a repo transaction is a short-term secured loan. In this example, Bank B is effectively lending \(\$ 20\) million to Bank A for 14 days and takes government bonds and money market instruments as security for the loan. When the repurchase takes place after 14 days, Bank B receives \(\$ 46,000\) more than it lent, which is the interest on the loan.

\section*{Importance of the repo market}

The repo market is important in the USA, European Union and UK for two reasons.
- The central bank uses the repo market to provide liquidity to commercial banks, and sets the interest rate (repo rate) for these transactions. The 14 -day 'gilt repo' rate is in effect the marginal cost of borrowing for banks that cannot obtain liquidity anywhere else. This rate is used by the central bank to influence other money market interest rates, as explained above.
- Commercial banks use the repo market for secured borrowing and lending between themselves. It is an alternative to the inter-bank market, where borrowing and lending is unsecured.

\subsection*{1.5 Comparison of the repo market and inter-bank market}

Interest rates in the inter-bank market are slightly higher than interest rates in the repo market. This is because repo transactions are a form of secured lending whereas inter-bank loans are unsecured.

A bank with a large quantity of short-dated government bonds or money market securities will therefore prefer to borrow in the repo market than in the inter-bank market, because borrowing costs are less. However, banks might not hold large quantities of short-dated government bonds or money market securities. If these banks need to borrow, they will use the inter-bank market.

For the purpose of your examination, the inter-bank market and inter-bank market interest rate (LIBOR) are more significant than the repo rate, because the inter-bank rate is more relevant for obtaining short-term liquidity and for hedging both interest rate risk and foreign exchange risk. (Hedging risks is described later.)

\section*{Money market instruments}
- Coupon-bearing and discount instruments
- Treasury bills
- Bills of exchange
- Certificates of deposit (CDs)
- Commercial paper (CP)

\section*{2 Money market instruments}

The largest money market is the inter-bank market for loans and deposits. There are also markets for dealing in other money market 'instruments' or securities. These include:
- Treasury bills
- Bills of exchange (including bankers' acceptances or BAs)
- Certificates of deposit (CDs)
- Commercial paper (CP).

\subsection*{2.1 Coupon-bearing and discount instruments}

There are two broad categories of money market instrument, coupon-bearing and discount instruments.
- Coupon-bearing instruments are instruments or securities on which interest is payable at a stated interest rate (or 'coupon') on a fixed amount of principal. When the instrument or security reaches its maturity date, its holder receives the initial principal plus interest in settlement. Examples of coupon-bearing instruments are certificates of deposit.
- Discount instruments are instruments or securities where the borrower undertakes to pay a fixed amount of principal at maturity, and the instrument is issued at a discount to its 'face value'. The difference between the discounted issue price and the eventual redemption price (face value) represents interest on the borrowing. Examples of discount instruments are Treasury bills, bills of exchange and commercial paper.

\subsection*{2.2 Treasury bills}

Treasury bills are short-dated securities issued by a government, when money is needed to meet a short-term financing requirement. Treasury bills are normally issued for 91 days (three months).

They are discount instruments. The government might issue 91-day Treasury bills with a face value of \(\$ 100\) million at a price of, say, 98.75 . It would raise \(\$ 98.75\) million from the issue and at the end of the 91 days it would pay \(\$ 100\) million to the holders of the bills. Interest for the three-month period would be \(\$ 1.25\) million on the \(\$ 98.75\) million raised, which is an interest rate of about \(5 \%\) (annual).

There is an active market in Treasury bills. Buyers of Treasury bills can re-sell them before they mature to other investors or banks. Treasury bills should be risk-free, because they are promises to pay by the government. A company with a short-term cash surplus might therefore decide to invest in Treasury bills, which can be re-sold at any time or held until the bills eventually mature.

\subsection*{2.3 Bills of exchange}

A bill of exchange is a form of promise to pay a stated amount of money at a date in the future (usually in several month's time). A bill has a drawer and a drawee.
- The bill is issued by the borrower, and is 'drawn on' the drawee. When the bill is drawn, it is a form of 'You Owe Me'. With the bill the drawer is stating that the drawee owes a specified amount of money.
- The bill is then 'accepted' by the drawee, who signs the bill to indicate acceptance. An accepted bill becomes an undertaking by the drawee to pay the specified amount of money at the specified date.

A bill of exchange that is drawn on and accepted by a bank is called a bank bill.
There is an active market in bank bills, especially bills that have been accepted by banks with high credit ratings. This means that the drawer of a bill is able to obtain short-term finance by selling the accepted bill in the money markets.

Bank bills might be used as a source of short-term finance by companies in two ways.
- As a method of financing foreign trade transactions. Trade finance is described in a later section.
- As a method of raising short-term finance by means of bankers' acceptances (BAs), as an alternative to bank borrowing or issuing commercial paper.

\section*{Bankers' acceptances (BAs)}

A company that intends to borrow amounts of money for a short-term over a period of time in the future might arrange a BA programme with a bank. Under the terms of the agreement, the bank undertakes to accept bills of exchange that are drawn on it by the company, up to a maximum amount. When the company needs short-term funding, it draws a bill on the bank. The bank accepts it and then sells it in the money market on behalf of the company at a discount.

The company therefore receives the money 'now'. The company must also pay the bank the face value of the bill when it reaches maturity, to enable the bank to settle the bill. The bill is therefore a form of short-term finance, and the interest cost is the difference between the discounted value of the bill when it is sold and the face value of the bill that must be paid in settlement

An advantage of BAs for a company is that if the programme is arrange with a topquality bank, the discount rate (interest rate) on the accepted bills might be fairly low - lower than on other forms of money market borrowing.

\section*{Example}

A company has arranged a bankers' acceptances programme with its bank. Under the terms of the arrangement, which lasts for one year, the company is able to draw bills on the bank up to a total value in issue at any time of \(\$ 25\) million.

The company might draw a bill on the bank for \(\$ 1,000,000\) with a settlement date in three months' time. The bank accepts the bill and sells it for the company in the bills market. The company might receive, say, \(\$ 985,222\), so that the discount on the bill is \$14,778.

The discount means that the rate of interest for the three months is about: \((\$ 14,778 / \$ 985,222) \times 12 / 3=6.0 \%\)

After three months when the bill reaches maturity, the bank pays the bill and the company pays \(\$ 1,000,000\) to the bank.

\subsection*{2.4 Certificates of deposit (CDs)}

A certificate of deposit is a certificate issued by a bank stating that the bank is holding a specified quantity of money as a term deposit, on which interest is being earned at a specified rate. The deposit cannot be removed from the bank until the end of the stated term, but it can be sold in a money market for CDs.

For example, an investor might place a deposit of \(\$ 20\) million with a bank for a fixed term of six months, and receive interest at \(5.5 \%\) on the deposit. It might be agreed that the bank should issue a certificate of deposit that the depositor holds. However, if the depositor needs access to money before the end of the six months, it can sell the CD on to another investor or a bank and receive immediate cash.

\subsection*{2.5 Commercial paper (CP)}

Large creditworthy companies have several ways of raising short-term finance, and might select the least-cost financing method. This might be borrowing a money market rates from a bank, arranging a BA programme or issuing commercial paper. The cheapest rate of financing might vary according to conditions in each of the money markets.

Commercial paper (CP) is an unsecured promissory note. A promissory note is a promise by the issuer of the note to pay a specific amount of money on a specified date. When a company issues CP it promises to pay the face value of the paper at a specified date in the future.

Non-financial companies issue CP through a bank, as part of a commercial paper programme. The bank issues the CP on behalf of the company and sells it to investors. All CP is negotiable, which means that it can be sold in the money market. In practice, however, investors buying CP normally hold it to maturity when they are paid the face value of the paper they have bought.

The company issuing the CP therefore receives immediate cash (at a discount to the face value of the paper) and makes a payment when the paper reaches maturity.

Only companies with a good credit rating are able to issue CP , and commercial paper is normally given a credit rating by one or more of the major credit rating agencies (Moody's, Standard \& Poor's and Fitch). The interest rate payable on CP varies with the term to maturity of the paper when it is issued, the credit rating for the paper and conditions in the market at the time of the issue.

\section*{Example}

A large company has arranged a commercial paper programme with a bank. The programme will last for two years. During that time the company may issue CP up to a total value of \(\$ 300\) million in issue at any time.

Initially, the company might issue \(\$ 50\) million of \(C P\) with a maturity date in three months time. The bank sells the paper to a number of investors, who buy the paper at a discount and will receive payment of the full face value after three months.

Investors buying the CP are able to re-sell it if they wish to do so at any time before maturity, through a bank that deals in the CP market.

The company can issue more CP at any time, up to the specified limit.

\section*{Example}

A company has arranged a commercial paper programme with a bank. It issues \(\$ 40\) million of paper with a maturity of three months. The interest rate is \(6.2 \%\)

\section*{Required}

Calculate the amount of money the company will receive from the CP issue.

\section*{Answer}

The annual interest rate is \(6.2 \%\), so for three months the interest rate is approximately \(6.2 \% \times 3 / 12=1.55 \%\).

Amount received from the issue \(\times 1.0155=\$ 40\) million.
Amount received from the issue \(=\$ 40\) million \(/ 1.0155=\$ 39,389,463\).
Interest payable will be \(\$ 40\) million - \(\$ 39,389,463=\$ 610,537\).

\section*{The treasury function}
- Role of the treasury function
- Cash flow forecasting and cash management
- Financing long-term and short-term investments
- Financial risk management

\section*{3 The treasury function}

\subsection*{3.1 Role of the treasury function}

In many large organisations, there is a treasury function or treasury department.
The role of the treasury department consists mainly of:
- managing the liquidity and cash flows of the organisation
- managing the foreign exchange positions and cash flows
- helping to obtain finance for the organisation
- managing the exposures to financial risk, by hedging currency exposures, interest rate exposures and other risk exposures.

\subsection*{3.2 Cash flow forecasting and cash management}

Cash flow forecasting is an important aspect of treasury management. A company must have enough cash (or access to borrowing, such as a bank overdraft facility) to meet its payment obligations. Cash forecasts are therefore made and revised regularly, to establish whether the organisation expects to have a cash surplus or to be short of cash.
- If a cash deficit is forecast, measures should be taken to ensure that cash will be available. It might be necessary to ask a bank for more finance. Some expenditure might be deferred. Alternatively, measures might be taken to speed up receipts from customers.
- If a cash surplus is forecast, the treasury department will consider how the surplus funds should be used. For example, if the surplus is expected to be temporary, how long will the surplus last and what is the most profitable method of investing the money (without risk) for that period?

Short-term cash forecasts can be prepared using receipts and payments cash budgets. Longer-term cash forecasts can be made by preparing an expected cash flow statement for the forecast period.

\section*{Cash management}

A centralised treasury department is able to manage the cash position of the group as a whole. It can manage total cash receipts, total cash payments and total net cash balances.

One technique for doing this is to pool bank accounts. All the bank accounts throughout the group for a particular currency might be pooled. At the end of each day, the balances in each account are transferred to a centralised cash account. (The cash is 'pooled'). The cash deficits in some accounts and cash surpluses in other accounts are therefore netted. In this way, the company can avoid interest charges on accounts that are in deficit, and transfer cash between accounts as required.

Pooling and netting of cash flows therefore improves cash management. However, for pooling and netting to be effective, the cash has to be managed by a central treasury department.

\subsection*{3.3 Financing long-term and short-term investments}

A company must be able to finance its planned long-term and short-term investments (including any proposed takeovers). Preference share capital is rare, and the options available to companies are therefore as follows:
- For long-term investments, there should be long-term finance. Long-term finance is either equity or debt. Bank loans might be classified as long-term debt, provided that it is a fixed loan or a revolving credit facility for several years. However, bank loans are typically for up to about seven years
- Short-term investments might be defined as working capital assets. Inventory and receivables should normally be financed by a combination of long-term finance and short-term liabilities (such as trade payables).

\section*{Sources of equity}

For many companies, the main source of new equity finance is retained profits. Profits are retained and reinvested to finance new investments and further growth.

Occasionally, a company might wish to raise extra capital by issuing new shares for cash. However, it is difficult for a small company to raise new finance in this way if its shares are not traded on a stock market and it has only a small number of individual shareholders.

A company whose shares are traded on the stock market might wish to issue new shares for cash. In some countries (such as the UK), it is a legal requirement for companies issuing new shares for cash to offer the shares to the existing shareholders in proportion to their existing shareholding. (The shareholders might agree to waive this right in certain circumstances and within certain limits).

A company will only be able to issue new shares for cash in this way if it has the confidence and support of its shareholders and other institutions in the stock market.

In a takeover bid, the shareholders in the target company might be offered new shares in the bidding company as the purchase consideration.

In some cases, a company whose shares are not traded on the stock market might be able to raise new capital by issuing shares, when the new equity finance is provided by specialist 'venture capital' organisations.

\section*{Sources of debt}

For most companies, the main source of debt capital is their bank or banks. Large companies, however, are able to raise debt capital by issuing bonds in the bond markets.

\section*{Sources of short-term finance}

Companies obtain short-term finance mainly from trade credit and bank lending. Large companies might have access to other sources of short-term borrowing, such as:
- commercial paper programmes (CP), or
- bankers' acceptance facilities (BA acceptances).

\section*{Trade finance}

When the company needs finance to support its foreign trade, the treasury function might be responsible for arranging the financing.

\subsection*{3.4 Financial risk management}

The treasury department is usually responsible for the management of financial risk. The main financial risks facing companies are:
- foreign exchange risk or currency risk
- interest rate risk
- credit risk
- market risk, where companies hold large quantities of market securities (such as shares and bonds) as assets. Market risk is the risk of an adverse movement in the market price of these assets.

Foreign exchange risk and interest rate risk are described in later chapters (although translation risk and transaction risk have been described in the context of overseas capital investment, in an earlier chapter.)

\section*{Credit risk}

Credit risk is the risk that a debt will not be paid and will become a bad debt, or that a customer will make debt payments later than scheduled. Credit control and credit risk management are aspects of day-to-day financial management, and you should already be familiar with credit management and debt collection systems.

Companies that borrow from banks are a credit risk to the bank, and a function of the treasury department in large companies might be to monitor the perceived credit risk of the company. Large companies might have a formal credit rating from one of the credit rating agencies (such as Moody's and Standard \& Poor's). The cost of borrowing for these companies depends on their credit rating.
- Banks will charge a higher 'spread' over the risk-free rate on loans to companies with a lower credit rating.
- If a company wants to issue new bonds, the interest payable on the bonds will depend on the credit rating attached to them.
The treasury management of a company that has debt with a credit rating might therefore be required to monitor the rating and maintain a dialogue with the credit rating agencies.

The link between credit ratings spreads over the risk-free rate and the cost of borrowing has been explained in the earlier chapter on the cost of capital.

Risk management methods in banks
- Value at Risk (VaR)
- Scenario analysis
- Stress testing

\section*{4 Risk management methods in banks}

For your examination, you are expected to have an awareness of the latest derivative products and risk management techniques. Financial risk management is at its most sophisticated in the banking industry, with extensive use of risk management techniques such as Value at Risk (VaR) modelling, scenario analysis and stress testing.

\subsection*{4.1 Value at Risk (VaR)}

Value at Risk models (VaR models) have been developed as a method of monitoring and managing exposures to risk. They are used mainly in banking and financial services, and cover risks in their positions on financial derivatives trading as well as risks on ordinary bank lending. Banks use VaR models for both their credit risk and market risk.

Market risk is the risk of losses to the bank from adverse movements in market prices, such as changes in share prices and changes in interest rates and currency exchange rates. Changes in market prices can affect banks through their direct holdings of shares, bonds and currencies, but can also result in losses on derivatives positions.

Value at Risk is a technique for measuring risk as a single number. VaR models produce a probability distribution for expected losses (or losses and gains), and this probability distribution can be used to identify the maximum loss that will be incurred within a given period of time at a given level of probability or confidence level.

For example, the credit risk VaR model used by a bank might predict that the maximum loss from bad debts in any one week will be, say, \(\$ 10\) million at the \(95 \%\) level of confidence. Similarly the market risk VaR model of a bank might predict that at the \(95 \%\) level of probability the maximum loss on the bank's market positions in any one day is \(\$ 500\) million.

Confidence levels of \(95 \%\) or \(99 \%\) are commonly used, so that Value at Risk can be defined as a statistical estimate of the level of loss on a portfolio that is expected to be equalled or exceeded will a specified, small probability.

\section*{Using VaR models}

VaR models can be used only by organisations whose employees have the expertise to construct and use reliable models. They were used initially by banks as a means of obtaining readily-understandable information about their exposures to credit risk and market risk.

They may also be used by banks under the Basel 2 rules for capital adequacy as a method of establishing their minimum capital requirements for credit risk and market risk.

VaR is also used to control risk. For example a bank might set VaR limits. A VaR limit is the maximum amount of Value at Risk that the bank will accept for a given period of time, at a given confidence level. For example, a bank might have a policy that its monthly VaR for credit risk must not exceed \(\$ 30\) million at the \(95 \%\) level of probability.

When the VaR model indicates that this level might be exceeded, or has been exceeded, management can take action to reduce the risk, for example by purchasing credit protection with credit default swaps or reducing the size of its loan portfolio.

Similarly, trading limits can be set with reference to market VaR for individual market traders in banks or for dealing desks. If a dealer or dealing desk exceeds its VaR limit, it will be required to change its 'positions' in the markets so that VaR falls back within the acceptable limit.

\subsection*{4.2 Scenario analysis}

Value at Risk measures risk under a given set of assumptions about what should normally be expected to occur. They are based on an 'official' version of what is likely to happen in the future, and so what might possibly go wrong.

An additional method of risk analysis is to challenge the 'official' view of the future and measurement of risk by creating alternative circumstances and market conditions that might happen, and looking at these realistic alternatives in detail.

The technique is therefore to create alternative 'scenarios' and assess the risk under each different scenario. Scenario planning or testing therefore makes management think in detail about the potential consequences of unexpected circumstances or events. Contingency plans can then be developed as necessary, to deal with problems that might arise if one of these scenarios does occur.

\subsection*{4.3 Stress testing}

Stress testing is a form of risk analysis that tests what will happen in the event that extreme conditions occur. For example, a bank might use stress testing to test the robustness of its market positions or capital adequacy and liquidity in the event of extreme conditions, such as a \(10 \%\) or \(20 \%\) fall in one day in the value of the US dollar. Stress testing can help management to understand the strength of their risk management systems, and their ability to function effectively in extreme conditions.

\section*{Finance of international trade}
- Problems for companies
- Letters of credit (documentary credits)
- Export factoring
- Forfaiting
- Countertrade

\section*{5 Finance of international trade}

\subsection*{5.1 Problems for companies}

Companies involved in international trade face larger financial problems and risk than companies that trade in only their domestic markets.
- Either the exporter or the importer, or both, are exposed to currency risk. Currency risk is described in another chapter.
- An exporter might be required to send goods abroad without knowing whether the customer will actually pay. If the customer refuses to pay, taking measures to obtain payment in another country can be time-consuming and difficult. Credit risk is therefore a major problem.
- Similarly, there can be credit risk for an importer, who is asked to pay for goods before the shipment has actually arrived (especially when goods are sent by sea). The risk is that the exporter will take the payment but fail to deliver the goods.
- Exporters often have to allow long periods of credit to their customers. This is because importers might ask for time to pay after the goods have been delivered, but delivery might take several weeks. Financing the receivables for export sales can therefore be difficult, especially for small exporters.

\subsection*{5.2 Letters of credit (documentary credits)}

Letters of credit, also called documentary credits, are a procedure for arranging the shipment of exports and the payment by the importer, that:
- provides protection against credit risk for both the exporter and the importer, and
- can be used by the exporter to obtain short-term finance.

When an exporter and importer agree to a transaction, the method of payment might be by letter of credit. Essentially, this system requires the exporter to provide specific documentation to the importer, giving evidence that the goods have been shipped as specified (and where appropriate that the goods have been properly insured and that the exporter has paid the freight charges).

On receipt of the documentary evidence, the importer arranges for payment by bill of exchange. The bill of exchange is one of the documents provided by the exporter,
stating the amount payable for the goods at a stated future date (the end of the credit period for the importer). The bill of exchange is then signed ('accepted') by a bank representing the importer, and on acceptance, it becomes an undertaking by the bank to make the payment.

The accepted bill is returned to the exporter's country. It may also be 'confirmed' by a bank in the importer's country. Confirmation is an additional guarantee of payment. The confirming bank undertakes to pay the bill in the event that the accepting bank does not.

A bill of exchange that has been accepted, or accepted and confirmed, can either:
- be held until its expiry date, when the accepting bank will make the payment on behalf of the importer, or
- sold before the expiry date, in the discount market for bank bills. The exporter can arrange for its bank to sell the bill in the discount market, in order to obtain the (discounted) payment sooner.

\subsection*{5.3 Export factoring}

Export factoring is the factoring of trade receivables, where the receivables are trade debts for export sales. An exporter can enter into an agreement with an export factoring organisation.
- The export factor issues invoices for the exporter, administers the exporter's export sales ledger and collects the payments from the foreign customers.
- The export factor provides credit insurance for 'approved' export sales. In 'nonrecourse' factoring, the factor accepts the risks of non-payment by the foreign customers. The exporter is therefore not exposed to the risks of non-payment and bad debts.
- The export factor provides an advance of money, typically about \(80 \%\) of the value of the unpaid invoices that the factor is responsible for collecting. The balance of the money, less the factor's fees and charges, is paid to the exporter when the foreign customers eventually pay.

Export factoring therefore reduces the administrative tasks of the exporter (invoicing, debt collection, book-keeping), provides credit insurance and also provides short-term finance.

The export factor charges a fee for its administration services and for the credit insurance, and charges interest on the money it advances to the exporter.

\subsection*{5.4 Forfaiting}

Forfaiting is a method of financing exports, used in some countries in continental Europe.

Forfaiting might be appropriate where payment for the exported goods will take place over a period of several years. Typically, the exports are exports of capital
goods such as machinery, and payment will be made by the importer over the economic life of the goods.
- The importer pays a proportion of the purchase price on delivery and agrees to pay the rest in instalments at regular intervals over a period of several years.
- The importer issues a series of bills of exchange or promissory notes, to coincide with the schedule of payments.
■ The importer finds a bank that is willing to 'avalise' the bills or promissory notes. Avalising means providing a guarantee of payment.
■ The exporter arranges for a bank in its own country to act as forfaiter. The forfaiting bank buys the promissory notes at a discount. The exporter therefore receives payment (a discounted payment). The forfaiting bank will probably hold the promissory notes or bills until their due date for payment, and then receive the payment from the importer.

\subsection*{5.5 Countertrade}

Countertrade is a method of arranging payment for exports when the buyer is in a country where there are foreign exchange restrictions.

Countertrade involves receiving payment in other goods for which there is an international market.

For example, an exporter might arrange to sell machinery to a company in a country where there are foreign exchange restrictions, and payments in foreign currency for imports are not permitted. The importer and exporter might therefore agree to payment in another type of goods. These goods could be any type of goods, but they must have a ready market. For example, in exchange for machinery, an importer might arrange to pay in coffee beans. These could then be sold in the commodity markets by the exporter, in order to obtain payment in cash.

Similarly an exporter of optical instruments to South America may agree to accept payment in cashew nuts, which it will then arrange to sell on the international markets.

\section*{Foreign exchange risk and currency risk management}

\author{
Contents \\ 1 Foreign exchange risk \\ 2 The FX markets: spot rates \\ 3 Forward FX contracts and forward rates \\ 4 Hedging exposure to foreign exchange risk \\ 5 Synthetic agreements for forward exchange (SAFEs) \\ 6 Money market hedge
}

\section*{Foreign exchange risk}
- The need for foreign exchange
- Currency transaction exposures
- Exchange rates and volatility
- Government measures to stabilise exchange rates

\section*{1 Foreign exchange risk}

\subsection*{1.1 The need for foreign exchange}

Many companies make foreign exchange transactions with their banks. The need for foreign exchange arises from international trade and international investment.
- A company buying goods from another country might be required to pay in a foreign currency, such as the domestic currency of the supplier. It must therefore obtain the foreign currency to make the payment.
- A company selling goods abroad might price the goods in the buyer's domestic currency, or in another currency such as US dollars. When the customer pays in the foreign currency, the company might sell the currency received in exchange for its own domestic currency
- A company investing abroad might need to obtain foreign currency to acquire or to make the investment.
If a company in Australia wants to buy goods from a supplier in Germany, and the purchase price is in euros, the Australian company has to buy euros from its bank in order to make the payment to the supplier.

Similarly, if a company in The UK sells goods to a customer in Africa, and the price is paid in US dollars, the company will probably sell the dollars it has received to its bank, in exchange for sterling. (Alternatively, the company could keep the dollars in a US dollar bank account, if it has one.) The company in Africa would also need to buy dollars from a bank in order to pay the UK supplier.

Many foreign currencies can be bought and sold freely in the foreign exchange markets (FX markets), which are operated world-wide by banks. On the other hand, some currencies do not have a liquid market, and foreign companies might be reluctant to accept payment in those currencies.

\subsection*{1.2 Currency transaction exposures}

Transactions in foreign currency give rise to currency transaction exposures. Transaction exposures were explained in the earlier chapter in international investment.

A transaction exposure occurs when a payment of a foreign currency is required at a future date, or when a receipt of a payment in a foreign currency will occur at a
future date. An exposure to risk occurs because the exchange rate could move adversely between 'now' and the time that the currency payment or receipt happens, with the result that either:
- it costs more to buy the foreign currency to make the currency payment, or
- there is less income when a currency payment is received and the currency is converted into domestic currency by selling it to a bank.

Currency transaction exposures can be analysed according to when the currency payment or receipt is expected to happen, and exposures can be measured in each foreign currency. An exposure is simply:
- the amount of payments that will be made in the currency, and
- the amount of the currency that will be received.

If payments and receipts of the same currency are expected at the same time, the net exposure is the difference between the expected receipt and the expected payment.

\subsection*{1.3 Exchange rates and volatility}

When currencies can be bought and sold in the FX markets, banks quote exchange rates at which they will deal. Exchange rates are quoted as a number of units of one currency (the variable currency) in exchange for one unit of the other currency (the base currency). For example, the sterling/US dollar exchange rate (GBP/USD) might be 1.9150 , meaning that each \(£ 1\) can be exchanged for US \(\$ 1.9150\) in the market.

Experience has shown that exchange rates can be very volatile. Volatility means that exchange rates can move up or down by large amounts, within a fairly short period of time. A notable example in recent years has been the euro/US dollar exchange rate. Since the euro was created in 1999 , when its value was about \(€ 1=\$ 1.20\), the exchange rate has ranged between about \(€ 1=\$ 0.80\) to about \(€ 1=\$ 1.35\). Exchange rate volatility creates foreign exchange risk for anyone involved in buying, selling, borrowing or investing foreign currency.

\section*{Example}

A German company sells goods to a US buyer for US\$330,000 when the exchange rate is \(€ 1=\$ 1.1000\). The US buyer is allowed three months' credit, and when the German company eventually receives the US dollars three months later, and exchanges them for euros, the exchange rate has moved to \(€ 1=\$ 1.2000\).
The original expectation would have been that the sale proceeds in euros would be \(€ 300,000\) ( \(\$ 330,000 / 1.1000\) ). However, during the time that it was exposed to the currency risk, the exchange rate has moved in an adverse direction, and the actual receipts are only \(€ 275,000\) ( \(\$ 330,000 / 1.2000\) ). The 'FX loss' has been \(€ 25,000\).

\section*{Conclusion}

This example illustrates several points about currency risk.
- Currency risk arises from exposure to the consequences of a rise or fall in an exchange rate. Here, the German company was exposed to the risk of a fall in the value of the US dollar.
- An exposure lasts for a period of time. Here, the exposure lasts from when the goods were sold on credit until the time that the customer eventually pays.
- Currency risk is a two-way risk, and exposure to risk can lead to either losses or gain from movements in an exchange rate. In this example, the exchange rate could have moved the other way. For example, if the exchange rate after three months had been \(\$ 1=€ 1\), the German company would have received \(€ 330,000\), which is \(€ 30,000\) more than it would have expected at the time of the sale. There would have been an 'FX gain' of \(€ 30,000\).

\subsection*{1.4 Government measures to stabilise exchange rates}

A government may try to stabilise the exchange rate for its currency. The purpose of having an exchange rate policy would be to create stable economic conditions for international trade. A stable exchange rate, with relatively little exchange rate volatility, should help to promote growth in the country's economy.

In the past, some governments were able to manage the exchange rate by dealing on the foreign exchange markets, using their official reserves of foreign exchange to either buy or sell domestic currency. By creating demand or supply for its currency in the markets, the government would try to move the exchange rate up or down against major currencies such as the dollar. However, the foreign exchange markets are now so large that very few countries are in a position to manage the exchange rate effectively in this way. (Countries such as China may be an exception.)

The most effective way for a government to manage its exchange rate today, if it wished to do so, would be to increase or reduce domestic interest rates on its currency. Raising or reducing interest rates should affect the demand for the currency from investors. For example, raising the interest rate should attract more investment into the currency, and by increasing demand for the currency, the foreign exchange value of the currency should increase.

There are several exchange rate policies that a government might adopt. These include:
- free floating ('benign neglect' of the exchange rate)
- managed floating of the currency
- a fixed exchange rate policy, with the exchange rate fixed against a major currency or a basket of world currencies
- a fixed exchange rate backed by a currency board system.

\section*{Free floating}

With a policy of free floating, the government does not have a policy about the exchange rate. Instead, it allows the currency to find its own market value in the foreign exchange markets.

\section*{Managed floating}

A policy of managed floating is to allow the currency to find its own level in the foreign exchange markets, but within target limits. (Targets may be set for the maximum and minimum exchange rate against, say, the US dollar or the euro.)

If the exchange rate threatens to go through the upper or lower target limit, the government will act to try to keep it within the policy limits, probably by raising or lowering interest rates.

\section*{Fixed exchange rate policy}

A government might try to fix its exchange rate against:
- another currency, such as the US dollar, or
- a basket of other world currencies, for example the US dollar, euro and yen.

The 'fixed rate' policy will normally permit some limited variations in the exchange rate.

For example, countries that wish to enter the eurozone in the European Community are expected to link their currency to the value of the euro for a period of time before they can be considered for 'eurozone membership'.

There are problems with fixing an exchange rate against another currency.
- Economic conditions in the two countries must remain similar; otherwise there will be too much pressure on the exchange rate to change. For example, the rate of inflation in both countries must be similar over a long period of time.
- The country's economy will be affected by any crisis in the economy of the other country, or by an increase in the volatility of the other country's currency.

\section*{Fixed exchange rate backed by a currency board}

A currency board system is another fixed exchange rate system. The government fixes its currency against the value of another currency (a 'hard' currency, such as the US dollar). Any new issues of domestic currency have to be backed by an amount of the 'hard currency' in the country's official reserves.

For example, a country with a currency board system might fix the exchange rate at 4 local currency units (LCUs) to the US dollar. If the country wants to increase its money supply (which will be necessary for economic growth), it will need to hold reserves of one US dollar for every increase of 4 LCUs in the money supply.

This 'backing' of a hard currency should help to stabilise the exchange rate for the country's own currency, which in turn should help the country to achieve economic stability.

A problem with a currency board system is that on occasions:
■ it might result in a shortage of domestic money supply, because of an insufficiency of the hard currency, or
- it might push up domestic interest rates (in order to attract more hard currency).

If the problem becomes too serious, the currency board system may break down. A currency board system has worked well for Hong Kong (whose dollar has been linked to the US dollar), but has not been so successful in other cases (such as Argentina).

\section*{The FX markets: spot rates}
- Bid and offer rates
- Spot rates
- Quoting exchange rates

\section*{2 The FX markets: spot rates}

You need to understand how currencies are bought and sold in the FX markets. This section of the chapter explains the basic 'rules' for buying and selling currency.

\subsection*{2.1 Bid and offer prices}

In an examination, you might be required to calculate the cash flows in a foreign currency exchange transaction. If so, you might be given one figure for the current exchange rate between two currencies. In practice, banks quote two rates: a bid rate and an offer rate.
- The bid rate is the rate at which the bank will buy the base currency.
- The offer rate is the rate at which the bank will sell the base currency.

Most exchange rates are quoted to four decimal places.
It is easy to get confused about which exchange rate should be applied to a particular transaction. The basic rule to remember is that the bank will use the rate that is more favourable to itself and less favourable to the customer.

\section*{Example}

A UK company needs \(\$ 10,000\) to pay a US supplier. The bank's current rates for sterling/US dollar (US\$/£1) are 1.9850-1.9855.

The British pound is the base currency, since the rate is quoted as \(£ 1=£ X\).
The company needs to buy US dollars in exchange for British pounds, in order to pay the US supplier. The bank is selling dollars and receiving British pounds in exchange. It will apply a rate of 1.9850 to the currency transaction with the company, because 1.9850 will give it more British pounds than the rate of 1.9855 . The cost of buying the dollars is therefore \(£ 5,037.78\) ( \(\$ 10,000 / 1.9850\) ).

\section*{Example}

The same UK company receives US \(\$ 15,000\) from a customer, and it wants to convert these dollar receipts into British pounds. The exchange rate is \(1.9850-1.9855\).

The bank will buy dollars at 1.9855 , and so the company will receive \(£ 7,554.77\) ( \(15,000 / 1.9855\) ) in exchange for the dollars.

Taking this transaction and the previous example together, the UK company's cash flows in British pounds from the two transactions would be as follows:
\begin{tabular}{lr} 
& \(£\) \\
\hline Buy \(\$ 10,000\) at 1.9850: pay & \((5,037.78)\) \\
Sell \(\$ 15,000\) at 1.9855: receive & \(7,554.77\) \\
\cline { 2 - 2 } Net receipts & \(2,516.99\) \\
\hline
\end{tabular}

It would be more sensible for the company to pay the \(\$ 10,000\) to the US supplier out of the \(\$ 15,000\) it receives from its customer. That would leave it needing to sell just US \(\$ 5,000\) at 1.9855 . The receipts in British pounds would then be \(£ 2,518.26\) (\$5,000/1.9855).

\section*{Example}
(a) A French company has received Aus \(\$ 7,000\) from a customer in Australia. The current exchange rate for the euro and the Australian dollar (Aus\$/€1) is \(1.5230-1.5240\).

How much will the company receive in euros for selling the Australian dollars?
(b) An Australian company needs to buy \(€ 12,000\) to pay a Spanish supplier. The exchange rate is the same as above. How much will it cost the company to obtain the euros that it needs?

Answer
(a) The bank is buying Australian dollars and selling euros. It will offer the rate more favourable to itself, which is 1.5240 . The company will receive \(\$ 7,000\) / \(1.5240=€ 4,593.18\).
(b) The Australian company needs to buy euros, and the bank will charge the rate most favourable to itself. To buy \(€ 12,000\), the Australian company will have to pay \(12,000 \times 1.5240=\) Aus \(\$ 18,288\).

\subsection*{2.2 Spot rates}

In the FX markets, banks trade currencies both spot and forward. A spot transaction is a transaction for the sale of one currency in exchange for another, for 'immediate' settlement. 'Immediate' settlement in practice usually means after two working days, so that if a company makes a spot transaction with a bank on a Monday, the actual exchange of currencies will happen on Wednesday, two working days later. For the purpose of the examination, however, you can treat spot transactions as transactions for immediate settlement.

Although currency can be bought and sold 'spot', spot transactions cannot be used to 'hedge' a currency exposure and foreign exchange risk.

\subsection*{2.3 Quoting exchange rates}

Normal practice in the foreign exchange markets is to quote exchange rates with the base currency first and the variable currency second. This means, for example, that if the sterling/US dollar rate is 1.9800 , we mean that the exchange rate is \(£ 1=\) US\$1.9800. Similarly, if the euro/US dollar rate is 1.3200 , we mean \(€ 1=\) US\$1.3200.

An examination question may use a different way of quoting exchange rates. For example, a question might state that the dollar/sterling rate is 1.9800 , when it means \(£ 1=\) US \(\$ 1.9800\). Or it might state that the dollar/euro rate is 1.3200 when it means \(€ 1=\) US\$1.3200.

The habit of examiners in the past to quote exchange rates in this way is unusual, because it goes against the conventions of the FX markets. It may therefore be important to read an examination question carefully, and make sure that you understand the meaning of a quoted exchange rate, and identify the base currency and the variable currency in the rate.

\section*{Forward FX contracts and forward rates}
- Forward contracts
- Forward rates
- Forward contracts and hedging exposure to FX risk
- Premiums and discounts
- Cross rates

\section*{3 Forward FX contracts and forward rates}

\subsection*{3.1 Forward contracts}

Banks trade foreign currencies both spot and forward. Spot transactions are for immediate settlement. A forward FX contract is a contract entered into 'now' for settlement at an agreed future date (or at any time between two agreed future dates). A bank will arrange a forward contract for settlement at any future date, but commonly-quoted forward rates are for settlement in one month, three months, six months and possibly one year.

For example, an Italian company can arrange a forward contract 'now' to sell a quantity of US dollars in exchange for euros in three months' time, at a rate of exchange that is agreed 'now'.

Forward exchange transactions are available in all the major traded currencies of the world, although settlement dates of more than one year forward are very unusual in any currencies except the most heavily-traded currencies such as the dollar-euro.

\subsection*{3.2 Forward rates}

Banks are able to quote forward exchange rates for currencies because of the money markets (short-term borrowing and lending markets). Forward exchange rates are different from spot rates, and they differ from spot rates because of the interest rate differences between the two currencies.

A forward rate can be higher or lower than the spot rate, depending on whether the interest rate on the variable currency is higher or lower than the interest rate on the base currency.

\section*{Example}

The current spot rate for the British pound and the US dollar (US\$/£1) is 1.8000. The interest rate on the British pound is \(6 \%\) per year and the interest rate on the US dollar is \(4 \%\) per year. The one year forward rate is the forward rate that a bank will quote now for trading dollars in exchange for pounds in one year's time. This forward rate is:

One year forward rate \(=\) Spot rate \(\times \frac{(1+\text { interest rate on the variable currency })}{(1+\text { interest rate on the base currency })}\)
In this example, the one year forward rate will be \(1.8000 \times\left[\frac{1.04}{1.06}\right]=1.7660\)

\section*{Forward rates do not predict future spot rates}

A one-year forward rate of \(£ 1=\$ 1.9660\) does not mean that the spot rate of exchange will be \(\$ 1.9660\) in one year's time. Forward rates do not attempt to predict what future spot rates will be. A forward rate represents the comparative investment value of the two currencies between 'now' and the settlement date for the forward contract.

However, the interest rate parity formula for predicting future exchange rates is also used to predict forward exchange rates. With this theory it is assumed that the forward rate does predict what the future spot rate will be. The interest rate parity formula was explained in the earlier chapter on international capital investment.

\subsection*{3.3 Forward contracts and hedging exposure to FX risk}

For companies, forward FX contracts can be used to hedge an exposure to currency risk (transaction risk). Currency risk will arise, for example, when a company expects to receive a quantity of a foreign currency in several months' time, which it will sell in exchange for its own domestic currency. If it plans to sell the foreign currency in a spot transaction, until it receives the currency, it is exposed to the risk that the exchange rate will move adversely and the currency will fall in value and be worth less than its current value.

For example, suppose that an Italian company expects to receive 5 million Japanese yen in three months' time, and the current exchange rate for euros against the yen (yen/€ 1 ) is 135.00 . At this rate, the Italian company would be able to exchange the yen for \(€ 37,037\) ( 5 million/135.0).
- However, there is a risk that the yen will fall in value during the three months, during which the company has an FX risk exposure arising from its future yen income. If the yen fell in value and after three months the spot rate is 150.00 , the yen income would be worth only \(€ 33,333\).
- On the other hand, if the yen strengthened in value, say to 120.00 spot after three months, the income would be worth \(€ 41,667\).

Although foreign exchange rates can move favourably as well as adversely, companies engaged in international trade usually prefer to avoid exposure to currency risk. They can 'hedge' currency exposures by arranging forward contracts to buy or sell currency. By fixing the exchange rate 'now' for a future currency purchase or sale transaction, the uncertainty or risk in the exchange rate is eliminated.

\section*{Example}

A British company needs to pay a German supplier \(€ 50,000\) in three months' time. A bank quotes the following forward rates for three months ( \(€ / £ 1\) ): \(1.4820-1.4830\).

If the company arranges a forward contract to buy the euros in exchange for British pounds, the forward rate will be 1.4820 . This is the rate that is more favourable to the bank, because it will receive more pounds in exchange for the euros. The cost to the company of obtaining \(€ 50,000\) in three months' time is \(£ 33,738.19\) (50,000/1.4820).

For the company, an advantage of arranging this forward contract to buy the euros is that:
- the company knows exactly how much it will have to pay in British pounds in three months' time, and
■ this forward contract eliminates the risk of an adverse movement in the exchange rate and allows the company to make financial plans (such as cash budgets) with greater certainty.
However, having made a forward exchange agreement, the company has a contractual obligation to carry out the terms of the agreement and buy the euros at 1.4820 at the settlement date. If the spot rate at this date is more favourable, say 1.6000 , the company cannot choose to ignore the forward contract and buy euros at the spot rate.

Hedging an exposure to currency risk with a forward contract gives protection against adverse exchange rate movements, but prevents the company from taking advantage of any favourable movement in the spot rate.

\subsection*{3.4 Premiums and discounts}

Forward rates are derived from current spot rates and interest rate differences between currencies. Spot rates change continually, whereas interest rate changes are less frequent. It is therefore common practice in the FX markets to quote forward rates as a premium or a discount to the spot rate.
- When the interest rate on the variable currency is lower than the interest rate on the fixed currency, the forward rate will be lower than the spot rate. The variable currency will be worth more forward than spot, and the forward rate is at a premium to the spot rate
- When the interest rate on the variable currency is higher than the interest rate on the fixed currency, the forward rate will be higher than the spot rate. The variable currency will be worth less forward than spot, and the forward rate is at a discount to the spot rate.

If you are given spot rates and the premium or discount for the forward rate, you should apply the following rule:
- Subtract a premium from the spot rate to derive the forward rate
- Add a discount to the spot rate to derive the forward rate.

A premium might be indicated by the letters ' pm ' and a discount by 'dis'.
These rules are summarised in the following table:
\begin{tabular}{lll}
\begin{tabular}{l} 
The interest rate on the \\
variable currency is:
\end{tabular} & Higher & Lower \\
\hline The forward rate is quoted & \begin{tabular}{l} 
Discount to the spot \\
rate (dis)
\end{tabular} & \begin{tabular}{l} 
Premium to the spot \\
rate \((\mathrm{pm})\)
\end{tabular} \\
\begin{tabular}{ll} 
Rule for obtaining the forward \\
rate:
\end{tabular} & \begin{tabular}{l} 
Add the discount to \\
the spot rate
\end{tabular} & \begin{tabular}{l} 
Subtract the premium \\
from the spot rate
\end{tabular}
\end{tabular}

\section*{Example}

A UK company expects to receive US \(\$ 75,000\) in six months from a US customer and it wishes to hedge the exposure to currency risk by arranging a forward contract. The following rates are available (US\$/ \(£ 1\) ):

GBP/USD
\begin{tabular}{lrrr} 
Spot & 1.7530 & - & 1.7540 \\
Six months forward & 240 & - & 231 pm
\end{tabular}

The dollar is quoted forward at a premium. The premium is shown in 'points' of price, so that \(240-231\) means \(0.0240-0.0231\).
The bank will apply the rate that is more favourable to itself. (If you need to work out which rate is more favourable, use the spot rates to do this). The company will be selling US dollars in exchange for pounds, and the higher rate will be used (the offer rate).
\begin{tabular}{lr} 
Spot rate & 1.7540 \\
Forward points (deduct premium) & \((0.0231)\) \\
& 1.7309 \\
\hline
\end{tabular}

The company can use a forward contract to fix its future income from the US dollars at \(£ 43,330.06\) ( \(75,000 / 1.7309\) ).

\section*{Example}

A US company needs to pay a Mexican supplier in three months. It will have to pay the supplier 10 million Mexican pesos. The company wants to arrange a forward contract to hedge its risk exposure. A bank quotes the following rates (pesos \(/ \$ 1\) ):
\begin{tabular}{lrrr} 
USD/MXP & & \\
\hline Spot & 11.2470 & - & 11.2485 \\
Three months forward & 340 & - & 360 dis
\end{tabular}

Using a forward contract, what will it cost the US company in dollars to pay the Mexican supplier in three months' time?

\section*{Answer}

The company will be given the more unfavourable rate (the more favourable rate to the bank). This is \(11.2470+\) discount \(0.0340=11.2810\).

The cost to the company of buying the pesos is \(10,000,000 / 11.2810=\) US\$886,446.24.

\subsection*{3.5 Cross rates}

The exchange rate for two currencies might be derived as a cross rate. This means that they are not traded directly on the FX markets, but are both traded through the US dollar. For example, if the exchange rate for the US dollar against the Hong Kong dollar (USD/HKD) is 8.1000 and the rate for the US dollar against the Canadian dollar (USD/CAD) is 1.2475, the cross rate for the Hong Kong dollar against the Canadian dollar (CAD/HKD) is \(8.1000 / 1.2475\), which is \(\mathrm{C} \$ 1=\) HK\$6.4930.

\section*{Example}

The three-month forward rate for the US dollar against the Swiss franc is 1.2166 (US\$1 = SwFr1.2166). The three-month forward rate for the US dollar against the British pound is \(£ 1=\) US \(\$ 1.8610\).

What is the three-month forward rate for the Swiss franc against the British pound (quoting the pound as the base currency)?

\section*{Answer}

US\$1 = SwFr 1.2166
\(£ 1=1.8610\)
Therefore \(£ 1=1.2166 \times 1.8610=\) SwFr 2.2641.

Hedging exposure to foreign exchange risk
- The purpose of hedging risk
- Methods of hedging exposures to foreign exchange risk
- Netting

\section*{4 Hedging exposure to foreign exchange risk}

\subsection*{4.1 The purpose of hedging risk}

The purpose of hedging an exposure to risk is to eliminate or reduce the possibility that actual events will turn out worse than expected. The purpose of hedging an exposure to currency risk is to remove (or reduce) the possibility that a future transaction involving a foreign currency will have to be made at a less favourable exchange rate than expected.

Exchange rates can move up or down, and spot rates could move favourably as well as adversely. However, many companies prefer to hedge their currency risks by fixing an exchange rate now for a future transaction, even if this means that it will not be able to benefit from any favourable movement in the exchange rate.

\subsection*{4.2 Methods of hedging exposures to foreign exchange risk}

The most important methods of hedging exposures to currency risk, which you will be expected to know for the examination, are:
- netting receipts and expenditure (or netting assets and liabilities)
- forward exchange contracts (described earlier)
- creating a money market hedge
- currency futures (described later)
- currency options (described later)
- possibly, currency swaps.

\subsection*{4.3 Netting}

Netting can be applied to cash flows in a foreign currency or to assets and liabilities denominated in a foreign currency.

\section*{Netting cash flows}

When a company expects to have future cash receipts in a foreign currency and future cash payments in the same currency at about the same time, it can use the receipts to make some or all of the payments. To the extent that future receipts match future payments, the foreign exchange risk is eliminated. Movements in the spot exchange rate will affect the netted receipts and payments equally. The loss
from the adverse movement affecting the cash receipts or payments will be offset by the gain from the favourable movement affecting the cash payments or receipts.

\section*{Example}

A UK company expects to receive US \(\$ 400,000\) in two months' time and to make payments of \(\$ 600,000\), also in two months. To hedge its currency exposures, the company can net \(\$ 400,000\) of receipts and payments, leaving a net exposure of just \(\$ 200,000\) in payments. This exposure might be hedged with a forward exchange contract.

Synthetic agreements for forward exchange (SAFEs)
- The nature of SAFEs
- The purpose of SAFEs

\section*{5 Synthetic agreements for forward exchange (SAFEs)}

\subsection*{5.1 The nature of SAFEs}

In a normal forward exchange contract, the two parties to the transaction agree to exchange a quantity of one currency for a quantity of a second currency, at a fixed rate of exchange. At settlement, an exchange of the two currencies takes place.

A synthetic agreement for forward exchange (SAFE) is a forward contract for the exchange of a notional amount of principal in the two currencies. They are similar to FRAs which are also contracts on a notional amount of principal. Like FRAs, SAFEs are settled by an exchange of cash based on movements in the exchange rate since the contract was made. (This type of contract, which is cash-settled on the basis of movements in prices or rate, is called a contract for difference.)

There are two types of SAFE:
- Non-deliverable forwards or NDFs. These are contracts that are cash-settled on the basis of movements in both the spot rate of exchange and also the size of the forward premium or discount. NDFs are also called FX forwards.
- Exchange rate agreements or ERAs. These contracts are cash-settled on the basis of movements in the size of the forward premium or discount only.

\subsection*{5.2 The purpose of SAFEs}

SAFEs were created originally by banks in response to rules on capital adequacy. These international rules, known as the Basel rules, require banks to maintain a minimum amount of capital against their exposures to potential losses from credit risk and market risk. 'Traditional' forward contracts require more capital than SAFEs.

SAFEs are now used as a substitute for traditional forward contracts in currency markets where the currencies are not freely-convertible or easily-convertible. In these currency markets, the settlement of traditional forward contracts would be difficult and often impractical.

SAFEs therefore enable a company to obtain a hedge against currency risk where the currency is not freely convertible. For example when capital controls and exchange restrictions on a currency are imposed by the government, the currency markets might trade non-deliverable forwards (NDFs) to allow companies to hedge exposures (or take exposures) in the currency. The cash settlement of the SAFE is often in a different currency that is freely tradeable, such as the US dollar.

A problem with SAFEs for currencies that are not freely convertible is that banks might have difficulty in hedging their own foreign currency exposures in these currencies, and they might compensate themselves for this risk by setting much wider spreads between the forward buying and selling rates for these contracts.

\section*{Money market hedge}
- Definition of a money market hedge
- Constructing a hedge for a future currency receipt
- Constructing a hedge for a future currency payment

\section*{6 Money market hedge}

\subsection*{6.1 Definition of a money market hedge}

A money market hedge is another method of creating a hedge against an exposure to currency risk. Instead of hedging with a forward exchange contract, a company can create a hedge by borrowing or lending short-term in the international money markets, to fix an effective exchange rate 'now' for a future currency transaction.

\subsection*{6.2 Constructing a hedge for a future currency receipt}

A company might expect to receive an amount of foreign currency at a future date, which it intends to exchange into its domestic currency. It wants to hedge its exposure to currency risk.

One way of hedging the risk is to make a forward exchange contract with a bank, in which it sells the future foreign currency receipts to the bank in exchange for domestic currency, at an exchange rate fixed 'now' by the forward contract.

Another way of hedging the risk is to establish a money market hedge. The money market hedge works by arranging a lending or borrowing transaction now, with a settlement date the same as the date when the future currency receipt or currency payment will occur. In the case of a hedge for a future receipt of foreign currency, it needs to borrow 'now' in the foreign currency, so that when the currency receipt actually occurs, it will be sufficient to pay the amount borrowed 'now' plus interest.
- To create a money market hedge, the company should borrow an amount of the currency immediately, for repayment at the same time that the future currency income will be received. The income in the currency will be used to repay the loan with interest. The amount borrowed should therefore, together with the accumulated interest for the borrowing period, equal the amount of the future currency income.
- Having borrowed the quantity of currency, the company should exchange it immediately (spot) for its domestic currency.
- The domestic currency obtained in this way can be used in the company's business. However, for examination purposes, you might be expected to assume that the domestic currency will be invested or deposited for the same period as the currency loan.
- At the end of the loan period, when the company uses its currency income to repay the currency loan, the deposit plus accumulated interest is an equivalent
amount in domestic currency. This can be used to calculate an effective forward interest rate for the hedge of the currency exposure.
A numerical example might help to clarify this technique.

\section*{Example}

A UK company expects to receive a payment of US \(\$ 800,000\) in three months' time. It wants to hedge this exposure to currency risk using a money market hedge.

Spot three-month interest rates currently available in the money markets are:
\begin{tabular}{lrr} 
& Deposits & Borrowing \\
\hline US dollar & \(4.125 \%\) & \(4.250 \%\) \\
British pound & \(6.500 \%\) & \(6.625 \%\)
\end{tabular}

The spot exchange rate (US/£1) is \(1.7770-1.7780\).

\section*{Step 1}

The UK company will be receiving US dollars in three months' time. It should therefore borrow US dollars for three months. The borrowing rate will be \(4.25 \%\) (the higher of the two quoted rates). This is an annual rate, and in answering an examination question, you should calculate the rate for the interest period as an appropriate fraction of the annual rate. Here, the interest for three months will be \(4.25 \% \times 3 / 12=1.0625 \%\) or 0.010625 .

The borrowed dollars plus accumulated interest after three months needs to be \(\$ 800,000\), therefore the amount of dollars borrowed should be:
\(\frac{\text { Final amount }}{(1+\text { interest rate for the period })}=\frac{\$ 800,000}{1.010625}=\$ 791,589\)

\section*{Step 2}

The company should sell the borrowed \(\$ 791,589\) in exchange for British pounds. The appropriate spot rate is 1.7780 . The company will receive \(£ 445,213\).

We now assume that this will be placed on deposit for three months. The interest rate on deposits for sterling is \(6.500 \%\). This is an annual rate, and the interest for three months is assumed to be \(6.5 \% \times 3 / 12=1.625 \%\) or 0.01625 .

After three months, the deposit plus accumulated interest will be \(£ 445,213 \times 1.01625\) \(=£ 452,448\).

\section*{Step 3}

At the end of three months, the company will receive US \(\$ 800,000\). Its three-month loan will mature, and the \(\$ 800,000\) is used to pay back the loan plus interest. The company has \(£ 452,448\) from its deposit (its short-term investment in British pounds).

The money market hedge has therefore fixed an effective exchange rate for the dollar receipts, which is calculated as \(\$ 800,000 / £ 452,448\). This gives an effective three-month forward rate of \(£ 1=\$ 1.7682\).

\subsection*{6.3 Constructing a hedge for a future currency payment}

To create a money market hedge for a future currency payment, a similar approach is required. To hedge a future payment in currency, a company should deposit an amount of the currency 'now' in the money market, so that the amount deposited plus interest will be sufficient to make the currency payment at the future date.
- A company with an obligation to make a payment in foreign currency at a future date should therefore buy a quantity of the currency now and place it on deposit until the payment is due to be made. The amount of currency placed on deposit, plus the accumulated interest, should equal the amount of the future payment.
- Buying the currency now spot will cost money. For the examination, it should usually be assumed that the company has to borrow in domestic currency to buy the foreign currency spot, and that the length of the loan period is the same as the deposit period for the foreign currency.
- At the end of the deposit period, the foreign currency deposit plus interest is used to make the currency payment. The domestic currency loan has accumulated interest, and the total amount now payable to settle the loan can be used to calculate the effective interest rate for the currency transaction.

Again, an example might help to clarify the method.

\section*{Example}

Suppose that a UK company is expecting to pay a supplier US \(\$ 500,000\) in six months' time, and it wants to fix an effective exchange rate for this transaction with a money market hedge.

Spot six-month interest rates currently available in the money markets are as follows:
\begin{tabular}{lrr} 
& Deposits & Borrowing \\
\hline US dollar & \(4.125 \%\) & \(4.250 \%\) \\
British pound & \(6.500 \%\) & \(6.625 \%\)
\end{tabular}

The spot exchange rate (US\$/£1) is \(1.7770-1.7780\).

\section*{Step 1}

The company should deposit US dollars for six months. The deposit rate will be \(4.125 \%\) (the lower of the two quoted rates). This is an annual rate, and in an examination the rate for the interest period is calculated as an appropriate fraction of the annual rate. Here, the interest for six months will be \(4.125 \% \times 6 / 12=2.0625 \%\) or 0.020625 .

The dollars placed on deposit plus accumulated interest after six months needs to be \(\$ 500,000\), therefore the amount of dollars placed on deposit for six months should be:
\(\frac{\text { Final amount }}{(1+\text { interest rate for the period })}=\frac{\$ 500,000}{1.020625}=\$ 489.896\)

\section*{Step 2}

These dollars should be bought with British pounds. The appropriate spot rate is 1.7770. The company will therefore pay \(£ 275,687\) to obtain the dollars.

We now assume that this money has to be borrowed for a six-month loan period. The interest rate on deposits for sterling is \(6.625 \%\). This is an annual rate, and the interest for six months will be \(6.625 \% \times 6 / 12=3.3125 \%\) or 0.033125 .

After three months, the loan plus accumulated interest will be \(£ 275,687 \times 1.033125=\) £284,819.

\section*{Step 3}

At the end of six months, the US deposit plus interest is used to make the payment of \(\$ 500,000\). The sterling loan is repayable with interest, and the amount payable can be used to calculate an effective exchange rate for the money market hedge.

The effective exchange rate is therefore \(£ 1=\$ 1.7555\) ( \(\$ 500,000 / £ 284,819\) )

\section*{Conclusion: forward exchange contracts and money market hedges}

In practice, a money market hedge should result in an effective exchange rate similar to the forward exchange rate. In the examination, however, one method of hedging might well result in a more favourable exchange rate than the other.

An examination question might give you a set of exchange rates and interest rates for two currencies, and details of a transaction that creates a currency risk exposure. The question might then ask you to compare a forward exchange contract with a money market hedge, and recommend which method of hedging is better.

You might also be required to compare a money market hedge with other methods of currency hedging, such as currency options and currency futures. These are explained in later chapters.

\section*{13}

\title{
Interest rate risk. Hedging with FRAs and swaps
}

\author{
Contents \\ 1 The nature of interest rate risk \\ 2 Hedging interest rate risk: FRAs \\ 3 Interest rate swaps \\ 4 Using interest rate swaps \\ 5 Currency swaps
}

\section*{The nature of interest rate risk}
- The effect of a change in interest rates
- Interest rate volatility and interest rate risk
- Short-term and long-term interest rates
- Money market interest rates: LIBOR

\section*{1 The nature of interest rate risk}

\subsection*{1.1 The effect of a change in interest rates}

Interest rates can move up or down, although economists are often able to predict the direction of future movements. A movement in interest rates can affect companies in either a positive or a negative way.
- If a company has borrowed at a variable rate of interest, it will have to pay higher interest costs if the interest rate goes up, and lower interest costs if the rate goes down.
- If a company has borrowed at a fixed rate of interest, for example by issuing bonds, it will continue to pay the same rate of interest even if market interest rates go down. However, competitors who have borrowed at a variable rate of interest, or competitors who decide to issue fixed rate bonds after the rate has fallen, will gain a competitive advantage.
- An investor in fixed rate bonds who expects to sell the bonds before their maturity will also be affected by a change in interest rates. A rise in interest yields will result in a fall in the price of existing fixed rate bonds. A fall in the market interest rate will send bond prices up.

Changes in interest rates are particularly significant for organisations that deal in financial assets and liabilities, such as banks and investment institutions. However, they can also be very important for other companies that borrow extensively, and have fixed rate or floating rate debts totalling hundreds of millions of dollars.

For example, if a company has borrowed \(\$ 500\) million from a syndicate of banks at a variable rate of interest, an increase in the annual interest rate of just \(0.25 \%\) will result in higher interest costs of \(\$ 1,250,000\) each year.

\subsection*{1.2 Interest rate volatility and interest rate risk}

Interest rate risk is particularly high when:
- interest rate changes are frequent (and sometimes large), and
- it is uncertain whether the next movement in rates will be up or down.

In other words, interest rate risk increases with interest rate volatility. Volatility is likely to be higher when expected inflation rates are high than when expected inflation rates are low.

\subsection*{1.3 Short-term and long-term interest rates}

A distinction is made between:
- short-term interest rates, which are money market interest rates
- long-term interest rates, which are bond yields.

Volatility in short-term rates affects short-term lending and borrowing, and also all variable rate lending, such as bank loans. Volatility in longer-term rates affects bond investors.

Note that yields on a corporate bond are affected by:
- interest rates for risk-free bonds (domestic government bonds)
- changes in the perceived credit risk of the bond issuer.

For example, suppose that a company's bonds which have been rated AA by a credit rating organisation are now downgraded to a rating of A+. The yield on the bond will increase to reflect the lower credit rating, and the market price of the bonds will fall. However, the increase in the bond yield is due to a credit risk factor rather than to interest rate risk.

\subsection*{1.4 Money market interest rates: LIBOR}

Short-term interest rates for borrowers are set at a margin above the base rate or official rate of the lending bank, or at a margin above a money market rate. The money markets are markets for wholesale borrowing and lending short-term, for periods ranging from overnight up to about 12 months. ('Wholesale' means borrowing and lending in large amounts.)

Each major financial centre has a money market and a 'benchmark' rate that the participants in the market use. In London, the benchmark rate of interest is the London Interbank Offered Rate or LIBOR.
- There are LIBOR rates for each maturity of lending, such as seven-day LIBOR, one-month LIBOR, three-month LIBOR and so on.
- London is a major international money market centre, and there are LIBOR rates in the major currencies as well as in sterling. For example, there is a US dollar LIBOR and a Swiss franc LIBOR. There is also a euro LIBOR, but the commonlyused benchmark rate for the euro is a rate called the euribor rate.
- In Paris, there are PIBOR rates; in Frankfurt there are FIBOR rates; and so on.

A company borrowing British pounds from a bank at a floating rate of interest might pay interest at a margin above LIBOR. For example, if interest is payable every six months, a borrower might pay interest at \(1.50 \%\) above the six-month sterling LIBOR rate.
(Basis points: \(1 \%=100\) basis points, and in the money market, interest rates may be stated as a number of basis points above LIBOR. So LIBOR plus \(1.50 \%\) might be stated as 150 basis points above LIBOR.)

When a company borrows at a variable rate of interest, it pays interest at the end of each interest period, which might be each month, or every three months, or every six months, and so on. The rate of interest payable for the period is decided by reference to the benchmark rate, such as three-month LIBOR, at the beginning of the interest period. As LIBOR rises or falls, the interest payable in each interest period also goes up or down.

\section*{Calculating the interest}

There are rules in the money markets about how interest should be calculated. The rules differ between currencies. For example:
- Interest on a sterling money market loan is calculated as:

Loan principal \(\times\) Annual interest rate \(\times(\) Number of days in the loan period \(/ 365\) )
- Interest on a US dollar money market loan is calculated as:

Loan principal \(\times\) Annual interest rate \(\times(\) Number of days in the loan period/360)

For your examination, you will normally be able to make the assumption that interest is calculated as:

Loan principal \(\times\) Annual interest rate \(\times(\) Number of months in the loan period/12).

\section*{Hedging interest rate risk: FRAs}
- Hedging methods
- Forward rate agreements (FRAs)
- The features of an FRA agreement
- How an FRA works

\section*{2 Hedging interest rate risk: FRAs}

\subsection*{2.1 Hedging methods}

Some organisations might wish to hedge their exposures to interest rate risk. They might also want to take advantage, if possible, from any favourable movements in interest rates. There are several ways in which risks can be hedged and opportunities to benefit from interest rate changes can be exploited.

The methods that you need to understand and might be required to use in your examination are:
- forward rate agreements or FRAs
- interest rate swaps
- interest rate options (described in a later chapter)
- interest rate futures (described in a later chapter)

All these methods can be used to deal with the possibility that 'spot' rates of money market interest will change, or current yields on bonds will change, and will be different at some time in the future.

\subsection*{2.2 Forward rate agreements (FRAs)}

A forward rate agreement (FRA) is a forward contract for an interest rate. FRAs are negotiated 'over-the-counter' with a bank. In some respects, an FRA is similar to a forward exchange rate. It is a contract arranged 'now' that fixes the rate of interest for a future loan or deposit period starting at some time in the future. For example, an FRA can be used to fix the interest rate on a six-month loan starting in three months' time.

Banks are able to quote forward rates for interest rates because there is a large and active money market, and banks are able to borrow and deposit funds short-term. As a result, if a bank can borrow for nine months at one rate of interest and deposit funds for three months at another rate of interest, it can work out a rate to quote to a customer that wants to borrow between the end of month 3 and the end of month 9 . A 'forward rate' can be fixed now that will guarantee the bank a profit on the transaction.

\section*{Example}

A bank can borrow dollars for nine months at \(5 \%\) and can deposit dollar funds for three months at \(4.75 \%\). Suppose that it borrows \(\$ 1\) million for nine months and places them on deposit for three months.
- After nine months, it will have to repay \(\$ 1\) million \(+(\$ 1\) million \(\times 5 \% \times 9 / 12)=\) \(\$ 1,037,500\). The interest payable is \(\$ 37,500\).
- After three months, its dollar deposit will grow to \(\$ 1\) million \(+(\$ 1\) million \(\times\) \(4.75 \% \times 3 / 12=\$ 1,011,875\). The interest received is \(\$ 11,875\).
- To break even by lending to a customer from the end of month 3 to the end of month 9 (six months), the bank would need to earn interest of \$25,625 (\$37,500 \(\$ 11,875)\).
- The (annual) interest rate on the month 6-month lending would therefore have to be: \(\frac{25,625}{1,011,875} \times \frac{12}{6} \times 100 \%=5.06 \%\)

This is not what banks would do in practice, but the example is intended to show that banks are able to use spot money market rates (which are rates for borrowing or depositing funds 'now') to derive interest rates for a future interest period, knowing that they will make a suitable return.

\subsection*{2.3 The features of an FRA agreement}

An FRA, like a forward exchange contract, is a binding agreement between a bank and a customer. It is an agreement that fixes an interest rate 'now' for a future interest period.
- An FRA for an interest period starting at the end of month 3 and lasting until the end of month 9 is a 3 v 9 FRA or a \(3 / 9\) FRA.
- Similarly, an FRA for a three-month period starting at the end of month 2 is a 2 v 5 FRA or a \(2 / 5\) FRA.

An FRA is an agreement that fixes a forward interest rate on a notional amount of money.

\section*{Buying and selling FRAs}

FRAs are bought and sold.
- If a company wishes to fix an interest rate (cost) for a future borrowing period, it buys an FRA. In other words, buying an FRA fixes a forward rate for shortterm borrowing.
- If a company wishes to fix an interest rate (income) for a future deposit period, it sells an FRA.. Selling an FRA fixes a forward rate for a short-term deposit.

The counterparty bank sells an FRA to a buyer and buys an FRA from a seller.

\section*{FRA rates}

A bank trading in FRAs will publish indicative rates. For example, prices for sterling FRAs might be quoted as follows:
\begin{tabular}{ll}
3 v 6 & \(5.27-5.23\) \\
4 v 7 & \(5.32-5.28\) \\
5 v 8 & \(5.39-5.34\) \\
9 v 12 & \(5.63-5.56\) \\
3 v 9 & \(5.40-5.36\) \\
4 v 10 & \(5.45-5.41\) \\
6 v 12 & \(5.50-5.45\)
\end{tabular}

A bank will always apply the rate that is more favourable to itself. For example, a company wanting to fix the interest cost with a 6 v 12 FRA, using the rates in the table above, would be quoted a rate of \(5.50 \%\) for the FRA.

\section*{Notional loans and deposits}

A forward exchange contract for currency is an agreement to buy and sell currency at a future date, when there will be an exchange of currencies between the two parties. An FRA is different. It is not an actual agreement to take out a loan or to make a deposit. An FRA is an agreement on a notional loan or deposit, not an actual loan or deposit. The size of the notional amount of principal (the notional loan or deposit) is specified in the FRA agreement.

\subsection*{2.4 How an FRA works}

An FRA works by comparing the fixed rate of interest in the FRA agreement with a benchmark rate of interest, such as LIBOR. The comparison takes place at the beginning of the notional interest period for the FRA.
- If the FRA rate is higher than the benchmark rate (LIBOR), the buyer of the FRA must make a payment to the seller of the FRA, in settlement of the contract
- If the FRA rate is lower than the benchmark rate (LIBOR), the buyer of the FRA receives a payment from the seller of the FRA, in settlement of the contract.

The amount of the payment is calculated from the difference between the FRA rate and the benchmark rate (LIBOR rate), applied to the notional principal amount for the FRA and calculated for the length of the interest period in the agreement.

\section*{Example}

Suppose that a company knows that it will need to borrow \(£ 5\) million in three months' time for a period of six months. (Alternatively, suppose that a company has a floating rate loan of \(£ 5\) million, with interest payable every six months, and the next interest period due to start in three months' time.)

The company can hedge its exposure to the risk of a rise in the six-month interest rate by buying a 3 v 9 FRA for a notional principal amount of \(£ 5\) million. If the
bank's FRA rates for 3 v 9 FRAs are \(5.40-5.36\), the rate applied to the agreement will be \(5.40 \%\). The benchmark rate of interest, or 'reference rate', will be the sixmonth sterling LIBOR rate.

\section*{Settlement of the FRA}

Suppose that at the end of month 3, six-month LIBOR is \(6.25 \%\). The FRA rate is lower; therefore the FRA rate is settled by a payment from the bank to the buyer of the FRA.
- The difference between the FRA rate and LIBOR is \(0.85 \%\). The payment to settle the FRA will therefore be based on an interest difference of: \(0.85 \% \times £ 5\) million \(\times\) \(6 / 12=£ 21,250\).
- The actual payment will be less than this, because the FRA is settled immediately, at the beginning of the notional interest period, and not at the end of the period. The \(£ 21,250\) is therefore discounted from an end-of-interest period value to a start-of-interest period value, using the reference rate of interest as the discount rate. This PV is the amount received in settlement of the FRA.

Suppose that at the end of month 3, six-month LIBOR is \(4.75 \%\). The FRA rate \((5.40 \%)\) is higher than the spot rate. Therefore the FRA rate is settled by a payment from the buyer of the FRA to the bank. The difference between the FRA rate and LIBOR is \(0.65 \%\).
- The payment to settle the FRA will therefore be based on this interest rate difference: \(0.65 \% \times £ 5\) million \(\times 6 / 12=£ 16,250\).
- Again, because the payment is at the beginning of the interest period and not at the end of the period, the \(£ 16,250\) should be discounted to a present value at the reference rate of interest. This PV is the amount of the payment in settlement of the FRA.

\section*{How an FRA fixes a forward interest rate}

Continuing the example, the company will presumably want to borrow \(£ 5\) million for six months from the end of month 3 . It will do so by arranging an ordinary short-term loan with a bank. The interest rate on the loan might be set at LIBOR + \(1 \%\).
- Suppose that at the end of month 3 , six-month LIBOR is \(6.25 \%\). The company will therefore borrow for six months at \(7.25 \%\). It will receive a payment from the FRA of \(0.85 \%\), so that the net cost of borrowing will be \(6.40 \%(7.25 \%-0.85 \%)\). This net effective interest rate is equal to the FRA rate of \(5.40 \%+1 \%\). The company has therefore been able to fix LIBOR at \(5.40 \%\) with the FRA.
- Suppose that at the end of month 3 , six-month LIBOR is \(4.75 \%\). The company will therefore borrow for six months at \(5.75 \%\). However, it must also make a payment of \(0.65 \%\) to settle the FRA, bringing the total cost of borrowing for the six months to \(6.40 \%(5.75 \%+0.65 \%)\). Again, this total effective rate is equal to the FRA rate of \(5.40 \%+1 \%\) and the FRA has therefore fixed the effective LIBOR rate at \(5.40 \%\).

\section*{Conclusion: using an FRA to hedge an interest rate risk exposure}

An FRA, like a forward exchange rate, therefore fixes an interest rate in advance.
- An FRA can therefore be used by a borrower to hedge an exposure to a future increase in the spot interest rate, or to protect a depositor against a future fall in the interest rate.
- However, the user of an FRA cannot benefit from any favourable movement in the interest rate, because the FRA fixes the rate and is a binding contract.

\section*{Example}

A company has forecast that due to an expected cash shortage, it will need to borrow \(\$ 20\) million for three months in two months' time. A bank quotes the following rates for FRAs:
```

2v 3 3.61-3.59
2v5 3.67-3.63
3v5 3.68-3.65

```

\section*{Required}

What would be the FRA agreement with the bank, and what rate would apply to the agreement?

If the company can borrow at LIBOR +50 basis points, what will be its effective rate of borrowing for the three months if US dollar LIBOR is \(4.50 \%\) at the start of the notional interest period for the FRA?

\section*{Answer}

The company needs a 2 v 5 FRA.

It needs to borrow; therefore the bank will quote the higher rate, 3.67.

In three months' time on the fixing data, if US dollar LIBOR is \(4.50 \%\) the bank will pay the equivalent of \((4.50 \%-3.67 \%)=0.83 \%\).
\begin{tabular}{lr} 
& \(\%\) \\
Three-month borrowing rate \((4.50 \%+50\) basis points & 5.00 \\
Less: settlement received in FRA agreement & \((0.83)\) \\
Effective borrowing rate & 4.17 \\
\hline
\end{tabular}

The effective borrowing rate is the same as the FRA rate of \(3.67 \%\) plus the 50 basis points borrowing margin that the company is required to pay on the money it borrows: \(3.67 \%+0.50 \%=4.17 \%\).

\section*{Interest rate swaps}
- The features of an interest rate swap
- Swap rates
- The effect of a coupon swap

\section*{3 Interest rate swaps}

\subsection*{3.1 The features of an interest rate swap}

An interest rate swap is an agreement between two parties, such as a company and a bank that deals in swaps, for a period of time that is usually several years. Swaps are therefore usually long-term agreements on interest rates.

In a swap agreement, the parties agree to exchange 'interest payments' on a notional amount of principal, at agreed dates throughout the term of the agreement.

The interest rate payments that are exchanged in a 'coupon swap' are as follows:
- One party to the swap pays a fixed rate (the swap rate).
- The other party pays interest at a reference rate or benchmark rate for the interest period, such as LIBOR.

The purpose of an interest rate swap is often to:
- swap a variable rate of interest payment (or receipt) into a fixed interest rate payment (or receipt)
- swap a fixed rate of interest payment (or receipt) into a variable rate of interest payment (or receipt).

\section*{Example: 'plain-vanilla-swap'}

For example, a company might arrange a four-year swap with a bank, for which the notional principal amount is \(\$ 20\) million and:
- the company pays interest every six months at a fixed rate of, say, \(4.25 \%\)
- the bank pays interest every six months at the six-month LIBOR rate for the period.

Since both parties pay interest every six months, if the payment dates coincide, the swap payments will simply be settled by a net payment for the difference in rates from one party to the other.

Over the life of the four-year swap, there will therefore be eight exchanges of interest payments. However, in an interest rate swap, there is no exchange of principal. The interest relates to a notional amount of principal, not an actual asset or liability.
- If the six-month LIBOR rate for one of the periods is, say, \(5.00 \%\), the exchange of payments would be settled by a payment from the bank to the company of \(0.75 \%\) interest ( \(5.00 \%-4.25 \%\) ) on \(\$ 20\) million for six months.
- If the six-month LIBOR rate for one of the periods is, say, \(3.00 \%\), the exchange of payments would be settled by a payment from the company to the bank of \(1.25 \%\) interest ( \(4.25 \%-3.00 \%\) ) on \(\$ 20\) million for six months.

The payments in a plain vanilla swap are at the end of each notional interest period, therefore the amounts payable are not discounted (unlike an FRA).

\subsection*{3.2 Swap rates}

Swap rates are the fixed rates that will be used by a bank in a coupon swap agreement. Swap rates might be quoted as follows:
\begin{tabular}{lcc}
\begin{tabular}{l} 
US dollar \\
Term
\end{tabular} & Bid & Ask \\
\hline Years & \(\%\) & \(\%\) \\
1 year & 4.03 & 4.06 \\
2 years & 4.17 & 4.20 \\
3 years & 4.29 & 4.32 \\
4 years & 4.63 & 4.66 \\
5 years & 4.91 & 4.94 \\
6 years & 5.14 & 5.17 \\
7 years & 5.33 & 5.36 \\
10 years & 5.73 & 5.76 \\
15 years & 6.16 & 6.19 \\
20 years & 6.35 & 6.38
\end{tabular}

These rates might be for a swap against three-month US dollar LIBOR. The lower rate (the bid rate) is the rate that the bank would pay in a swap. The higher rate (the ask rate) is the fixed rate that the bank would receive in a swap. Banks dealing in swaps make a profit or 'turn' from the difference between the bid and ask rates. Swap rates for the major international currencies are quoted for terms of up to 30 years.

\subsection*{3.3 The effect of a coupon swap}

In a coupon swap, one party pays a fixed rate of interest and the other pays 'the floating', which is the variable reference rate of interest, such as six-month LIBOR. For a company with a loan or bonds in issue, the effect of arranging a swap can therefore be:
- to swap from fixed rate interest liabilities to floating rate liabilities, or
- to swap from floating rate liabilities to fixed rate liabilities.

\section*{Example}

A company has a bank loan of \(£ 10\) million on which it pays variable rate interest at LIBOR \(+1 \%\). The loan has five more years to maturity. The company is worried about the risk that interest rates will soon rise, and it wants to set a limit on its interest costs.

It might therefore arrange a five-year swap with a bank, with interest rates to coincide with the interest payments on its bank loan.

The bank might quote rates of \(5.34-5.39\) for a five-year swap in sterling.
The company will receive the floating rate in the swap, to offset the floating rate payments on its bank loan. It will pay the fixed rate, and the rate will therefore be \(5.39 \%\).

The swap therefore alters the net interest payments for the company as follows:
\begin{tabular}{lr} 
& \(\%\) \\
\hline Loan payments & (LIBOR + 1) \\
Swap & LIBOR \\
Receive the floating & \((5.39)\) \\
Pay the fixed & \((6.39)\) \\
\cline { 2 - 2 } Net interest cost &
\end{tabular}

The company had a floating rate liability of LIBOR \(+1 \%\), and has now changed this into a net fixed interest liability of \(6.39 \%\).

On each interest payment date, the company will pay LIBOR \(+1 \%\) in interest on its bank loan, and under the swap agreement will receive or pay the difference between LIBOR for the period and the fixed rate of \(5.39 \%\).

You might see that an interest rate coupon swap is similar in concept to an FRA, but is for a longer period of time and covers more than one interest period.

The company might subsequently change its mind. For example, after two years, it might decide that it wants a floating rate liability again. If so, it can go back to a floating rate liability by arranging with the bank to cancel the swap and agreeing a cancellation payment (for the value of the swap at the date of cancellation).

\section*{Example}

A company has \(5 \%\) bonds in issue with a nominal value of 40 million euros. The bonds have ten more years to maturity. The company wants to exchange its fixed rate liability for a floating rate liability in euros. A bank quotes the following rate for a ten-year swap: \(4.22-4.25\).

By arranging a swap, what will be the effective interest cost for the company?

\section*{Answer}
\begin{tabular}{lr} 
& \(\%\) \\
\hline Cost of the bonds & \((5.00)\) \\
Swap & 4.22 \\
Receive the fixed & (LIBOR) \\
Pay the floating & (LIBOR + 0.78) \\
Net interest cost &
\end{tabular}

\section*{Using interest rate swaps}
- Swapping interest rate liabilities
- Obtaining fixed rate liabilities
- Credit arbitrage

\section*{4 Using interest rate swaps}

Interest rate swaps are used to manage interest rates on liabilities (and assets, in the case of investment institutions and banks). They can therefore be a method of hedging exposures to interest rate risk.

\subsection*{4.1 Swapping interest rate liabilities}

Some large companies use interest rate swaps to manage their net interest liabilities (in each currency). For example, a company that borrows extensively, through a combination of bank loans and bond issues, might have a policy that:
- \(25 \%\) of its debts should be at a fixed rate
- \(25 \%\) of its debt should be at a floating rate, and
- the remaining \(50 \%\) may be at a fixed or floating rate, or a mixture of fixed and floating, depending on the judgement of the finance director or treasury department.

The company might then use interest rate swaps to alter its net liabilities, within the company's policy guidelines, between fixed rate and floating rate. It might move towards more floating rate liabilities if interest rates are expected to fall, and towards fixed rate liabilities when interest rates are expected to rise.

The advantage of using swaps is that a company can alter its net liabilities from fixed to floating rate or floating to fixed rate, without having to alter or re-negotiate its actual loans or bond issues. For example, a company with fixed rate bonds can swap from fixed to floating rate liabilities with a swap, without having to redeem the bonds early and negotiate a floating rate loan with a bank.

\subsection*{4.2 Obtaining fixed rate liabilities}

Many companies are unable to obtain fixed rate debt. Fixed rate interest liabilities come from issuing bonds. Medium-term bank loans are invariably at a floating rate. If a company is too small to issue bonds, or does not have the credit status to issue bonds, it must borrow from banks to obtain debt finance.

If a company wants fixed rate liabilities, but can only borrow from a bank, it can obtain a loan at a floating rate and swap into a fixed rate net liability.

\section*{Example}

A company borrows from its bank for five years at LIBOR plus 150 basis points. It wants its interest rate liabilities to be fixed, so it makes a five-year swap transaction with a bank, in which it pays a fixed rate of \(5.8 \%\) and receives LIBOR.

As a result of the swap, the company's net interest obligations are fixed at \(7.3 \%\).
\begin{tabular}{lr} 
& \(\%\) \\
\hline Loan payments & (LIBOR + 1.50) \\
Swap & LIBOR \\
Receive the floating & \((5.80)\) \\
Pay the fixed & \((7.30)\) \\
Net interest cost &
\end{tabular}

\subsection*{4.3 Credit arbitrage}

At one time, swaps were sometimes used to obtain a lower interest rate on borrowing. This was possible because swaps banks were able to identify opportunities for 'credit arbitrage'. These opportunities arose because of anomalies in the rates of interest at which different companies could borrow.

When an opportunity for credit arbitrage exists, one of the following situations will occur:

\section*{Situation 1}

Two companies want to borrow. They can both borrow at either a fixed rate or a floating rate. Company A has to pay a higher rate of interest than Company B. However, the difference in borrowing costs between the two companies is less for fixed rate borrowing than for variable rate borrowing.

For example:
\begin{tabular}{lrr} 
& \begin{tabular}{r} 
Fixed rate \\
borrowing cost
\end{tabular} & \begin{tabular}{r} 
Variable rate \\
borrowing cost
\end{tabular} \\
\hline Company A & \(7.25 \%\) & \\
LIBOR \(+1.5 \%\) \\
Company B & \(6.50 \%\) & LIBOR \(+0.5 \%\) \\
Difference & \(0.75 \%\) & \\
& & \\
\hline
\end{tabular}

Credit arbitrage is possible using an interest rate swap if Company A wants to borrow at a variable rate of interest and Company B wants to borrow at a fixed rate.

\section*{Situation 2}

Two companies want to borrow. They can both borrow at either a fixed rate or a floating rate. Company C has to pay a higher rate of interest than Company D. However, the difference in borrowing costs between the two companies is more for fixed rate borrowing than for variable rate borrowing.

For example:
\begin{tabular}{lrr} 
& \begin{tabular}{r} 
Fixed rate \\
borrowing cost
\end{tabular} & \begin{tabular}{r} 
Variable rate \\
borrowing cost
\end{tabular} \\
Company C & \(7.75 \%\) & \begin{tabular}{r} 
LIBOR \(+1.5 \%\)
\end{tabular} \\
Company D & \(-6.50 \%\) & LIBOR \(+0.5 \%\) \\
Difference & \(-1.25 \%\) & \\
\hline
\end{tabular}

Credit arbitrage is possible using an interest rate swap if Company C wants to borrow at a fixed rate of interest and Company B wants to borrow at a variable rate.

\section*{Example}

A bank is aware that two companies can borrow at the following rates:
\begin{tabular}{lrr} 
& Fixed & Floating \\
\hline Company A & \(5.50 \%\) & LIBOR \(+0.50 \%\) \\
Company B & \(6.40 \%\) & LIBOR \(+1 \%\)
\end{tabular}

Company A wants to borrow at a floating rate, and can do so at LIBOR \(+0.50 \%\). Company B wants to borrow at a fixed rate, and can do so at \(6.40 \%\). However, an opportunity for credit arbitrage exists, because company A can borrow at a fixed rate \(0.90 \%\) less than company B, but at a floating rate only \(0.50 \%\) less than company B. The opportunity for credit arbitrage totals \(0.40 \%(0.90 \%-0.50 \%)\).

A bank might therefore propose the following arrangement:
- Company A should borrow at a fixed rate, by issuing bonds at \(5.50 \%\). Company \(B\) should borrow at a floating rate, by obtaining a bank loan at LIBOR \(+1 \%\).
- Company A should enter into a swap with the bank in which it receives \(5.20 \%\) fixed and pays LIBOR.
- Company B should enter into a swap with the bank in which it pays \(5.25 \%\) fixed and receives LIBOR.

The net interest cost of each company would be as follows:
\begin{tabular}{lrr} 
& Company A & Company B \\
\hline & \(\%\) & \(\%\) \\
Borrowing cost & \((5.50)\) & \((\) LIBOR +1\()\) \\
Swap & & \\
\(\quad\) Receive & (LIBOR) & LIBOR \\
\(\quad\) Pay & \((5.25)\) \\
Net cost & \((\) LIBOR +0.30\()\) & \(-(6.25)\) \\
\hline
\end{tabular}

Company A reduces its net borrowing cost by \(0.20 \%\) below the cost of borrowing directly at a floating rate, and company B reduces its fixed rate cost by \(0.15 \%\) below the cost of issuing bonds at \(6.40 \%\). The bank makes a profit of \(0.05 \%\) from the difference between its fixed rates in the swap for receiving ( \(5.25 \%\) from company B) and paying ( \(5.20 \%\) to company A).

\section*{Currency swaps}
- The nature of currency swaps
- Reasons for using currency swaps
- FX swaps

\section*{5 Currency swaps}

\subsection*{5.1 The nature of currency swaps}

Interest rate swaps are swaps in the same currency, usually between a fixed rate and a floating rate of interest, with interest calculated on a notional amount of principal.
Currency swaps are similar, but with some significant differences:
- The swap is between two different currencies. One party pays interest on an amount of principal in one currency. The other party pays interest on an equivalent amount of principal in a different currency.
- The interest rates that are swapped need not be a fixed rate in exchange for a floating rate. A currency swap can be between a fixed rate in one currency and a (different) fixed rate in the other currency.
- There is an actual exchange of principal. There must be an exchange of principal at the end of the swap, at a rate of exchange that is fixed at the beginning of the swap. (There might also be an actual exchange of principal at the beginning of the swap, but this is not usual.)

\section*{Example}

A UK company has taken an opportunity to borrow US \(\$ 180\) million in the bond markets, by issuing a seven-year bond. However, it wants to have its interest liabilities in sterling, not dollars. It might therefore arrange a seven-year currency swap in which the agreed exchange rate is \(£ 1=\mathrm{US} \$ 1.80\).
- For the seven years of the swap, the UK company will receive fixed rate interest in US dollars from the swap counterparty. The interest received on each interest payment date will be interest for the period at the agreed swap rate for US dollars, on \(\$ 180\) million.
- The UK company will pay interest in the swap on \(£ 100\) million, also at a fixed rate agreed in the swap.
- The interest received in US dollars can be used to meet the dollar interest liabilities on the bonds. This leaves the company with net interest obligations in sterling.
- At the end of the swap, there is an exchange of principal. The UK company will receive US \(\$ 180\) million from the swap counterparty and in exchange must pay \(£ 100\) million. It will use the US \(\$ 180\) million to redeem the dollar bonds.

The effects of the currency swap may be summarised as follows:
\begin{tabular}{lll} 
& Interest & Principal payments (end of the swap) \\
\hline Bonds & Pay dollars & Pay dollars (\$180 million) \\
Currency swap & Receive dollars & Receive dollars (\$180 million) \\
& Pay sterling & Pay sterling \\
Net effect & Pay sterling & Pay sterling
\end{tabular}

The effect of the currency swap has therefore been to borrow in one currency, but swap the interest and loan principal repayment liabilities into a different currency.

Currency swaps are therefore used to hedge long-term currency risk.

\subsection*{5.2 Reasons for using currency swaps}

As the previous example shows, currency swaps can be used to swap interest payment and principal repayments from one currency into another, and so are a method of hedging long-term currency risk exposures.

Companies can use currency swaps to borrow at a favourable interest rate in the international loans or bonds markets, and swap their liabilities into a currency of their preference. This could enable them to borrow more cheaply than borrowing directly in the currency of their preference.

If the currency swap involves an exchange of interest at a fixed rate in one currency for a floating rate in the other currency, the swap can also be used to hedge interest rate risk.

However, currency swaps are far less common than interest rate swaps. Interest rate swaps are used extensively in the financial markets, particularly by banks and other financial institutions.

\subsection*{5.3 FX swaps}

An FX swap is similar to a currency swap, but with one important difference.
- In a currency swap, the two swap counterparties exchange a series of interest payments in the two currencies over the life of the swap.
- In an FX swap, there is no exchange of interest payments. There is an exchange of principal at the start of the swap and a re-exchange at the end of the swap, but there are no cash flows during the term of the swap.

The reason for having FX swaps is that a company might find it difficult to raise debt in a particular currency. For example, a company might want to issue bonds in a currency for which the bond market is illiquid and not popular with investors. The company might want to issue bonds to obtain a liability in the currency for the purpose of hedging its currency exposures.

To create its liability in the currency, the company might:
- Issue bonds in another currency, such as US dollars
- Arrange an FX swap for the immediate exchange of the dollars for the other currency. The company would then have a liability to repay the currency at the end of the term of the swap.

\section*{Futures and hedging with futures}

\section*{Contents}

1 Financial futures
2 The role of the futures exchange
3 Open positions and closing positions
4 Ticks and tick values
5 Basis and basis risk
6 Hedging risk exposures with futures
7 Currency futures
8 Short-term interest rate futures (STIRs)
9 Hedging with bond futures and stock index futures

\section*{Financial futures}
- The nature of futures contracts
- Commodity futures and financial futures
- Futures exchanges
- Settlement dates
- Futures prices

\section*{1 Financial futures}

\subsection*{1.1 The nature of futures contracts}

A future is forward contract for the purchase or sale of a standard quantity of an item, for settlement or delivery at a specified future date. It is therefore a contract to buy or sell a quantity of an item at a future settlement date, at a price agreed 'now'. Futures contracts have some special features.
- They are standardised contracts. Every futures contract for the purchase/sale of a particular item is identical to every other futures contract for the same item, with the only exception that their settlement dates/delivery dates may differ.
- They are traded on an exchange, rather than negotiated 'over-the-counter'.

\subsection*{1.2 Commodity futures and financial futures}

Commodity futures are futures contracts for the sale and purchase of standard quantities of standard commodities, such as wheat, oil, copper, gold, rubber, soya beans, coffee, cotton, sugar, and so on.

Financial futures are futures contracts for the sale and purchase of a financial item, such as a quantity of currency, a notional portfolio of shares, a quantity of notional government bonds, a notional three-month deposit, and so on. There are currency futures, short-term interest rate futures, bond futures and stock index futures.

\subsection*{1.3 Futures exchanges}

Futures are traded on futures exchanges. Each futures exchange has its own particular futures contracts. Only in a few cases do two futures exchanges compete for business by offering trading in similar futures contracts.

The major futures exchanges include the Chicago Board of Trade (CBOT) and Chicago Mercantile Exchange (CME), Eurex and LIFFE.

\subsection*{1.4 Settlement dates}

Futures are traded on a futures exchange for just four settlement dates each year. These are usually in March, June, September and December. Futures are therefore referred to by their settlement date, such as March Eurodollar futures or September S\&P500 futures.

The actual settlement date for contracts in each month is specified by the futures exchange. In examination questions it is normally assumed (for simplicity) that contract settlement dates are at the end of the month.

\subsection*{1.5 Futures prices}

The price for futures is the price at which buyers and sellers agree to make a transaction with each other.

Dealing in futures contracts continues throughout the trading day on the futures exchange, and the prices at which futures are bought and sold vary with each transaction. However, the exchange authorities receive information about the prices of transactions that have been made, and up-to-the minute price information is published to the market. Buyers and sellers of futures are therefore aware of what current market prices are.

\section*{The role of the futures exchange}
- Counterparty to all trades
- Margin

\section*{2 The role of the futures exchange}

The futures exchange regulates the market that it provides. It establishes rules of conduct and provides the systems in which trading can take place. In addition, the exchange provides security to the market by virtually eliminating credit risk for its participants.

\subsection*{2.1 Counterparty to all trades}

An important feature of trading on a futures exchange is that when a buyer and seller agree a transaction in futures and report the transaction to the exchange, the exchange takes on the role of counterparty to the buyer and the seller in the transaction. This means for example that if X makes a transaction to sell 20 December currency futures to Y :
- the futures exchange will become the buyer of 20 December futures from X and
- the exchange will also become the seller of 20 December futures to Y .

Both X and Y have a contract with the exchange, and not a contract with each other. X is therefore not relying on the good credit standing of Y to honour the contract, and \(Y\) is not relying on the good credit standing of \(X\). They both rely on the credit standing of the exchange itself. Since the credit status of the exchange is high, the credit risk is minimised.

The exchange protects itself against credit risk from participants in the exchange, by means of a system of margin payments. Margin payments are explained below.
(Note: Strictly speaking, the exchange itself is not the counterparty to every transaction. The exchange is represented by a clearing house, that acts as counterparty to every transaction. For the LIFFE futures exchange in London, for example, the clearing house is the London Clearing House or LCH).

\subsection*{2.2 Margin}

After someone has bought or sold futures at an agreed price, the market price of the futures will move up or down. The buyer or seller of the futures will make a gain or a loss on the futures position, depending on whether the market price has moved favourably or adversely. If the price moves adversely by a large amount, a person might have a large loss on his futures position. A problem for the futures exchange is how to prevent someone with a large loss from refusing to settle the contract and pay for the loss. This problem is overcome by a requirement for every position in futures to be covered by a cash deposit with the exchange. This cash deposit is called a margin.
- When two parties agree to the sale/purchase of a quantity of futures, both parties are required to pay a cash deposit, called an initial margin, to cover the risk of short-term losses on their position.
- If a position goes into loss because the market price moves adversely, the exchange will call for an additional cash payment of variation margin to cover the loss.

In this way, all loss positions have been covered by cash payments, and there is no credit risk for the exchange from non-payment of losses by the buyers or sellers of futures contracts.

\section*{Example}

A company arranges to buy futures on coffee at a contract price of \(\$ 2,000\) per tonne. Having bought the futures, it must pay a deposit or initial margin to the futures exchange. The seller of the coffee futures must also pay initial margin.

If the 'spot' price of coffee then falls, and the price of coffee futures falls too, the company will be making a loss on its futures position. To cover these losses, the exchange will ask the company to pay an additional variation margin.

Margin protects the futures exchange against bad debt risk from buyers or sellers of futures who makes losses on their position.

\section*{Open positions and closing positions}
- Open positions: long and short positions
- Settlement
- Closing positions

\section*{3 Open positions and closing positions}

\subsection*{3.1 Open positions: long and short positions}

When a person has bought futures without previously having sold any, he establishes a long position in the futures. A long position in futures therefore represents an undertaking to buy a quantity of futures at a future date. For example, if a company buys 10 March US dollar-euro currency futures, it is 'long' 10 March futures.

It is possible to sell futures without having any (in other words, without having a long position). A position in which a person has sold futures, without previously buying them, has a short position. For example, someone can sell 100 September S\&P500 Index futures, without already 'owning' 100 futures or even without owning an equivalent quantity of shares in companies in the S\&P500 Index.

Short and long positions are both open positions in futures, and at any time, the total amount of long positions is always equal to the total of short positions. This is because the number of futures contracts sold and the number purchased must be equal.

\subsection*{3.2 Settlement}

Someone with an open position in futures can keep the position open until the futures contract reaches its settlement date. At settlement, the holder of a long position will require the futures exchange to settle the contract for the purchase of the underlying items in the futures contract. Similarly, at settlement the holder of a short position will be required to make the sale of the underlying item to the futures exchange.

Some futures contracts are cash-settled, and some are settled by physical delivery. For example, short-term interest futures are always cash settled, whereas bond futures are settled by the actual delivery of a quantity of bonds from the seller to the buyer.

\subsection*{3.3 Closing positions}

Open positions are not usually kept open until settlement, and so only a few futures contracts are actually settled at their settlement date. Instead, positions are usually closed before settlement.
- The holder of a long position can close the position by selling an equal number of the futures, for the same settlement date, before the settlement date. For example, a long position in 30 June Eurodollar futures can be closed at any time before settlement in June by selling 30 June Eurodollar futures.
- The holder of a short position can close the position by buying an equal quantity of the futures, at any time before settlement. For example, a short position of 25 September FTSE100 futures can be closed at any time before settlement in September by buying 25 September FTSE100 futures.

When a long position is closed, the original buying price and the selling price to close the position will be different, and there will be an overall gain or loss from the difference between the buying and selling prices. Similarly, when a short position is closed, the original selling price and the buying price to close the position will be different, and there will be an overall gain or loss from the difference between the selling and buying prices.

Dealing in futures therefore results in gains or losses on the futures position, which arise because of movements in the market price of the futures.

\section*{Example}

A company wants to hedge a currency transaction exposure. The exposure relates to a transaction that will occur in November, and the company needs to sell 20 euro currency futures. There are futures with settlement dates in September and December, but not November.

It should therefore sell futures for the next settlement date after the expected transaction, which in this example is December. Suppose the company sells 20 futures at a rate of \(1.3500(\$ 1.3500=€ 1)\).

In November when the transaction occurs, the company should close its position. It opened the position by selling 20 December futures, and it should close the position by buying 20 December futures. There will be a cash profit or loss on closing the position.

The company will use the spot market for the transaction in euros. The gain or loss on the futures position, taken with the spot price, will result in a net exchange rate close to (but probably not equal to) the rate of 1.3500 that was obtained in the futures contract.

This will be illustrated in more detail later.

\section*{Ticks and tick values}

■ Ticks
- The value of a tick

\section*{4 Ticks and tick values}

\section*{\(4.1 \quad\) Ticks}

A tick is the minimum price movement for a futures contract. For example:
- US dollar-euro currency futures are contracts for \(€ 125,000\) in exchange for US dollars. They are priced at the EUR/USD rate (\$ per €1), and the size of a tick is US\$0.0001.
- Short-term interest futures are priced up to a theoretical maximum of 99.9999 and each tick is 0.0001 in price. A tick represents an interest rate of \(0.01 \%\) per annum.
- Stock index futures are priced in terms of the stock index itself. For example, DAX Index futures might be priced at 8200 or the Hang Seng Index futures at 21750. Price movements are in units of the index itself.

\subsection*{4.2 The value of a tick}

Since every futures contract is for a standard quantity of the underlying item, each movement in price of one tick has the same value, representing a gain or loss to a buyer and a corresponding loss or gain to a seller. For example:
- The value of one tick for a US dollar-euro currency future ( \(€ 125,000\) in exchange for US dollars) is: 125,000 euros \(\times \$ 0.0001\) per euro \(=\$ 12.50\).
This can be proved easily. If the price of the contract is \(\$ 1.2000\), the cost of 125,000 would be \(\$ 150,000\). If the exchange rate rises by one tick to \(\$ 1.2001\), the cost of \(€ 125,000\) would be \(\$ 150,012.50\). This upward price movement would represent a gain of \(\$ 12.50\) to someone with a short position in the futures and a loss to the holder of a long position.
- A Eurodollar future is a short-term interest future representing a three-month deposit of US \(\$ 1,000,000\). The value of one tick for a Eurodollar future is therefore: \(\$ 1,000,000 \times 3 / 12 \times 0.0001=\$ 25\).
- For stock index futures, each one tick of price has a fixed value. The value of a tick for FTSE100 futures, for example, is \(£ 10\).

\section*{Example}

Tick values can be used to calculate the gain or loss on a futures position. For example, suppose that a company buys 20 June Eurodollar futures at 9650 and sells 20 June futures later at 9672 to close the position. There is a gain of 0.0022 ticks on each contract, the value of each tick is \(\$ 25\), and the total gain is therefore:

20 contracts \(\times 22\) ticks \(\times \$ 25\) per tick \(=\$ 11,000\).

\section*{Basis and basis risk}
- Basis
- Basis risk

\section*{5 Basis and basis risk}

\subsection*{5.1 Basis}

The futures price for an item is never the same as the spot market price or current market price, except at settlement date for the contract. For example, a company might sell September sterling/US dollar futures in April at a price of 1.9800 when the current spot exchange rate is 1.9600 . The difference of 0.0200 or 200 points is called 'basis'.

As a futures contract moves towards settlement, the basis should gradually reduce in size, until it is 0 at settlement date. If the basis is not 0 at settlement, dealers in the market would be able to make an instant guaranteed profit by:
- buying or selling in the futures market, and simultaneously
- selling or buying in the cash market (spot market).

However, most futures positions are closed before settlement. When a futures position is closed, there will still be some basis, and the futures price and the current spot market price will be different.

For your examination, you might be required to estimate what the basis should be when a position is closed. To do this, we have to make the simplified assumption that the basis will reduce in a straight line from the time the futures position is opened to 0 at settlement date. Using this assumption, we can estimate the expected basis at the intended date for closing the position.

\section*{Example}

A company sells September sterling/US dollar futures at the end of April at a price of 1.9800 when the current spot exchange rate is 1.9600 . There are exactly five months to settlement date, assuming the settlement date for the September futures contract is at the end of that month. The company expects to close the position at the end of July, after three months.

The basis is 200 at the end of April. Assuming that basis falls to zero at a steady rate, it should be expected to fall by 200 between the end of April and the end of September, which is by 40 each month. (This is the total basis of 200 points divided by 5 months.) At the end of July, after three months, basis should have fallen by 120, and the expected basis when the position is closed in July would therefore be 80 (200 - 120). We would therefore estimate that the futures price will be higher than the spot rate by 80 .
(The futures price will be higher than the spot rate because it was higher when the futures were originally sold and the short position in the futures was established.)

\subsection*{5.2 Basis risk}

In practice, basis does not fall at an even rate.
Basis risk is the risk that when a futures position is closed, the size of the actual basis will be different from the expectation of what the basis should be. In the example above, when the position in September currency futures is closed in July, the futures price might be 1.9790 and the current spot price might be 1.9740, giving a basis of 50 . This is less than the expected basis of 80 that was estimated when the futures position was opened, by a difference of 30 .

The significance of basis is explained later.

\section*{Hedging risk exposures with futures}
- Creating a hedge with futures
- Imperfect hedges
- Basis and imperfect hedging

\section*{6 Hedging risk exposures with futures}

\subsection*{6.1 Creating a hedge with futures}

Futures can be used to hedge a short-term exposure to currency risk, interest rate risk or market risk for share prices. They can hedge short-term exposures only, because virtually all futures trading is in contracts with a settlement date within the next 12 months.

The concept of hedging with futures is to create a position in futures that is the opposite of the exposure that is being hedged. By creating an opposite position in futures:
- any loss on the position that is being hedged will be matched by a gain on the futures position
- however, any gain on the position that is being hedged will be matched by a loss on the futures position.

A useful rule to remember is that:
- a long position in futures will gain from an increase in the futures price and lose from a fall in the price
- a short position in futures will gain from a fall in the futures price and lose from a rise in the price.

The settlement date for the futures must be later than the date of the transaction that is being hedged. For example, if a company is hedging a position in interest rates for the euro, and intends to close the position in August, it should open a position in September futures. If it opened a position in June futures, the futures would reach settlement in June, and the position would no longer be hedged.

\section*{Example}

A US company expects to receive 2,000,000 euros from a customer in April, in four months' time, and it wishes to hedge the exposure with currency futures. On receipt of the euros in April, the company will exchange them into US dollars. The company's exposure is to a fall in the value of the euro.

Each US dollar/euro currency future is for \(€ 125,000\). The US company will want to sell euros in exchange for US dollars, therefore it will sell June futures to hedge its position. It will sell 16 contracts (which is \(€ 2,000,000 / € 125,000\) per contract).

If the euro falls in value against the US dollar, the value of the \(€ 2,000,000\) receivable will fall. However, the price of the futures will also fall and since the company has a short position in the futures, it will gain from the fall in price. The gain on the futures position will offset the fall in value in the dollars.

\subsection*{6.2 Imperfect hedges}

With futures, it is not usually possible to hedge a position exactly. Futures are contracts for a standard quantity of an underlying item, whereas the position for which the hedge is required might not be an exact multiple of the value of a futures contract.

\section*{Example}

A French company expects to receive US \(\$ 1,200,000\) in July, in three months' time, and it wants to hedge its exposure with currency futures. It will exchange the dollar income in July for euros, and so will be selling dollars and buying euros. Its exposure is therefore to a fall in the value of the dollar.

A futures contract is for 125,000 euros. It will create a hedge with futures by purchasing September futures. We need to calculate the equivalent value of \(\$ 1,200,000\) in euros. Suppose the company can buy September futures at 1.2350 ( \(\$ 1.2350\) per \(€ 1\) ).
- This represents 971,660 euros (US\$1,200,000/1.2350).
- 971,660 euros is equivalent to 7.8 contracts ( \(€ 971,660 / € 125,000\) per contract).

The company cannot buy a fraction of a future, and so must buy 7 or 8 contracts. If it buys 7 contracts, it will be hedging \(€ 875,000\) and if it buys 8 contracts, it will be hedging \(€ 1,000,000\). In both cases, the hedge is imperfect.

In this example, the company will probably buy 8 contracts, because 7.8 is nearer 8 than 7.

Having bought futures to create a long position, the company will gain from a rise in the futures price (a rise in the exchange rate, which means a weaker dollar). This will offset the fall in the value of the future dollar income of \(\$ 1,200,000\) due to a fall in the value of the dollar.

\section*{Note on buying or selling futures as a hedge}

You need to understand how a hedge is created, by buying or by selling futures. The basic principle is that the futures position that provides a hedge against the risk must be the 'opposite' of the underlying position, so that a loss on the underlying position will be offset by a gain on the futures position (and a gain on the underlying position results in a loss on the futures position).

In this example, the company will receive US dollars. This is the underlying position. The risk is that dollars will fall in value against the euro. The hedge is created by paying US dollars in the futures contracts, which is the opposite of the underlying position. Paying US dollars means receiving/buying euros, so the hedge is constructed by purchasing dollar/euro futures.

\subsection*{6.3 Basis and imperfect hedging}

A hedge with futures is also imperfect because of basis and basis risk. When a hedge is created with futures, the current spot price will differ from the futures price by the basis. Over time, since basis falls towards 0 , changes in the futures price will be larger than changes in the current spot market price. This means that the hedge created by the futures position will not provide a gain or loss that matches exactly the loss or gain on the transaction or position that is being hedged.

The effect of basis on hedging is described in more detail later.
Hedging with currency futures: the approach summarised
You might find the following guidelines useful in deciding how to create a hedge with currency futures:
\begin{tabular}{|ll} 
Step & \begin{tabular}{l} 
Comment
\end{tabular} \\
\(\mathbf{1}\)\begin{tabular}{ll} 
Identify the underlying transaction \\
that you are trying to hedge.
\end{tabular} & \begin{tabular}{l} 
Does the company need to buy a quantity of a \\
currency at the future date or sell a quantity of a \\
currency? Which currency will the company need to \\
buy or sell?
\end{tabular} \\
What is the currency risk in the \\
underlying transaction?
\end{tabular}\(\quad\)\begin{tabular}{l} 
Will the company have to pay more if a currency \\
goes up in value, or if it goes down in value? Or will it \\
receive less if a currency goes up in value or down in \\
value? \\
It is useful to think of the currency in which the \\
futures are denominated.
\end{tabular}

\section*{Currency futures}
- Features of currency futures
- Hedging currency exposures with futures

\section*{\(7 \quad\) Currency futures}

\subsection*{7.1 Features of currency futures}

Currency futures are contracts for the purchase/sale of a standard quantity of one currency in exchange for a second currency. Futures contracts are priced at the exchange rate for the transaction.

Most currency futures are contracts for the major international currencies. For example:
\begin{tabular}{lr} 
Currency future (CME) & Amount \\
\hline US dollar - euro & EUR 125,000 \\
US dollar - yen & 12.5 million yen \\
US dollar - British pound & \(£ 62,500\) \\
US dollar - Swiss franc & SFr125,000 \\
US dollar - Canadian dollar & C \(\$ 100,000\)
\end{tabular}

An examination question on currency futures should tell you the size of each futures contract.

\subsection*{7.2 Hedging currency exposures with futures}

The principles of hedging with futures have been explained already. However, several examples will be used to illustrate how hedging works. Study each example carefully: they get progressively more difficult.

Example: perfect hedge, no basis
It is July. A US company must pay 750,000 euros to a Spanish supplier in November, and wishes to hedge its currency exposure with currency futures. In this example, it is assumed that there is no basis, and the futures price and the current spot market exchange rate are always equal. The price of December futures in July is 1.2100 ( \(\$ 1.2100\) per \(€ 1\) ). The value of a tick is \(\$ 12.50\). (This is \(€ 125,000 \times \$ 0.0001\) per \(\$ 1\).)

The exposure is to the risk of a rise in the value or cost of the euro between July and November. The underlying position is that the US company will pay euros in November. So to create a futures hedge against the risk of a rise in the cost of the euro, it should buy euro currency futures. If the cost of the euro goes up, the company will have to pay more to obtain the 750,000 euros, but this higher cost will be offset by a gain on the futures position.

This logic can be set out in the three-step approach described earlier, as follows:
\begin{tabular}{|lll|}
\hline Step & & Comment \\
\(\mathbf{1}\) & \begin{tabular}{l} 
Identify the underlying transaction \\
that you are trying to hedge.
\end{tabular} & \begin{tabular}{l} 
The US company will be paying 750,000 euros
\end{tabular} \\
\(\mathbf{2}\) & \begin{tabular}{l} 
What is the currency risk in the \\
underlying transaction?
\end{tabular} & \begin{tabular}{l} 
The cost of obtaining the euros will go up. The \\
cost/value of the euro will rise against the dollar.
\end{tabular} \\
\(\mathbf{3}\) & \begin{tabular}{l} 
Work out a futures position that \\
creates a profit if there is a 'loss' on \\
the underlying transaction.
\end{tabular} & \begin{tabular}{l} 
If a loss will be incurred on the underlying transaction \\
if the value of the euro goes up, the futures position \\
should create a profit if the value of the euro goes up.
\end{tabular} \\
& \begin{tabular}{l} 
So buy euro currency futures, since a profit will be \\
made if the value of the euro goes up and the futures \\
can be sold at a higher price.
\end{tabular} \\
\hline
\end{tabular}

The US company should therefore buy 6 December contracts ( \(€ 750,000 / € 125,000\) per contract) at a price of 1.2100 . The company is therefore 'long' 6 December contracts.

Suppose the spot exchange rate in November when the US company must make the payment in euros is 1.2240 . The cost of buying the euros has gone up.

The long' futures position is closed by selling 6 December contracts at 1.2240 , because in this example the futures price is also 1.2240 . The position is closed at a profit, as follows:
\begin{tabular}{lr} 
Open futures position: buy at & 1.2100 \\
Close position: sell at & 1.2240 \\
\hline Gain & 0.0140 \\
\hline
\end{tabular}

Gain \(=140\) ticks per contract at \(\$ 12.50\) per tick.
Total gain on futures position \(=6\) contracts \(\times 140\) ticks \(\times \$ 12.50=\$ 10,500\).

The US company has to pay US1.2240 to obtain the euros to make the payment in November, but the net cost is calculated by taking the cost of the spot transaction and the gain or loss on the futures position.
\begin{tabular}{lr} 
& \(\$\) \\
\hline Spot market: buy \(€ 750,000\) at 1.2240 & 918,000 \\
Less gain on futures position & \((10,500)\) \\
\cline { 2 - 2 } Net cost of \(€ 750,000\) & 907,500 \\
\hline
\end{tabular}

The net cost is \(\$ 907,500\), so the effective exchange rate secured by futures hedge \(=\) US\$907,500/€750,000
\(=\mathrm{US} \$ 1.2100 / € 1\).

This is the spot rate and the futures price at the time the position was opened.

\section*{Conclusion}

With a perfect hedge and no basis, futures can therefore fix the effective exchange rate at the spot rate when the hedge was created. However, in practice, the hedge is likely to be imperfect and there is basis.

\section*{Example: imperfect hedge but no basis}

It is June. A UK company expects to receive US \(\$ 1,000,000\) from a customer in August, and wishes to hedge its currency exposure with currency futures. In this example, it is assumed that there is no basis, and the futures price and the current spot market exchange rate are always equal. The price of September futures in June is 1.8200 . The value of one tick is \(\$ 6.25\).

The exposure is to the risk of a fall in the value of the US dollar between June and August. The company will receive US dollars. It can create a hedge with futures by selling dollars and buying British pounds. The UK company should therefore buy the currency futures, which are denominated in British pounds.

This logic can be set out in the three-step approach described earlier, as follows:
\(\left.\left.\begin{array}{|lll}\text { Step } & & \begin{array}{l}\text { Comment } \\ \text { Identify the underlying transaction } \\ \text { that you are trying to hedge. }\end{array}\end{array} \begin{array}{l}\text { The UK company will be receiving } \$ 1 \text { million. }\end{array}\right\} \begin{array}{l}\text { What is the currency risk in the } \\ \text { underlying transaction? }\end{array} \begin{array}{l}\text { The value of the dollar will fall and the income will be } \\ \text { worth less in sterling. Futures are denominated in } \\ \text { sterling, so it is better to state that the risk is that the } \\ \text { value of sterling will increase. }\end{array}\right\}\)

At a rate of 1.8200 , the sterling equivalent of \(\$ 1,000,000\) is \(£ 549,451\). Each futures contract is for \(£ 62,500\), therefore the company would want to buy 8.79 contracts. The company will probably decide to buy 9 contracts, although the hedge is imperfect.

Suppose that in August when the dollars are received, the dollar has actually strengthened in value and the exchange rate is 1.7800 (and the futures price is also 1.7800 ). The company will sell the \(\$ 1,000,000\) and close its futures position. The position for the company is as follows:
\begin{tabular}{lr} 
Open futures position: buy at & 1.8200 \\
Close position: sell at & 1.7800 \\
\hline Loss & 0.0400 \\
\hline
\end{tabular}

Loss \(=400\) ticks per contract at \(\$ 6.25\) per tick.
Total loss on futures position \(=9\) contracts \(\times 400\) ticks \(\times \$ 6.25=\$ 22,500\).
\begin{tabular}{lr} 
& \(\$\) \\
\hline Received from customer & \(1,000,000\) \\
Loss on futures & \((22,500)\) \\
\cline { 2 - 2 } Net receipt in dollars & 977,500 \\
\hline Sell dollars at 1.7800 (spot rate) & \\
Income in British pounds & \(£ 549,157\)
\end{tabular}

Effective exchange rate secured by futures hedge \(=\) US\$1,000,000/£549,157 \(=\) \$1.8210/£1.

This is close to the spot rate and futures price at the time the position was opened. Ignoring basis, the futures hedge has therefore fixed the effective exchange rate close to the spot rate and futures price when the hedge was created. However, in this example, the exchange rate moved favourably, and the gain from the increase in value of the dollar income was offset by a loss on the futures position.

\section*{Example: imperfect hedge and basis}

A Netherlands company expects to pay US \(\$ 1,200,000\) to a US supplier in late November. It is now late July, and the current spot exchange rate is \(€ 1=\$ 1.2200\). The current spot price for December dollar-euro currency futures is \(\$ 1.2170\). The company wants to hedge its exposure with currency futures.

The logic of establishing a hedge with currency futures can be set out in the threestep approach described earlier, as follows.
\begin{tabular}{|lll}
\hline Step & & Comment \\
\(\mathbf{1}\) & \begin{tabular}{l} 
Identify the underlying transaction \\
that you are trying to hedge. \\
What is the currency risk in the \\
underlying transaction?
\end{tabular} & \begin{tabular}{l} 
The European company will be paying \(\$ 1.2\) million. \\
The cost/value of the dollar will increase and the cost \\
inse. Futures are denominated in euros, \\
so it is better to state that the risk is that the value of \\
the euro will fall.
\end{tabular} \\
& \begin{tabular}{l} 
Work out a futures position that \\
creates a profit if there is a 'loss' on \\
the underlying transaction.
\end{tabular} & \begin{tabular}{l} 
if the euro falls in value, the futures position should \\
create a profit if the value of the euro falls. \\
So sell euro currency futures, since a profit will be \\
made if the value of the euro falls and the futures
\end{tabular} \\
& \begin{tabular}{l} 
position can be closed by buying euro futures at a \\
lower price than the futures were originally sold.
\end{tabular} \\
\hline
\end{tabular}

The company will sell futures to create a 'short position' to hedge the currency risk.
- The equivalent value of \(\$ 1,200,000\) at the current futures price is 986,031 euros (\$1,200,000/1.2170).
- The number of contracts required is therefore 7.9 contracts ( \(€ 986,031 / € 125,000\) per contract).
- The company will probably create the hedge with 8 contracts (by selling 8 December futures).

In this example, the basis is 30 points in July (1.2200-1.2170). The December futures contract reaches settlement in five months' time, therefore basis should fall by 6 points each month ( 30 basis points \(/ 5\) months). By the end of November, four months later, the basis should therefore have fallen by 24 points, from 30 to 6 .

Suppose that in November when the dollars are paid, the spot rate has moved to 1.2040 and the futures price is 1.2030 . (The actual basis is 10 .) The company will close its futures position.
\begin{tabular}{ll} 
Open futures position: sell at & 1.2170 \\
Close position: buy at & 1.2030 \\
& \\
\hline
\end{tabular}

Gain \(=140\) ticks per contract at \(\$ 12.50\) per tick.

Total gain on futures position \(=8\) contracts \(\times 140\) ticks \(\times \$ 12.50=\$ 14,000\).
\begin{tabular}{lr} 
& \(\$\) \\
\hline Payment to customer & \(1,200,000\) \\
Gain on futures & \(14,000)\) \\
\cline { 2 - 2 } Dollars to be purchased spot & \(1,186,000\) \\
\cline { 2 - 2 } Buy dollars at 1.2040 spot & \\
Cost in euros & \(€ 985,050\)
\end{tabular}

Effective exchange rate secured by futures hedge \(=\) US\$1,200,000/€985,050 = US\$1.2182/€1.

This is fairly close to the spot rate and futures price at the time the position was opened. In this example, the expected basis in November was 6 points and the actual basis was 10 points, and the gain on the futures position was therefore 4 points or ticks per contract more than expected. This has affected the value of the hedge by 8 contracts \(\times 4\) ticks \(\times \$ 12.50=\$ 400\).

\section*{Short-term interest rate futures (STIRs)}
- Features of short-term interest rate futures
- Hedging short-term interest rate exposures with STIRs
- Hedging examples

\section*{8 Short-term interest rate futures (STIRs)}

\subsection*{8.1 Features of short-term interest rate futures}

A short-term interest rate future (STIR) is a contract for the purchase and sale of a notional deposit, usually a three-month bank deposit. An examination question should give you the contract size for any STIR and the length of the deposit period. The most commonly-traded STIRs include the following:
\begin{tabular}{lrr} 
Contract & \begin{tabular}{r} 
Underlying amount \\
= three-month deposit of:
\end{tabular} & \begin{tabular}{r} 
Tick value \\
\(\mathbf{( 0 . 0 0 0 1}\) of price)
\end{tabular} \\
\hline Eurodollar & US \(\$ 1,000,000\) & \(\$ 25\) \\
Euribor & \(€ 1,000,000\) & \(€ 25\) \\
Short sterling & \(£ 500,000\) & \(£ 12.50\)
\end{tabular}

\section*{Prices}

The futures price for STIRs is the annual interest rate. However, the rate is deducted from 100, which means that:
- A rate of \(4 \%\) per year is indicated by a futures price of \(96.0000(100-4)\)
- A rate of \(5.2175 \%\) is indicated by a futures price of 94.7825
- A price of 93.5618 represents an annual interest rate for the three-month deposit of \(6.4382 \%\).

A reason for pricing STIRs in this way is that:
- when interest rates go up, the value of a future will fall, and
- when interest rates fall, the price of the future will rise.

In this way, prices for STIRs move in the same way as cash market prices for bonds and other interest products.

\subsection*{8.2 Hedging short-term interest rate exposures with STIRs}

STIRs can be used to hedge exposures to the risk of a rise or fall in short-term interest rates. Using short-term interest rate futures is similar to using currency futures to hedge a currency exposure. However, the following rules need to be applied.
- If the aim is to hedge against the risk of an increase in the short-term interest rate, the hedge is created by selling futures. If the interest rate does go up, futures prices will fall, and there will be a profit on the short position in STIRs
- If the aim is to hedge the risk of a fall in the short-term interest rate, the hedge is created by buying futures.
Short-term interest rate futures are futures for three-month deposits. If a company wishes to hedge an interest rate risk for a different interest period, such as two months, four months or six months, the number of futures to create the hedge should be adjusted by a factor: (Interest period to be hedged/3 months).

\section*{Example 1: number of futures for the hedge}

It is now the end of October. A company expects to borrow \(\$ 5\) million for six months from February, in four months' time and is concerned about the risk of a rise in the eurodollar interest rate. It decides to hedge the exposure with eurodollar futures. Each eurodollar future is for a three-month deposit of \(\$ 1,000,000\).

The company will hedge the position by selling March futures. The number of contracts required for the hedge for a six-month loan is:
\(\frac{\$ 5,000,000}{\$ 1,000,000 \text { per contract }} \times \frac{6 \text { months }}{3 \text { months }}=10\) contracts
If changes in the three-month interest rate and changes in the six-month interest rate are similar during the period that the hedge is in place, hedging with three-month STIRs will be an effective method of hedging exposures to movements in the sixmonth interest rate.

\section*{Example 2: number of futures for the hedge}

It is now the end of July. A company expects to borrow \(£ 10.5\) million for two months from the end of October, in three months' time and is concerned about the risk of a rise in the sterling interest rate. It decides to hedge the exposure with sterling futures. Each future is for a three-month deposit of \(£ 500,000\).

The company will hedge the position by selling November futures. The number of contracts required for the hedge for a two-month loan is:
\(\frac{\$ 10,500,000}{£ 500,000 \text { percontract }} \times \frac{2 \text { months }}{3 \text { months }}=14\) contracts

\subsection*{8.3 Hedging examples}

Example: perfect hedge, no basis
A company will need to borrow 8 million euros from the end of May. It is now January. The company is concerned about the risk of a rise in the euribor rate (the benchmark interest rate for the euro) and it wishes to hedge its position with
futures. The current spot euribor rate is \(3.50 \%\) (for both three months and six months) and the current June euribor futures price is the same, 96.50 .

The value of 1 tick for a euribor futures contract is \(€ 25(€ 1,000,000 \times 0.0001 \times 3 / 12)\).

\section*{Required}
(a) How should the company hedge its interest rate exposure if it plans to borrow the 8 million euros for (1) three months or (2) six months.
(b) Suppose that in May when the company borrows the 8 million euros, the three-month and six-month spot euribor rate is \(4.25 \%\) and the June futures price is the same, 95.75 (100-4.25). Calculate the effective annual interest rate that the company has secured with its futures hedge if it borrows the 8 million euros for (1) three months or (2) six months.

\section*{Answer}

The exposure is to the risk of a rise in the euribor rate. Therefore the company should:
- sell 8 June euribor futures ( \(€ 8,000,000 / € 1,000,000\) per contract) if it is hedging a three-month loan exposure, and
- sell 16 June contracts ( 8 contracts \(\times 6\) months \(/ 3\) months) if it is hedging a sixmonth loan exposure.

In May, the futures position will be closed. The selling price is 95.75 .
\begin{tabular}{lr} 
Open futures position: sell at & 96.50 \\
Close position: buy at & 95.75 \\
Gain & \(\underline{00.75}\) \\
\hline
\end{tabular}

Gain \(=75\) ticks per contract at \(€ 25\) per tick.
Total gain on futures position:
(a) Hedging the three-month rate \(=8\) contracts \(\times 75\) ticks \(\times € 25=€ 15,000\).
(b) Hedging the six-month rate \(=16\) contracts \(\times 75\) ticks \(\times € 25=€ 30,000\).

The company will borrow 8 million euros at \(4.25 \%\).
\begin{tabular}{lrr}
\multicolumn{4}{c}{ (a) Hedging the three- } \\
month rate
\end{tabular} \begin{tabular}{r} 
(b) Hedging the \\
six-month rate
\end{tabular}

The net effective cost can be converted into an effective annual interest rate that has been achieved by the hedge with futures.
(a) Net effective interest rate for hedge of three-month rate \(=\)
\[
\left(\frac{70,000}{8,000,000}\right) \times \frac{12}{3}=0.035 \text { or } 3.50 \% .
\]
(b) Net effective interest rate for hedge of six-month rate \(=\)
\[
\left(\frac{140,000}{8,000,000}\right) \times \frac{12}{6}=0.035 \text { or } 3.50 \% .
\]

Given a perfect hedge and no basis, the hedge fixes the effective interest rate at the interest rate when the futures position was opened.

\section*{Example: imperfect hedge and no basis}

A UK company will need to borrow \(£ 4.75\) million for three months from the beginning of September. It is now April. The company is concerned about the risk of a rise in the LIBOR rate and wishes to hedge its position with futures. The current spot three-month LIBOR \(5.45 \%\) and the current futures price is the same, 94.55 .

\section*{Required}

How should a hedge for the interest rate exposure be created, and what will be the effective interest rate for the loan from September if the spot LIBOR rate is \(5.14 \%\) in early September and the September futures price is the same, 94.86 ?

The value of one tick for a 'short sterling' future is \(£ 12.50\) ( \(£ 500,000 \times 0.0001 \times 3 / 12\) ).

\section*{a \\ Answer}

The exposure is to the risk of a rise in the LIBOR rate; therefore the company should sell September short sterling futures. The number of contracts to sell is 9.5 contracts ( \(£ 4.75\) million/ \(£ 500,000\) per contract).. The company should sell either 9 or 10 futures, but the hedge will be imperfect. In this example, the company sells 10 September contracts.

The selling price in April is 94.55 . In early September, the futures position will be closed.
\begin{tabular}{lr} 
Open futures position: sell at & 94.55 \\
Close position: buy at & 94.86 \\
& \(\underline{00.31}\) \\
\hline
\end{tabular}

Loss \(=31\) ticks per contract at \(£ 12.50\) per tick.
Total loss on futures position \(=10\) contracts \(\times 31\) ticks \(\times £ 12.50=£ 3,875\)

The company will borrow \(£ 4.75\) million for three months at \(5.14 \%\).
\begin{tabular}{lr} 
& \(£\) \\
\hline Interest cost: \(£ 4.75\) million \(\times 3 / 12 \times 5.14 \%\) & \(61,037.50\) \\
ADD: loss on futures position & \(3,875.00\) \\
\hline Total effective cost & \(64,912.50\) \\
\hline
\end{tabular}

Effective interest rate for hedge of three-month rate
\(=\left(\frac{64,912.50}{4,750,000}\right) \times \frac{12}{3}\)
\(=0.0547\) or \(5.47 \%\)

Given an imperfect hedge and no basis, the hedge fixes the effective interest rate at a rate close to the interest rate when the futures position was opened ( \(5.45 \%\) ).

\section*{Example: perfect hedge and basis}

A company will need to borrow US \(\$ 20\) million for three months from the end of October. It is now the end of June. The company is concerned about the risk of a rise in the US\$ LIBOR rate and wishes to hedge its position with futures. The current spot three-month US\$ LIBOR is \(4.30 \%\) and the current December eurodollar futures price is 96.30 .

The value of 1 tick for a Eurodollar future is \(\$ 25\).

\section*{Required}

How should a hedge for the interest rate exposure be created, and what will be the effective interest rate for the loan from October if the spot US\$ LIBOR rate is \(4.10 \%\) and the December futures price at this date is 96.06 ?

Answer

The exposure is to the risk of a rise in the US\$ LIBOR rate; therefore the company should sell December Eurodollar futures. The number of contracts to sell is \$20 million \(/ \$ 1,000,000\) per contract \(=20\) contracts.

The selling price at the end of June is 96.30 . The current spot three-month interest rate is \(4.30 \%\), equivalent to 95.70 . The basis is therefore 60 points. This should be expected to reduce in size by 10 points each month in the six months between the end of June and the end of December when the futures reach their settlement date. By the end of October when the futures position is closed 4 months later, the expected basis should be 20 points. (This is 60 points minus ( 4 months \(\times 10\) points per month.)

At the end of October, the futures position will be closed.
\begin{tabular}{lr} 
Open futures position: sell at & 96.30 \\
Close position: buy at & 96.06 \\
& \begin{tabular}{l}
00.24 \\
\hline
\end{tabular}
\end{tabular}

Gain \(=24\) ticks per contract at \(\$ 25\) per tick.
Total gain on futures position \(=20\) contracts \(\times 24\) ticks \(\times \$ 25=\$ 12,000\).
The company will borrow \(\$ 20\) million for three months at \(4.10 \%\).
\begin{tabular}{lr} 
& \(\$\) \\
\hline Interest cost: \(\$ 20\) million \(\times 3 / 12 \times 4.10 \%\) & 205,000 \\
Less: Gain on futures position & \((12,000)\) \\
\cline { 2 - 2 } Net effective cost & 193,000 \\
\hline
\end{tabular}

Effective interest rate for hedge of three-month rate \(=\left(\frac{193,000}{20 \text { million }}\right) \times \frac{12}{3}\) \(=0.0386\) or \(3.86 \%\).

At the end of October, the actual basis is 16 points, because the spot three-month rate is \(4.10 \%\) ( 95.90 ) and the futures price is 96.06 . ( \(96.06-95.90=0.16\) or 16 points).

The value of 16 points or ticks on the futures position would be 20 contracts \(\times 16\) ticks \(\times \$ 25=\$ 8,000\). This represents an effective interest rate on borrowing \(\$ 20\) million for three months of \([(\$ 8,000 / \$ 20\) million \() \times(12 / 3)=] 0.0016\) or \(0.16 \%\). This is the difference between the effective interest rate secured by the futures position and the interest rate in the futures contract when the position was opened in June (3.70\%).

\section*{Conclusion}

The effectiveness of a hedge is reduced when the hedge is imperfect and because of basis and basis risk.

\section*{Example: perfect hedge and basis}

A company will need to borrow \(£ 60\) million for four months from the end of April. It is now the end of November. The company is concerned about the risk of a rise in the LIBOR rate and wishes to hedge its position with futures. The current spot three-month LIBOR is \(5.50 \%\) and the current June sterling interest rate futures price is 94.85 . The company is able to borrow at LIBOR \(+0.75 \%\).

One tick is \(0.01 \%\) and the value of a tick is \(£ 6.25\). The nominal three-month deposit in a sterling futures contract is \(£ 500,000\).

\section*{Required}
(a) How should a hedge for the interest rate exposure be created?
(b) What will be the expected effective interest rate for the loan from the end of April? The expected basis at the close-out date should be estimated, but basis risk should be ignored for the purpose of this example.
(c) What will be the actual effective cost of borrowing the \(£ 60\) million at the end of April if LIBOR at that time is \(5.75 \%\) ?

\section*{Answer}
(a) The exposure is to the risk of a rise in the LIBOR rate; therefore the company should sell June futures. The number of contracts to sell is:
\(\frac{£ 60 \text { million }}{£ 500,000 \text { per contract }} \times \frac{4 \text { months }}{3 \text { months }}=160\) contracts

The company should sell 160 June STIRs at 94.85, which 'fixes' the interest rate at \(5.15 \%\) for the end of June.
(b)

Basis at the end of November is the difference between the futures price (94.85) and the LIBOR spot rate ( \(5.5 \%\) or 94.50 ). Basis is therefore 35 points ( \(94.85-94.50\) ).

There are seven months from the end of November to settlement of the futures contract at the end of June, so basis is expected to fall at the rate of 5 basis points (0.05) per month. By the end of April when the futures position will be closed, expected basis is 0.10 ( 2 months \(\times 0.05\) ) and the futures price is expected to be higher by the spot rate by this amount.

Selling the futures contracts therefore 'fixes' the LIBOR interest rate at a rate close to \(5.25 \%\) for the end of April (the \(5.15 \%\) in the futures price plus the expected basis of \(0.10 \%\) ).

Since the company can borrow at LIBOR \(+0.75 \%\) the total effective borrowing rate for the four months that the company hopes to 'lock in' with the futures - given no basis risk - is \(5.25 \%+0.75 \%=6.00 \%\).
(c)

If the spot LIBOR rate at the end of April is \(5.75 \%\) or 94.25 , the expected futures price will be 94.35 ( 10 basis points higher, given no basis risk).

The futures position will be closed as follows:
\begin{tabular}{lr} 
November: sell at & \(\%\) \\
April: buy to close at & 94.85 \\
Profit & 94.35 \\
\hline
\end{tabular}

Profit per contract \(=(0.50 / 0.005\) per tick \() \times £ 6.25\) per tick \(=£ 625\).
Profit on 160 contracts \(=£ 625 \times 160=£ 100,000\).

The company will borrow \(£ 60\) million for 4 months at an interest rate of \(6.50 \%\) (LIBOR + 0.75\%).

The interest cost of the loan will be \(£ 60\) million \(\times 6.50 \% \times 4 / 12=£ 1,300,000\).
\begin{tabular}{lr} 
& \(£\) \\
Interest cost of loan & \(1,300,000\) \\
Gain on futures position & \((100,000)\) \\
\hline Net borrowing cost & \(1,200,000\) \\
\hline
\end{tabular}

The effective net interest cost is \(£ 1,290,000\). On a loan of \(£ 60\) million for four months, this represents an effective interest rate of:
\(\frac{£ 1,200,000}{£ 60 \text { million }} \times \frac{12}{4}=6.00 \%\)

This actual effective borrowing rate of \(6.00 \%\) is the same as the expected effective borrowing rate calculated in part (b).

Hedging with bond futures and stock index futures
- Hedging with bond futures
- Hedging with stock index futures

\section*{\(9 \quad\) Hedging with bond futures and stock index futures}

\subsection*{9.1 Hedging with bond futures}

Bond futures can be used to hedge against the risk of a change in long-term interest rates (bond yields) and bond prices. For example, suppose that an investment institution is holding \(\$ 200\) million of US Treasury bonds, and it wants to protect this bond portfolio against any rise in bond yields and fall in bond market prices to the end of December.

It can do this by selling December bond futures for US Treasuries. If the interest yield goes up before the end of December:
- bond prices in the bond markets will fall
- however, the institution will make an offsetting gain on its position in bond futures.

The net effect should be to protect the overall value of the institution's investment portfolio.

\subsection*{9.2 Hedging with stock index futures}

Stock index futures can be used to hedge against the risk of a rise or fall in a stock market index. Typically, an investment institution might hold a large portfolio of equity investments that are closely represented by a particular stock market index. The institution can use futures to hedge against the risk of a fall in the index over the next few months. It can do this by selling stock index futures.

\section*{Example: perfect hedge and basis}

An investment institution holds a portfolio of shares in UK companies, and the portfolio is currently valued at \(£ 10\) million. The shares in the portfolio are closely represented by the composition of the FTSE100 stock market index.

The institution can hedge against the risk of a fall in the stock market using stock index futures. For the FTSE100 futures contract, one point of the index has a tick value of \(£ 10\). The institution should sell futures to hedge its position, and the number of contracts it should sell is:

Current value of the portfolio
Futures price \(\times\) Tick value

For example, if the futures price for the FTSE100 index is 6000 , to hedge a portfolio worth \(£ 15\) million, the institution would sell \(£ 15\) million \(/(6000 \times £ 10)=250\) futures. If the actual index fell by 1 point to 5999 , the value of the portfolio would fall by \(1 / 6,000\) or \(£ 2,500\). There would be an offsetting gain on the futures position of 250 contracts \(\times 1\) tick \(\times £ 10\) per tick \(=£ 2,500\).

The position has been successfully hedged.

\section*{Options and hedging with options}

\section*{Contents}

1 Features of options
2 Buying options and selling (writing) options
3 Exercising options
4 Factors influencing option values
5 Hedging with options
6 Currency options
7 Interest rate options
8 Caps, floors and collars

\section*{Features of options}
- Definition of an option
- Financial options, commodity options and real options
- Call options and put options
- Expiry date: American-style and European-style options
- OTC and exchange-traded options

\section*{1 Features of options}

\subsection*{1.1 Definition of an option}

An option is something that gives its holder the right, but not the obligation, to take a particular course of action at sometime in the future. Typically, an option gives its holder the right, but not the obligation, either to buy or to sell a quantity of a particular item on or before a specified date in the future, at a price that is fixed in the contract.

\subsection*{1.2 Financial options, commodity options and real options}

There are different types of option.
- A financial option is a contract that gives its holder the right, but not the obligation, either to buy or to sell a quantity of a financial item on or before a specified date in the future, at a price that is fixed in the contract. There are financial options in currencies, interest rates, share prices and stock index values. For example, an option in shares of company Z might give its holder the right to sell 1,000 shares in company Z on 31st March at a price of \(\$ 10\) per share.
- With a commodity option, the option holder has the right to buy or sell a quantity of a specified commodity, such as a quantity of wheat, or a quantity of a metal such as gold or copper.

Real options are somewhat different. Real options relate to the choices facing a company when it is considering whether to invest in a new capital project. The company has the following choices (which are not obligations):
- to make a further investment or additional investment if the project is a success
- to abandon the investment after it has been started, if it now appears that it will not be successful
- to wait before investing, instead of investing immediately.

These types of option are called real options, which were described in an earlier chapter.

\subsection*{1.3 Call options and put options}

Financial options are either call options or put options.
- A call option gives its holder the right to purchase the underlying item in the option agreement.
- A put option gives its holder the right to sell the underlying item in the option agreement.

For example, a call option on 1,000 shares in company Z gives its holder the right to buy 1,000 shares in company Z at the price agreed in the option contract, and a put option on 1,000 shares in company Z would give its holder the right to sell 1,000 shares in company Z at the agreed price.

\subsection*{1.4 Expiry date: American-style and European-style options}

An option agreement has an expiry date, after which the option lapses and the agreement comes to an end.
- An American-style option can be exercised by its holder at any time on or before the expiry date.
- A European-style option can be exercised only at the expiry date for the option and not before.

The terms 'American' and 'European' do not refer to the countries where these types of option are available. Both types of option agreement are made throughout the world.

For example, if a company holds an American-style call option to buy US \(\$ 500,000\) in exchange for euros at a rate of 1.2000 (US\$/€1) with an expiry date of 20 September, the company can exercise its right to buy the \(\$ 500,000\) at the agreed rate at any time up to and including 20th September. However, if the option is not exercised by that date, it will lapse (cease to exist).

\subsection*{1.5 OTC and exchange-traded options}

Some financial options are arranged directly between buyer and seller. Directlynegotiated options are called over-the-counter options or OTC options. Examples of OTC options include borrowers' and lenders' options, caps, floors and collars. Currency options might also be arranged in OTC agreements.

Some options are traded on an exchange. Traded share options and some currency options are exchange-traded. In addition, there are options on futures contracts, and all options on futures are traded on the futures exchange where the underlying futures are traded.

\section*{Buying options and selling (writing) options}
- Exercise price or strike price
- Rights and obligations of buyer and seller
- Option premium = option price
- Options: a zero-sum game

\section*{2 Buying options and selling (writing) options}

\subsection*{2.1 Exercise price or strike price}

The exercise price for an option is the price at which the holder can:
- buy the underlying item, in the case of a call option, or
- sell the underlying item, in the case of a put option.

With OTC options, the exercise price is agreed between the option buyer and the option seller. With exchange-traded options, options are available for buying or selling at a limited range of fixed strike prices, and buyers and sellers agree on the price at which they will make a transaction in the options at one of these prices.

For example, the following table shows exercise prices that might be available on the CME exchange for US dollar/euro currency options on a day in August, and the prices of the most recent transactions in those options.
\begin{tabular}{lcrcrrr} 
& \multicolumn{3}{c}{ CALLS } & \multicolumn{3}{c}{ PUTS } \\
\begin{tabular}{l} 
Expiry: \\
Strike price:
\end{tabular} & Sep & Dec & Mar & Sep & Dec & Mar \\
\hline 12200 & 0.34 & 2.03 & 3.22 & 1.62 & 3.15 & 3.91 \\
12300 & 0.18 & 1.62 & 2.81 & 2.16 & 3.53 & 4.49 \\
12400 & 0.09 & 1.33 & 2.44 & 3.06 & 4.20 & 5.12 \\
12500 & 0.04 & 1.18 & 2.11 & 3.66 & 4.90 & -
\end{tabular}

\section*{Notes}
(a) A strike price of 12200 indicates an exchange rate of \(€ 1=\$ 1.2200\).
(b) At this date, currency options were being traded at just four strike prices, for expiry dates in September, December and March.

\subsection*{2.2 Rights and obligations of buyer and seller}

Options are bought and sold. The seller of an OTC option is often called the option writer. Selling an OTC option is often called 'writing an option'. For exchangetraded options, it is more usual to refer to 'sellers' of options, who have a short position in the options.
- The option buyer or option holder has the right to exercise the option but is not obliged or contractually required to do so.
- On the other hand, the seller or writer of the option is contractually obliged to sell or buy the underlying item if the option is exercised by its holder.

\subsection*{2.3 Option premium = option price}

Options are bought and sold at a price, which is called the option premium. This is paid by the buyer of the option to the option seller/writer when the option agreement is made. The option writer therefore receives the premium no matter whether the option is subsequently exercised or not.

The table of currency options above shows current prices for US dollar/euro traded currency options. For example, September call options in \(\$ / €\) were being traded at a price of 0.34 . In this particular example, this premium price is stated in US cents per euro. Each traded option on the CME exchange is for 125,000 euros in exchange for US dollars, and the cost of one September call option was therefore \(\$ 425(125,000 \times\) \(\$ 0.0034\) ).

Similarly, the premium for a December put option at a strike price of \(\$ 1.25\) was \(\$ 6,125(125,000 \times \$ 0.0490)\).

\subsection*{2.4 Options: a zero-sum game}

Options can be described as a zero-sum game between the option buyer and the option seller. Any gain for the buyer is matched by an equal loss for the seller. Similarly, any gain for the seller is matched by an equal loss for the buyer. The combined sum of their profits and losses is always zero.

\section*{Example}

A company buys a European-style call option on 1,000,000 euros at a strike price of 1.2500 (US\$/€1). The option premium is 2.4 US cents per euro, therefore the cost of the option is \(\$ 24,000\).

What would be the net gain or loss for the option buyer and the option writer if the exchange rate at expiry is:
(a) \(\$ 1.2350=€ 1\),
(b) \(\$ 1.2670=€ 1\), and
(c) \(\$ 1.2930=€ 1\).

\section*{Answer}
(a) The option will not be exercised, because it is cheaper to buy the 1 million euros at the spot market price of \(\$ 1.2350\). The option will lapse.
(b) The option will be exercised, because it is cheaper to buy euros at the strike price of \(\$ 1.2500\) than the spot rate of \(\$ 1.2670\).
(c) The option will be exercised because it is cheaper to buy euros at the strike price of \(\$ 1.2500\) than the spot rate of \(\$ 1.2930\).
\begin{tabular}{|c|c|c|c|}
\hline Option buyer & (a) spot price 1.2350 & (b) spot price & (c) spot price 1.2930 \\
\hline & & \$ & \$ \\
\hline Spot price & Not & 1.2670 & 1.2930 \\
\hline Exercise price & exercised & 1.2500 & 1.2500 \\
\hline Gain from exercising option (per €1) & & 0.0170 & 0.0430 \\
\hline & \$ & \$ & \$ \\
\hline Total gain on option ( \(€ 1,000,000\) ) & 0 & 17,000 & 43,000 \\
\hline Cost of option & \((24,000)\) & \((24,000)\) & \((24,000)\) \\
\hline (Loss)/gain to the buyer & \((24,000)\) & \((7,000)\) & 19,000 \\
\hline
\end{tabular}

The option writer would make a gain of \(\$ 24,000\) if the spot rate at expiry is \(\$ 1.2350\), a net gain of \(\$ 7,000\) if the spot rate is 1.2670 and a net loss if the spot rate is 1.2930 .

\section*{Exercising options}
- In-the-money, out-of-the-money and at-the-money options
- Exercising call options
- Exercising put options

\section*{3 Exercising options}

\subsection*{3.1 In-the-money, out-of-the-money and at-the-money options}

Options are in-the-money, at-the-money or out-of-the-money.
- An option is in-the-money when its exercise price (strike price) is more favourable to the option holder than the current market price of the underlying item.
- An option is at-the-money when its exercise price (strike price) is exactly equal to the current market price of the underlying item.
- An option is out-of-the-money when its exercise price (strike price) is less favourable to the option holder than the current market price of the underlying item.

When an option is first written, it might be in-the-money, at-the-money or out-of-the-money.
- An in-the-money option is worth more than an out-of-the-money option, and so it has a higher premium.
- An at-the-money option or an out-of-the-money option should still have some value, because there is some possibility that the market price of the underlying item will move in the option holder's favour before the expiry date, and so there is a possibility that the option will be in-the-money at expiry.

However, an option that is deeply out-of-the-money, with an exercise price that is very much worse than the current market price of the underlying item (from the option holder's point of view) is most unlikely to be exercised, and so will have a value of 0 or close to 0 .

\section*{An option will only be exercised if it is in-the-money.}

When an option is exercised, the value of the option is the difference between the exercise price and the current market price of the underlying item.

\subsection*{3.2 Exercising call options}

A call option will only be exercised if the market price of the underlying item is higher than the exercise price for the option. For example, a European call option on 1,000 shares in company Z with a strike price of \(\$ 10\) per share will only be exercised if the market price of the share is above \(\$ 10\) at expiry.

\subsection*{3.3 Exercising put options}

Similarly, a put option will only be exercised if the market price of the underlying item is lower than the exercise price for the option. For example, a European put option on 2,000 shares in company \(X Y\) with a strike price of \(\$ 8\) per share will only be exercised if the market price of the share is below \(\$ 8\) at expiry.

\section*{Hedging with options}
- Main features of hedging with options
- Creating a hedge

\section*{4 Hedging with options}

\subsection*{4.1 Main features of hedging with options}

Financial options can be used to hedge exposures to the risk of adverse movements in exchange rates, interest rates, bond prices and share prices. Hedging with options differs from hedging with forward contracts, money market hedges, FRAs and futures, in several important ways.
- The hedge has a cost. A hedge should normally be created by buying options rather than selling/writing options, and the option buyer must pay the premium to obtain the options to create the hedge.
- An option does not have to be exercised. It will only be exercised if it is in-themoney. If it is out-of-the-money, the option holder will let the option lapse and buy or sell the underlying item in the cash market, at the more favourable market price. This means that an option holder can use the options as a protection against adverse movements in the market rate, but can take advantage of any favourable movement in the market rate.

\section*{Example}

An investor holds 5,000 shares in another company, XYZ , which have a current market price of \(\$ 6.00\). The investor will want to sell the shares in a few months' time, in April, but not before then. He is concerned that the share price might fall between now and April, but is also aware of the possibility that it could rise.

The investor can hedge the exposure to market risk (share price risk) by purchasing a put option on 5,000 shares in \(X Y Z\), for expiry in April. Suppose the strike price of the option is \(\$ 6.00\), so that the option is at-the-money when written.
- If the share price falls below \(\$ 6.00\) before April, say to \(\$ 5.50\), the investor can exercise the option and sell at the strike price of \(\$ 6.00\).
- On the other hand, if the share price rises above \(\$ 6.00\), say to \(\$ 7.00\), he can let the option lapse and sell the shares at the market price of \(\$ 7.00\).

\subsection*{4.2 Creating a hedge}

The rule for creating a hedge with options is as follows:
- To hedge against the risk of a rise in the price of the underlying item, buy call options
- To hedge against the risk of a fall in the price of the underlying item, buy put options

\section*{Currency options}
- Features of currency options
- Hedging with currency options
- Gains or losses on options and the effective exchange rate
- Imperfect hedges

\section*{5 Currency options}

\subsection*{5.1 Features of currency options}

The main features of currency options have already been described.
- A currency option gives its holder the right to buy (call option) or sell (put option) a quantity of one currency in exchange for another, on or before a specified date, at a fixed rate of exchange (the strike rate for the option).
- Currency options can be purchased over-the-counter or on an exchange. Examination questions are more likely to feature exchange-traded options than OTC options. Currency options are traded on some exchanges, notably the Philadelphia Stock Exchange, and options on currency futures are traded on the CME exchange in Chicago.
- Traded currency options are for a standard quantity of one currency in exchange for another currency, and strike prices are quoted as exchange rates. The premiums are normally quoted as an amount in one currency per unit of the other currency. For example, traded options on currency futures for US\$ - \(£\) are for \(£ 62,500\) and are priced in US cents per \(£ 1\).

\subsection*{5.2 Hedging with currency options}

The rules for using exchange-traded currency options to hedge an exposure to currency risk are as follows:
- Establish what the exposure is. The company will have a future intention or obligation to buy or sell a quantity of currency in exchange for another. Establish whether the currency will be bought or sold, how much will be bought or sold, and when.
- Decide whether a call option or a put option is required to hedge the exposure.
- Decide on the strike price for the option. For exchange-traded options, there will normally be a small range of strike prices to choose from.
- Calculate how many option contracts must be purchased to create the hedge.

\section*{Example}

A US company expects to pay 1 million euros to a supplier in Belgium. It is now November and the payment is due in March. The company wants to use currency options to hedge the exposure. Each currency option is for 125,000 euros.

The company will need to buy euros to make the payment to the supplier; therefore it wants to hedge against the risk of a rise in the value of the euro (= a fall in the value of the dollar). The company should therefore buy call options.

We shall suppose that the company chooses a strike price of 1.2400 (US\$/ \(€ 1\) ) for the options, and that the premium for a March call option at this strike price is 3.43 US cents per euro.

The company should buy 8 call option contracts ( \(€ 1,000,000 / € 125,000\) per contract). The cost of the premium will be \(\$ 34,300(1,000,000 \times \$ 0.0343)\).

\subsection*{5.3 Gains or losses on options and the effective exchange rate}

The effective exchange rate that is obtained from a hedge with options on currency futures depends on what happens when the settlement date arrives, and whether the option is exercised or not.

The following example illustrates this point:

\section*{Example}

Continuing the previous example, a US company expects to pay \(€ 1\) million at the end of March and buys 8 call options on March euro currency futures at a strike price of 1.2400 .
We can look at the position of the US company if the spot exchange rate (US\$/€1) in March is:
(a) \(\$ 1.2000\)
(b) \(\$ 1.2200\)
(c) \(\$ 1.2500\)
(d) \(\$ 1.2800\)
(e) \(\$ 1.3000\)

Answer

The call option will be exercised if the futures price at the end of March is over 1.2400. At the end of March when the futures contract reaches settlement, the futures settlement price should be the current spot exchange rate.
\begin{tabular}{|c|c|c|c|c|c|}
\hline \multirow[t]{3}{*}{} & \multicolumn{5}{|c|}{Spot rate at expiry} \\
\hline & \multirow[t]{2}{*}{\[
\begin{array}{r}
1.2000 \\
\text { Do not } \\
\text { exercise }
\end{array}
\]} & 1.2200 & \multirow[t]{2}{*}{\[
\begin{array}{r}
1.2500 \\
\text { Exercise }
\end{array}
\]} & \multirow[t]{2}{*}{\[
\begin{array}{r}
1.2800 \\
\text { Exercise }
\end{array}
\]} & \multirow[t]{2}{*}{\[
\begin{array}{r}
1.3000 \\
\text { Exercise }
\end{array}
\]} \\
\hline & & Do not exercise & & & \\
\hline & \$ & \$ & \$ & \$ & \$ \\
\hline Cost of buying & 1,200,000 & 1,220,000 & 1,240,000 & 1,240,000 & 1,240,000 \\
\hline €1 million & & & & & \\
\hline Cost of premium & 34,300 & 34,300 & 34,300 & 34,300 & 34,300 \\
\hline Total cost & 1,234,300 & 1,254,300 & 1,274,300 & 1,274,300 & 1,274,300 \\
\hline Effective exchange rate & \$1.2343 & \$1.2543 & \$1.2743 & \$1.2743 & \$1.2743 \\
\hline
\end{tabular}

The hedge has enabled the company to fix a maximum effective exchange rate of \(\$ 1.2743\) for buying the euros, but at the same time allowing the company to obtain a lower effective exchange rate if the spot price is below the strike price, and the option is not exercised.

\subsection*{5.4 Imperfect hedges}

When a hedge is created with exchange-traded currency options, the hedge is often imperfect, for reasons similar to those already explained for hedging with futures.
When a hedge with exchange-traded currency options is imperfect, and the options are exercised, the un-hedged or over-hedged amount of currency should be bought for sold in the spot currency market, at the spot rate, at the same time that the options are exercised.

\section*{Example: imperfect hedge}

A company in Belgium expects to pay US\$2 million to a supplier in Arabia. It is now November and the payment is due in March. The current spot rate is 1.2100 . The company wants to use currency options to hedge the exposure. Each currency option is for 125,000 euros and the value of 1 tick ( 0.0001 ) is \(\$ 12.50\).

The company will need to buy US dollars and sell euros to make the payment to the supplier; therefore it wants to hedge against the risk of a rise in the value of the dollar and a fall in the value of the euro. Since the contracts are for euros, the company should therefore buy put options.

We shall suppose that the company chooses a strike price of \(\$ 1.2200=€ 1\) for the options, and that the premium for a March put option at this strike price is 2.75 US cents per euro.
- At a strike price of \(\$ 1.2200, \$ 2,000,000\) is the equivalent of \(1,639,344\) euros.
- To hedge \(1,639,344\) euros, the company needs to buy 13.1 put options \((1,639,344\) euros/125,000 euros per contract).

The company will probably buy 13 put options. The cost of the premium will be 13 contracts \(\times 125,000 \times \$ 0.0275=\$ 44,687.50\). The company will buy these dollars spot at \(\$ 1.2100\), and so the cost of buying the options (in euros) will be \(€ 36,931.82\).

\section*{(a) The spot exchange rate moves in the option holder's favour}

Suppose that the exchange rate moves in the time to expiry such that the company does not want to exercise the options to sell futures at 1.2200 because the euro is worth more than 1.2200. In this example, suppose that the spot rate in March when the dollars must be paid is \(\$ 1.2500=€ 1\). The company will let the option lapse and purchase the dollars spot at \(\$ 1.2500\). The effective exchange rate is as follows:
\begin{tabular}{lr} 
& \(€\) \\
\hline Purchase \(\$ 2,000,000\) at \(\$ 1.2500\) & \(1,600,000.00\) \\
Cost of option premium & \(36,931.82\) \\
\cline { 2 - 2 } Total cost & \(1,636,931.82\) \\
\cline { 2 - 2 } Effective exchange rate for \(\$ 2\) million & \(\$ 1.2218=€ 1\) \\
\((\$ 2\) million/€1,636,931.82) &
\end{tabular}

\section*{(b) The spot exchange rate moves adversely}

Suppose that the exchange rate moves in the time to expiry such that the company wants to exercise the options. In this example, suppose that the spot rate in March when the dollars must be paid is \(\$ 1.1800=€ 1\). The company will exercise the option to sell the 13 futures contracts at \(\$ 1.2200\).
\begin{tabular}{lr} 
& \(\$\) \\
\hline November: sell options at & 1.2200 \\
March: close by buying at & 1.1800 \\
\cline { 2 - 2 } Profit per contract & 0.0400 \\
\hline
\end{tabular}

Profit on futures position \(=13\) contracts \(\times 400\) ticks \(\times \$ 12.50\) per tick \(=\$ 65,000\)
\begin{tabular}{|c|c|}
\hline & \$ \\
\hline Payment to be made in \$ & 2,000,000 \\
\hline Gain on futures & \((65,000)\) \\
\hline \multirow[t]{2}{*}{Dollars to be purchased at the spot rate} & 1,935,000 \\
\hline & \(€\) \\
\hline Cost of buying \$1,935,000 spot at 1.1800 & 1,639,830.51 \\
\hline Original cost of option premium & 36,931.82 \\
\hline Total cost & 1,676,762.33 \\
\hline Effective exchange rate for \(\$ 2\) million (\$2 million/€1,676,762.33) & \$1.1928 = € 1 \\
\hline
\end{tabular}

\section*{Interest rate options}
- Features of interest rate options
- Types of interest rate option
- Borrowers' options
- Lenders' options
- Swaptions

\section*{6 Interest rate options}

\subsection*{6.1 Features of interest rate options}

An interest rate option is an option on a notional loan or deposit (or an option on an interest rate future), where the loan or deposit period begins:
- on the expiry date for the option for a European-style option, or
- on or before the expiry date for the option, for an American-style option.

The option guarantees a maximum or a minimum rate of interest for the option holder, and interest rate options are therefore sometimes called interest rate guarantees or IRGs.
- A call option guarantees a maximum rate of interest.
- A put option guarantees a minimum rate of interest.

The maximum or minimum rate of interest guaranteed by the option is the strike rate for the option, in comparison with an agreed benchmark rate of interest, such as LIBOR or euribor.

An interest rate option is for a notional loan or deposit. If it is exercised, an actual loan or deposit is not created. Instead, the option is 'cash-settled' by a payment from the writer of the option to the option holder.

\subsection*{6.2 Types of interest rate option}

Many interest rate options are arranged over-the-counter (OTC). These include:
- borrowers' options and lenders' options
- caps, floors and collars.

Options on interest rate futures are traded on the futures exchanges where the interest rate futures are also traded.

\subsection*{6.3 Borrowers' options}

A borrower's option guarantees a maximum borrowing rate for the option holder. The strike rate for the option is compared with an agreed reference rate or benchmark interest rate, such as LIBOR.
- It the reference rate of interest is higher than the strike rate when the option reaches expiry, the option will be exercised. The option writer must make a payment to the option holder for the difference between the actual interest rate (reference rate) and the strike rate for the option.
- It the reference rate of interest is lower than the strike rate when the option reaches expiry, the option holder will let the option lapse.

The premium for the option might be expressed either:
- as an actual percentage of the notional principal amount, or
- as an annual rate of interest on the notional principal amount.

A borrower's option can be used to fix a maximum effective borrowing rate for a future short-term loan, but allow the option holder to benefit from any fall in the interest rate up to the expiry date for the option.

\section*{Example}

A company intends to borrow US \(\$ 10\) million in four months' time for a period of three months, but is concerned about the volatility of the US dollar LIBOR rate. The three-month US\$ LIBOR rate is currently \(3.75 \%\), but might go up or down in the next four months. The company therefore takes out a borrower's option with a strike rate of \(4 \%\) for a notional three-month loan of US \(\$ 10\) million. The expiry date is in four months' time. The option premium is the equivalent of \(0.5 \%\) per annum of the notional principal. For simplicity, we shall suppose that the company is able to borrow at the US dollar LIBOR rate.
(a) If the three-month US dollar LIBOR rate is higher than the option strike rate at expiry, the option will be exercised. If the three-month LIBOR rate is \(6 \%\), the company will exercise the option, and the option writer will pay the option holder an amount equal to the difference between the strike rate for the option \((4 \%)\) and the reference rate ( \(6 \%\) ). The payment will be based on \(2 \%\) of \(\$ 10\) million for three months. (This payment is discounted because a borrower's option is settled at the beginning of the notional interest period, and not at the end of the interest period).
(b) If the three-month US dollar LIBOR rate is lower than the option strike rate at expiry, the option will not be exercised. For example, if the LIBOR rate after four months is \(3 \%\), the option will not be exercised and will lapse.

These possible outcomes are summarised in the table below, assuming (for the purpose of illustration) a spot LIBOR rate at the option expiry date of (a) \(6 \%\) and (b) \(3 \%\).
\begin{tabular}{lrr} 
& \begin{tabular}{c} 
LIBOR rate at expiry \\
\(6 \%\)
\end{tabular} & \begin{tabular}{r}
\(3 \%\) \\
\\
\\
\\
Exercise the option
\end{tabular} \\
\hline & Do not exercise \\
\hline Borrow for three months at & 6.00 & \(\%\) \\
Receive from option writer & \((2.00)\) & 3.00 \\
Cost of option premium & 0.50 & - \\
Effective interest cost (\% annual rate) & -4.50 & 0.50 \\
\hline
\end{tabular}

If the borrower can borrow at the reference rate of interest, a borrower's option sets the maximum borrowing cost at the strike rate plus the option premium cost.

\subsection*{6.4 Lenders' options}

A lender's option guarantees a minimum deposit rate (savings rate) for the option holder. In all other respects, it is similar to a borrower's option. The strike rate for the option is compared with an agreed reference rate or benchmark interest rate, such as LIBOR.
- It the reference rate of interest is lower than the strike rate when the option reaches expiry, the option will be exercised. The option writer must make a payment to the option holder for the difference between the actual interest rate (reference rate) and the strike rate for the option.
- It the reference rate of interest is higher than the strike rate when the option reaches expiry, the option holder will let the option lapse.

\subsection*{6.5 Swaptions}

A swaption is an option on a swap. It gives its holder the right, but not the obligation, to enter into a swap agreement at a future date, on terms that are fixed now.

Swaptions are not commonly used, particularly by companies, because of the cost of the option premium.

\section*{Caps, floors and collars}
- Caps
- Floors
- Collars
- Zero cost collars
- Creating a short-term collar with options on interest rate futures

\section*{7 Caps, floors and collars}

Borrowers' and lenders' options are options for one interest period only. Companies that borrow might seek to hedge their interest rate risk for a number of consecutive interest periods. For example, a company borrowing from a bank might have a fiveyear loan with interest payable every six months, at a variable rate of interest. The loan would therefore have ten consecutive interest periods, with the interest rate fixed for just the first six-month period. The company might want to hedge its exposure to the risk of a high interest rate in the next nine interest periods.

Caps, floors and collars are OTC interest rate options that allow a company to hold interest rate options for a series of consecutive interest periods, instead of having to arrange a number of separate single-period borrowers' or lenders' options.

\subsection*{7.1 Caps}

An interest rate cap is a borrower's option on a series of consecutive interest periods. For example, a company might buy a cap fixing the maximum six-month LIBOR rate at the strike rate for the option, for nine consecutive interest periods, each expiring at six-monthly intervals.

\section*{Example}

A company borrows \(£ 8\) million for five years from a bank at a variable rate of interest, with interest payable every six months at six-month LIBOR plus \(0.50 \%\). The LIBOR rate for the first six months of the loan is \(5 \%\), therefore the company pays interest for this first six months at \(5.5 \%\).

If the company wants to hedge the risk of a rise in the LIBOR rate for the next nine interest periods, it can arrange a cap for a notional principal amount of \(£ 8\) million. It might select a strike rate of \(5 \%\). Alternatively, it might select a strike rate of \(6 \%\), because the premium would be cheaper.

Suppose the strike rate for the cap is \(5 \%\), for the next nine interest periods. The exercise dates for the cap can be arranged to coincide with the start of each sixmonth interest period for the loan. The cap will fix the maximum effective rate of interest on the loan for the next none periods.
- The cap fixes the maximum effective LIBOR rate at \(5 \%\).
- The company will borrow at an effective maximum rate of \(5.5 \%\) (strike rate + \(0.50 \%\) ).
- However, the company has to pay the premium for the cap. This might be \(1 \%\) per year of the notional principal amount. The cost of the premium would make the total maximum effective rate \(6.50 \%\) per year.

For each notional interest period of the cap, the cap operates in a very similar way to a borrower's option.
- If the reference rate of interest for the interest period (LIBOR or euribor) is higher than the strike rate for the cap, the writer of the option must make a payment to the option holder for the difference.
- If the reference rate of interest for the interest period is lower than the strike rate for the cap, the option in the cap will not be exercised.

\section*{Example}

Returning to the previous example, suppose that the six-month LIBOR rates at the first five exercise dates in the cap are as follows:
```

2nd interest period, Year 1 (1)
1st interest period, Year 2 (2)
2nd interest period, Year 2 (3)
1st interest period, Year 3 (4)
2nd interest period, Year 3 (5)

```
```

LIBOR = 4.0%

```
LIBOR = 4.0%
LIBOR = 4.5%
LIBOR = 4.5%
LIBOR = 5.25%
LIBOR = 5.25%
LIBOR = 5.75%
LIBOR = 5.75%
LIBOR = 6.0%
```

LIBOR = 6.0%

```

The strike rate for the cap is \(5 \%\). The outcome for each interest period will be as follows:
\begin{tabular}{lrrrrr}
\begin{tabular}{l} 
Interest period/ \\
expiry date
\end{tabular} & LIBOR & & \begin{tabular}{r} 
Borrow at LIBOR \\
\(\mathbf{+ 0 . 5 0 \%}\)
\end{tabular} & \begin{tabular}{r} 
Receipt \\
from cap
\end{tabular} & \begin{tabular}{r} 
Net interest \\
rate
\end{tabular} \\
\hline\((1)\) & \(4.0 \%\) & Do not exercise & \(4.50 \%\) & - & \(4.50 \%\) \\
\((2)\) & \(4.5 \%\) & Do not exercise & \(5.00 \%\) & - & \(5.00 \%\) \\
\((3)\) & \(5.55 \%\) & Exercise & \(5.75 \%\) & \((0.25) \%\) & \(5.50 \%\) \\
\((4)\) & \(5.75 \%\) & Exercise & \(6.25 \%\) & \((0.75) \%\) & \(5.50 \%\) \\
\((5)\) & \(6.0 \%\) & Exercise & \(6.50 \%\) & \((1.00) \%\) & \(5.50 \%\)
\end{tabular}

The cap therefore sets an effective interest rate of \(5.50 \%\) (the strike rate \(+0.50 \%\) ) and also allows the option holder to benefit from a lower LIBOR rate, by choosing not to exercise the option when this rate is more favourable than the strike rate for the option. However, the effective interest rates are actually higher than in the table above, because the cost of the option premium needs to be added.

With interest rate caps (and also with floors and collars), the payment by the option writer when a cap is exercised is at the end of the notional interest period, not when the option is exercised. The payment is therefore not discounted. In the example above, the payment by the option writer for the fifth expiry date would be:
\((6.00-5.00) \% \times £ 8\) million \(\times 6 / 12=£ 40,000\). This would be made at the end of the six-month interest period.

\subsection*{7.2 Floors}

Interest rate floors are similar to caps, except that they are a series of successive lenders' options. They can be used to fix a minimum interest rate for a series of interest rate periods.

\subsection*{7.3 Collars}

A major disadvantage of caps and floors is the high cost of the premiums. A company might want to hedge its exposure to the risk of a rise or fall in an interest rate for a number of consecutive interest periods, but might not want to pay the cost of the premium for the cap or the floor.

This can be done by arranging an interest rate collar.
- As an alternative to a cap, a company can arrange a collar in which it:
- buys a cap at one strike rate, and simultaneously
- sells a floor at a different, lower strike rate.

The effect of this collar is to fix a maximum interest cost through the mechanism of the cap, but also to fix a minimum interest cost through the mechanism of the floor.

The cost of the cap premium and the income from the sale of the floor are the net premium cost for the collar.
Collar premium = Cap premium cost - Floor premium income.
- Similarly, as an alternative to a floor, a company can arrange a collar in which it:
- buys a floor at one strike rate, and simultaneously
- sells a cap at a different, higher strike rate.

The effect of this collar is to fix a minimum interest income through the mechanism of the floor, but also to fix a maximum interest income through the mechanism of the cap.
The cost of the floor premium and the income from the sale of the cap are the net premium cost for the collar.
Collar premium \(=\) Floor premium cost - Cap premium income.
A collar therefore has the effect of fixing the reference rate of interest between a high and a low limit.

\subsection*{7.4 Zero cost collars}

Even the cost of a collar can be quite high. However, it might be possible to arrange a zero cost collar. This is collar for which the premium is zero.

For example, if a collar consists of buying a cap and selling a floor, the premium value of the cap and the premium value of the floor would be equal, leaving no premium to pay for the collar.

When a zero cost collar is possible - and it is by no means always possible to arrange a collar for no cost - the strike rate for the cap and the strike rate for the collar will be very close to each other, if not the same. In effect, this would fix the interest period at a single rate for the full term of the collar, or would restrict the range between maximum and minimum interest rate to a very narrow limit.

\subsection*{7.5 Creating a short-term collar with options on interest rate futures}

An interest rate collar can be created with options on interest rate futures.
You need to remember that an interest rate future is a contract on a notional interest-earning deposit.

To create a collar with options on interest rate futures to fix a maximum and a minimum cost of borrowing, you should:
- Buy put options on interest rate futures for the maximum interest rate in the collar. If the spot interest rises above this put option rate and the futures price therefore falls below it, the options will be exercised to sell the futures at the exercise price and make a profit from closing the futures position.
- Sell call options on interest rate futures for the minimum interest rate in the collar. If the spot interest falls below the call option rate and the futures price therefore rises above it, the call option holder will exercise option on the futures at the exercise price. The company must buy the futures at that price and make a loss on the futures position.
- The net cost of the collar is the premium paid for the put options minus the premium received from selling the call options.

\section*{Example}

It is now the end of March and the spot LIBOR rate is. A company needs to borrow \(\$ 120\) million at the end of July for 2 months. It can borrow at LIBOR plus \(1 \%\) and it wants to create an interest rate collar so that its effective borrowing cost will be within the range \(5 \%\) to \(6 \%\).

Futures on three-month dollars are for \(\$ 1\) million each.

Current option prices are as follows, at the end of March. Premiums are shown as annual \% rates. A three-month dollar short-term interest rate future is for a deposit of \(\$ 1,000,000\).
\begin{tabular}{lcccc} 
& \multicolumn{2}{c}{ CALLS } & \multicolumn{2}{c}{ PUTS } \\
\begin{tabular}{l} 
Expiry: \\
Strike price:
\end{tabular} & Jun & Sep & Jun & Sep \\
\hline 9500 & 0.225 & 0.305 & 0.165 & 0.215 \\
9600 & 0.025 & 0.080 & 0.490 & 0.750
\end{tabular}

Required
(a) Explain how the collar should be created and how many options contracts should be used.
(b) Calculate the premium cost of the collar.
(c) Calculate the effective cost of borrowing the \(\$ 120\) million if the LIBOR rate at the end of July is \(6 \%\) and he futures price at this date is also \(6 \%\). (In other words, basis is ignored for the purpose of this question.)

\section*{a \\ Answer}
(a)

The borrowing period will begin at the end of July, therefore options on September futures should be used.

The company borrows at LIBOR plus \(1 \%\) and wants the collar to establish an effective borrowing range of \(5 \%\) to \(6 \%\). This means that the collar needs to establish a borrowing range for LIBOR of \(4 \%\) to \(5 \%\), with a maximum LIBOR cost of \(5 \%\) and a minimum LIBOR cost of \(4 \%\).

The company should therefore buy put options at the \(5 \%\) interest rate of 9500 and sell call options at the \(4 \%\) rate of 96.00 .

The company intends to borrow \(\$ 120\) million for 2 months; therefore the number of option contracts is:
[ \(\$ 120\) million/ \(\$ 1\) million per future \(] \times[2\) months \(/ 3\) months \(]=80\) contracts .
(b)
\begin{tabular}{lc} 
& \begin{tabular}{c} 
Premium \\
\(\%\) per
\end{tabular} \\
Cost of September put options at 9500 & year \\
Income from September call options at 9600 & 0.215 \\
Net cost of collar & 0.080 \\
\cline { 2 - 3 } & \\
\hline
\end{tabular}

The cost of the premium in \(\$\) is:
80 contracts \(\times \$ 1\) million \(\times 0.135 \% \times 3 / 12=\$ 27,000\).
(c)

At the end of July if the spot LIBOR rate is \(6 \%\), the company will borrow \(\$ 120\) million for two months at 7\%.

The put options on dollar interest rate futures will also be exercised at 9500, and if the futures price is \(6 \%\) (9400), there will be a profit of 100 ticks ( \(9600-9500\) ) on closing the futures position.

The profit on the futures will be: 80 contracts \(\times 100\) ticks \(\times \$ 25=\$ 200,000\).

The net effective cost of borrowing is therefore as follows:
Interest on \(\$ 120\) million for 2 months at 7\% 1,400,000
Gain on futures
Cost of collar
Total effective cost
This represents an effective interest rate of:
\(\frac{\$ 1,227,000}{\$ 120 \text { million }} \times \frac{12 \text { months }}{2 \text { months }}=0.6135\) or \(6.135 \%\)

This is the maximum borrowing cost established by the collar (6\%) plus the cost of the premium ( \(0.135 \%\), see earlier calculation).

\section*{16}

\section*{Option pricing and delta hedging}

\section*{Contents}

1 Factors influencing option values
2 The Black-Scholes option pricing model for call options
3 Value of a put option: put-call parity
4 The forex variant for pricing FX options
5 Delta hedging

\section*{Factors influencing option values}
- Option premium and option value
- Intrinsic value and time value
- The main factors affecting time value
- The five factors affecting option prices

\section*{1 Factors influencing option values}

\subsection*{1.1 Option premium and option value}

The option premium is the price paid by an option buyer to the option seller for an option. The premium represents the value of the option at the time it is written.

Between the time the option is written and the option's expiry date, the value of the option will change. The value will go up or down, according to changes in the likelihood that the option will be exercised, and if it is exercised, what the size of the gain for the option holder is likely to be.

Every option that has not yet expired therefore has some value, unless it is deeply out-of-the-money.
- The value of an option can be calculated by the option writer when the option is written, and this value is the premium charged for the option to the option buyer.
- After the option has been written but before its expiry, options can be re-valued to a current value. This revaluation process is now required by international accounting standard IAS 39, for financial reporting for financial instruments.

You need to be able to use the Black-Scholes option pricing model to calculate the value of a European-style call option. This model is explained later.

\subsection*{1.2 Intrinsic value and time value}

The value of an option can be analysed into an intrinsic value and a time value.
Intrinsic value + Time value \(=\) Total value of the option
- Only in-the-money options have intrinsic value. Intrinsic value is the difference between the strike price for the option and the market price of the underlying item. If an option is at-the-money or out-of-the-money, its intrinsic value is 0 .
- Time value is the value placed on the option that relates to the probability that the option will become in-the-money before expiry, or will become even more in-the-money before expiry, and if so by how much it is likely to be in-themoney at expiry.

\subsection*{1.3 The main factors affecting time value}

Several factors influence the time value of an option:
- The length of time remaining to expiry. If there are two identical options with the same strike price but different times to expiry, the option with the longer time to expiry is more valuable. This is because there is more time for the option to become in-the-money, or to become even more deeply in-the-money in the period remaining to expiry.
- The size of the difference between the strike price for the option and the current market price of the underlying item. An option that is at-the-money will be worth more than a similar option that is out-of-the-money. This is because in the time remaining to expiry, an at-the-money option is more likely than an out-of-the-money option to become in-the-money. For the same reason, an option that is slightly out-of-the-money is worth more than an option that is deeply out-of-the-money.
- The volatility in the market price of the underlying item. An option is more valuable when there is higher volatility in the price of the underlying item. This is because when volatility is greater, the probability is greater that the market price will move up or down by a large amount in the remaining time to expiry of the option.
- The risk-free rate of interest. Another factor in the calculation of an option price is the risk-free rate of interest. This is included in the Black-Scholes option pricing model.

\section*{Example}

Compare the following two call options on shares:
\begin{tabular}{lcc} 
& \begin{tabular}{c} 
Option on shares of \\
Company \(\mathbf{X}\)
\end{tabular} & \begin{tabular}{c} 
Option on shares of \\
Company Y Y
\end{tabular} \\
Exercise price & 850 & 750 \\
Current market value of shares & 825 & 500 \\
Time remaining to expiry & 1 year & 3 months \\
Volatility of share price (annual) & \(35 \%\) & \(10 \%\)
\end{tabular}

The value of the options on shares of Company X will be worth much more than the options on shares of Company Y, even though both options are currently out-of-themoney.
- The strike price on the Company \(X\) options is only 25 higher than the current market value, and the annual volatility is quite high relative to the current market price. There is also one year to the expiry date, during which time the price of the shares might rise.
- The strike price on the Company Y options is much higher than the current market price, indicating that the options are deeply out-of-the-money. The share price volatility is much lower than for Company X and there are only three months to expiry of the options. It is difficult to imagine that these options will be in the money by the expiry date, and the options are probably worthless.

\subsection*{1.4 The five factors affecting option prices}

There are five factors that affect options prices:
- The current price of the underlying item (such as the current market price of the share)
- The strike price or exercise price for the option
- The volatility in the market price of the underlying item
- The time remaining to expiry of the option
- The risk-free interest rate.

These five factors are all elements in the Black-Scholes option pricing model.

Information about the market value of traded options in company shares are published regularly in the financial press. The published prices indicate the significance for the option price of the current market price of the underlying, the exercise price and the time to expiry

\section*{Example}

Traded options in shares of publishing company Pearson Group were being traded at the following prices during early June, when the actual share price on the London Stock Exchange was 866 pence. Premium prices are shown in pence per share.
\begin{tabular}{lrrrrrr} 
& \multicolumn{3}{c}{ CALLS } & \multicolumn{3}{c}{ PUTS } \\
\begin{tabular}{l} 
Expiry: \\
Strike price:
\end{tabular} & Jun & Sep & Dec & Jun & Sep & Dec \\
\hline 860 & 17.5 & 34.0 & 44.5 & 10.0 & 22.0 & 29.5 \\
880 & 8.0 & 23.5 & 33.5 & 21.0 & 32.0 & 39.0
\end{tabular}

\section*{Notes}
(a) The premium varies with the difference between the strike price and the current market price for the shares. Call options with a higher strike price have a lower value. Put options with a higher exercise price have a higher value.
(b) Premiums vary with time to expiry. For each exercise price, the premium for both call options and put options is higher for a later expiry date.

\section*{The Black-Scholes option pricing model for call options}
- The model formulae
- Notes on the model

Using the Black-Scholes model

\section*{2 The Black-Scholes option pricing model}

The Black-Scholes option pricing model is used to derive the price of a Europeanstyle call option. This model, or variants of it, is used by banks to calculate the premium for an over-the-counter (OTC) option. The model is quite complex if you are not familiar with mathematics, but you might be required to calculate an option value with the model in your examination.

\subsection*{2.1 The model formulae}

Formulae for the Black-Scholes model are given in the formula sheet in the examination. There are three formulae.
\(\mathrm{c}=\mathrm{Pa}_{\mathrm{a}} \mathrm{N}\left(\mathrm{d}_{1}\right)-\mathrm{P}_{\mathrm{e}} \mathrm{N}\left(\mathrm{d}_{2}\right) \mathrm{e}^{-\mathrm{rt}}\)
where
\[
\mathrm{d}_{1}=\frac{\ln \left(\mathrm{P}_{\mathrm{a}} / \mathrm{P}_{\mathrm{e}}\right)+\left(\mathrm{r}+0.5 \mathrm{~s}^{2}\right) \mathrm{t}}{\mathrm{~s} \sqrt{\mathrm{t}}}
\]
\(\mathrm{d}_{2}=\mathrm{d}_{1}-\mathrm{s} \sqrt{\mathrm{t}}\)
\(\mathrm{P}_{\mathrm{a}}=\) the price of the underlying item, such as the current share price (stock option) or current exchange rate (currency option)
\(N\left(d_{1}\right)=\) the probability in a normal distribution that the value is less than \(d_{1}\) standard deviations above the mean. (This is explained later. \(\mathrm{N}\left(\mathrm{d}_{2}\right)\) is also explained later.
\(\mathrm{P}_{\mathrm{e}}=\) the exercise price or strike rate for the option
\(\mathrm{e} \quad=\) the constant e . You will need a calculator that calculates values to the power of \(e\).
\(\mathrm{r}=\) the risk-free interest rate (as an annual rate: for example, \(5 \%=0.05\) )
\(\mathrm{t}=\) time to expiry of the option, in years. For example, if the option has three months to expiry, \(\mathrm{t}=0.25\).
\(\mathrm{s}=\) standard deviation of the value/returns of the underlying item. This should be the annual volatility, for consistency with the values of r and t .

\subsection*{2.2 Notes on the model}

If you are not familiar with mathematics, the following notes might be useful.
- To calculate the price of a European-style call option, it is necessary to calculate values for \(d_{1}\) and \(d_{2}\). To calculate \(d_{2}\), it is necessary to know the value of \(d_{1}\). The starting point is therefore to calculate the value of \(d_{1}\).
- The first item in the formula for the value of \(d_{1}\) is \(\ln \left(\mathrm{P}_{\mathrm{a}} / \mathrm{P}_{\mathrm{e}}\right)\). The letters ' \(\ln\) ' mean 'normal logarithm of'. Normal logarithms are logarithms to the base of the constant e. You need a calculator that can work out natural logarithms, and you need to make sure that you can use the natural logarithm function on the calculator.
(Note: a natural logarithm is a number expressed as a value to the power of e. The constant ' e ' has a value of 2.71828 . This means, for example, that the natural logarithm of 4 is 1.3863 because 4 is equal to 2.71828 to the power of 1.3863 ).

The standard deviation is a measurement of volatility of the price of the underlying item. In the case of a stock option, it is the standard deviation of the annual returns from the share. An annual standard deviation of \(15 \%\), for example, would be 0.15 in the formula. Remember that the standard deviation is the square root of the variance. If an examination gives you the variance of the returns, remember to take the square root to obtain the standard deviation.

Having calculated values for \(\mathrm{d}_{1}\) and \(\mathrm{d}_{2}\), the final step is to calculate the option price. To do this, you need to establish values for \(N\left(d_{1}\right)\) and \(N\left(d_{2}\right)\). These are values obtained from normal distribution tables. These statistical distribution tables are provided in the examination, as a standard normal distribution table. These tables also explain how the tables should be used to find the values for \(N\left(d_{1}\right)\) and \(N\left(d_{2}\right)\).

The rules are as follows:
- Having established a value for \(\mathrm{d}_{1}\left(\right.\) or \(\left.\mathrm{d}_{2}\right)\), find the corresponding value in the normal distribution tables. For example, if \(\mathrm{d}_{1}=1.75\), look for the value in the row 1.7 and the column 0.05 - this value is 0.4599 .
- If the value of \(d_{1}\) is positive, add 0.5 to the value you have obtained from the table. Similarly, if the value of \(\mathrm{d}_{2}\) is positive, add 0.5 to the value you have obtained from the table.
- If the value of \(d_{1}\) is negative, subtract the value you have obtained from the table from 0.5 . Similarly, if the value of \(d_{2}\) is negative, subtract the value you have obtained from the table from 0.5 .

This gives you the value for \(\mathrm{N}\left(\mathrm{d}_{1}\right)\) ( or \(\mathrm{N}\left(\mathrm{d}_{2}\right)\) ).
The option price is calculated as:
- the value of \(\mathrm{P}_{\mathrm{a}}\) multiplied by the value for \(\mathrm{N}\left(\mathrm{d}_{1}\right)\)
- minus the value of \(\mathrm{P}_{\mathrm{a}}\) multiplied by the value for \(\mathrm{N}\left(\mathrm{d}_{2}\right)\) and multiplied by the value of \(e^{-r t}\).

The value of \(e^{-r t}\) is the inverse of the constant \(e\) to the power of rt. In other words, \(e^{-r t}\) is \(1 / \mathrm{e}^{\text {rt. }}\). Use your calculator to calculate the value of \(\mathrm{er}^{\text {rt, }}\), then calculate the inverse of your answer.

\subsection*{2.3 Using the Black-Scholes model}

The formulae might seem very complex. The following example shows how they should be applied:

\section*{Example}

The current share price of BS Company is \(\$ 16\). What should be the price of a European-style call option on the company's shares at an exercise price of \(\$ 16.50\), if the expiry date is in six months, the standard deviation of annual returns on the share is \(12 \%\) and the risk-free rate of return is \(7 \%\) per year?

\section*{Answer}

Step 1. Calculate \(\mathrm{d}_{1}\)
\[
\mathrm{d}_{1}=\frac{\ln (16.00 / 16.50)+\left[0.07+\left(0.5 \times 0.12^{2}\right) 0.5\right.}{0.12 \sqrt{0.5}}
\]
\(\ln \left(\frac{16}{16.50}\right)=\ln 0.9697\). Using a calculator, the natural logarithm of 0.9697 can be obtained. This is -0.03077 .
\(\mathrm{d}_{1}=\frac{-0.03077+0.0386}{0.12 \sqrt{0.5}}\)
\(=0.00783 / 0.08485\)
\(=+0.092\)

Step 2. Calculate \(\mathrm{d}_{2}\)
\(\mathrm{d}_{2}=\mathrm{d}_{1}-s \sqrt{\mathrm{t}}=0.092-(0.12) \sqrt{0.5}\)
\(=+0.0071\).

\section*{Step 3. Obtain values for \(\mathbf{N}\left(\mathbf{d}_{1}\right)\) and \(\mathbf{N}\left(\mathbf{d}_{2}\right)\)}
(a) \(\quad \mathrm{N}\left(\mathrm{d}_{1}\right)\). When \(\mathrm{d}_{1}=0.092\), the corresponding value in the normal distribution table is 0.3212 . The value of \(d_{1}\) is positive, so add \(0.50 . \mathrm{N}\left(\mathrm{d}_{1}\right)=0.3212+0.50=0.8212\).
(b) \(\quad \mathrm{N}\left(\mathrm{d}_{2}\right)\). When \(\mathrm{d}_{2}=0.007\), the corresponding value in the normal distribution table is 0.0279 . The value of \(\mathrm{d}_{2}\) is positive, so add \(0.50 . \mathrm{N}\left(\mathrm{d}_{2}\right)=0.0279+0.50=0.5279\).

\section*{Step 4. Calculate the option price (for a call option)}

It might help to calculate the value of \(\mathrm{e}^{-r t}\) first. The value of rt is \((0.07)(0.5)=0.035\).

The value of \(\mathrm{e}^{-0.035}\) is the same as \(1 / \mathrm{e}^{0.035}\). Using a calculator, the value of \(\mathrm{e}^{0.035}\) can be obtained: this is 1.0356 . Therefore \(\mathrm{e}^{-0.035}=1 / 1.0356=0.9656\).

The exercise price \(\left(\mathrm{P}_{\mathrm{e}}\right)\) is 16.50 . The option price can now be calculated as follows:
Optionprice \(=\mathrm{P}_{\mathrm{s}} \mathrm{N}\left(\mathrm{d}_{1}\right)-\mathrm{P}_{\mathrm{e}} \mathrm{N}\left(\mathrm{d}_{2}\right) \mathrm{e}^{-\mathrm{rt}}\)
\(=16(0.8212)-16.50(0.5279)(0.9656)\)
\(=13.14-8.16\)
\(=4.98\)

The option price is \(\$ 4.98\). This is high relative to the current market price, but the high option value is due to the fact that the strike price is close to the current market price, and the returns on the share are fairly volatile.

\section*{Example}

In this similar example, the values of \(\mathrm{d}_{1}\) and \(\mathrm{d}_{2}\) are negative.

The current share price of CB Company is 400 . What should be the price of a European-style call option on the company's shares at an exercise price of 440, if the expiry date is in six months, the standard deviation of annual returns on the share is \(30 \%\) and the risk-free rate of return is \(6 \%\) per year?

\section*{Answer}

Step 1. Calculate \(\mathrm{d}_{1}\)
\[
\mathrm{d}_{1}=\frac{\ln (400 / 440)+\left[0.06+\left(0.5 \times 0.30^{2}\right)\right] 0.5}{0.30 \sqrt{0.5}}
\]
\(\ln (400 / 440)=\ln 0.9091\). Using a calculator, the natural logarithm of 0.95238 can be obtained. This is -0.0953 .
\(\mathrm{d}_{1}=\frac{-0.0953+0.0525}{0.30 \sqrt{0.5}}\)
\(=-0.0428 / 0.2121\)
\(=-0.202\)

\section*{Step 2. Calculate d \({ }_{2}\)}
\(\mathrm{d}_{2}=\mathrm{d}_{1}-s \sqrt{\mathrm{t}}=-0.202-(0.30) \sqrt{0.5}\)
\(=-0.414\)
Step 3. Obtain values for \(N\left(d_{1}\right)\) and \(N\left(d_{2}\right)\)
\(\mathrm{N}\left(\mathrm{d}_{1}\right)\). When \(\mathrm{d}_{1}=-0.202\), the corresponding value in the normal distribution table is about 0.0801 . The value of \(d_{1}\) is negative, so subtract 0.0801 from \(0.50 . \mathrm{N}\left(d_{1}\right)=0.5000-\) \(0.0801=0.4199\)
\(\mathrm{N}\left(\mathrm{d}_{2}\right)\). When \(\mathrm{d}_{2}=-0.414\), the corresponding value in the normal distribution table is about 0.1613 . The value of \(d_{2}\) is negative, so subtract 0.1613 from \(0.50 . \mathrm{N}\left(\mathrm{d}_{2}\right)=0.5000-\) \(0.1613=0.3387\).

\section*{Step 4. Calculate the option price (for a call option)}

It might help to calculate the value of \(e^{-r t}\) first. The value of rt is \((0.06)(0.5)=0.030\).
The value of \(\mathrm{e}^{-0.030}\) is the same as \(1 / \mathrm{e}^{0.03}\). Using a calculator, the value of \(\mathrm{e}^{0.03}\) can be obtained: this is 1.03045 . Therefore \(\mathrm{e}^{-0.03}=1 / 1.03045=0.9704\).

The exercise price \(\left(\mathrm{P}_{\mathrm{e}}\right)\) is 440 . The option price can now be calculated as follows:
Optionprice \(=\operatorname{PaN}\left(\mathrm{d}_{1}\right)-\mathrm{P}_{\mathrm{e}} \mathrm{N}\left(\mathrm{d}_{2}\right) \mathrm{e}^{-\mathrm{rt}}\)
\(=400(0.4199)-440(0.3387)(0.9704)\)
\(=167.96-144.62\)
\(=23.34\).

\section*{Value of a put option: put-call parity}
- Value of a put option: put-call parity pricing model
- Value of a put option: alternative put-call parity pricing model

\section*{3 Value of a put option: put-call parity}

\subsection*{3.1 Value of a put option: put-call parity pricing model}

The Black-Scholes model calculates the value of a European-style call option. It can also be used to calculate the value of a European-style put option with the same expiry date and exercise price, using the put-call parity pricing model.

The value of a European-style put option is calculated as:
Value of put + Current market price of underlying item = Value of call + Present value of the strike price.
\(\mathrm{V}_{\text {PUt }}+\mathrm{MV}=\mathrm{V}_{\text {Call }}+\mathrm{P}_{\text {Strike }}\)

\section*{Example}

Shares in BS Company have a current market value of \(\$ 16\). The value of a call option with a strike price of \(\$ 16.50\) and an expiry date in six months is \(\$ 4.98\). The risk-free rate of return is \(7 \%\) per year, so the present value of the exercise price is \(\$ 16.50 \times\) \(1 /(1.07)^{0.5}=\$ 16.50 \times 0.9667=\$ 15.95\).

Applying the put-call parity pricing model:
Value of put \(+\$ 16=\$ 4.98+\$ 15.95\)
Value of put \(=\$ 4.93\).

\subsection*{3.2 Value of a put option: alternative put-call parity pricing model}

An alternative formula for the put-call parity is given in the formula sheet in the examination. Allowing for rounding differences, it gives the same valuation for a put option as the formula described above.

The alternative formula is:
\(\mathrm{p}=\mathrm{c}-\mathrm{P}_{\mathrm{a}}+\mathrm{P}_{\mathrm{e}} \mathrm{e}^{-\mathrm{rt}}\)
where
\(p=\) the price of a put option
\(\mathrm{c}=\) the price of a call option with the same exercise price and expiry date
\(P_{a}\) is the current market price of the underlying item
\(\mathrm{P}_{\mathrm{e}}\) is the exercise price of the option
\(r=\) the risk free rate of interest
\(t=\) the time to expiry in years

\section*{Example}

The same example used above to demonstrate put-cal parity will be used to show how the alternative formula is used.

Shares in BS Company have a current market value of \(\$ 16\). The value of a call option with a strike price of \(\$ 16.50\) and an expiry date in six months is \(\$ 4.98\).

The risk-free rate of return is \(7 \%\) per year, and the option has six months to expiry, so \(\mathrm{rt}=(0.07)(0.5)=0.035\). The value of \(\mathrm{e}^{0.035}\) is 1.03562 so \(\mathrm{e}^{-0.035}\) is 0.9656 .
\(p=4.98-16+(16.5)(0.9656)\)
\(\mathrm{p}=4.91\).
This differs from the valuation above using the alternative formula by 0.02 , due to mathematical rounding differences.

\section*{The forex variant for pricing FX options}
- The Grabbe variant of the Black-Scholes model
- Initial problems with applying the formula
- Using the forex variant for pricing FX options

\section*{4 The forex variant for pricing FX options}

\subsection*{4.1 The Grabbe variant of the Black-Scholes model}

A variant of the Black-Scholes model can be used to calculate a price for Europeanstyle at-the-money currency options. It is sometimes called the Grabbe variant of the Black-Scholes model. The formulae are provided in the formula sheet for your examination, and you need to be able to use them.

\section*{The meaning of 'at-the-money'}

An at-the-money currency option is an option whose exercise rate or strike rate is equal to the current forward exchange rate for a forward contract whose settlement date is the expiry date for the option. For example, a six-month at-the-money currency option is a currency option whose stroke rate is the current six-month forward rate.

\section*{The formulae for the pricing model}

There is a formula for calculating the price of a call option (c) and another formula for the price of a put option (p). The formulae are as follows.
\(\mathrm{c}=\mathrm{e}^{-\mathrm{rt}}\left[\mathrm{F}_{0} \mathrm{~N}\left(\mathrm{~d}_{1}\right)-\mathrm{XN}\left(\mathrm{d}_{2}\right)\right]\)
or
\(\mathrm{p}=\mathrm{e}^{-\mathrm{rt}}\left[\mathrm{XN}\left(-\mathrm{d}_{2}\right)-\mathrm{F}_{0} \mathrm{~N}\left(-\mathrm{d}_{1}\right)\right]\)
where
\(\mathrm{d}_{1}=\frac{\ln \left(\mathrm{F}_{0} / \mathrm{X}\right)+\mathrm{s}^{2} \mathrm{t} / 2}{\mathrm{~s} \sqrt{\mathrm{t}}}\)
\(\mathrm{d}_{2}=\mathrm{d}_{1}-\mathrm{s} \sqrt{\mathrm{t}}\)
Some of these symbols have the same meaning as in the basic Black-Scholes formula.
X is the current spot exchange rate
\(F_{0}\) is the forward exchange rate for a settlement date at the expiry of the currency option. This is also the exercise rate or strike rate for an at-the-money option.
\(\ln \left(\mathrm{F}_{0} / \mathrm{X}\right)\) is the \(\log\) normal value of \(\mathrm{F}_{0} / \mathrm{X}\)
s is the annualised standard deviation of the exchange rate (and \(\mathrm{s}^{2}\) is the annual variance of the exchange rate)
\(t\) is the time period to expiry of the option in years
\(r\) is the risk-free interest rate. The risk-free rate is normally assumed to be an interbank money market rate such as LIBOR.

\section*{The risk-free interest rate}

With currency options, there are two currencies and each currency has a different risk-free interest rate. It is therefore important to know which risk-free interest currency to use in the option pricing formula. The interest rate for the base currency should be used. This is the currency in which the option is priced.

\section*{Example}

To calculate a price for traded options in sterling (British pounds), you should use the three-month sterling LIBOR rate for pricing three-month options, the six-month sterling LIBOR rate for pricing six-month options and the nine-month sterling LIBOR rate for pricing nine-month options. These money market rates are all expressed as annual percentage rates, and should be used as the rate for \(r\) in the option pricing model.

\section*{The annual standard deviation (volatility) of the exchange rate}

The standard deviation of the exchange rate, which is the measure of the volatility of the exchange rate, is an annual standard deviation. If an examination question gives you a standard deviation (or 'volatility') for any time period other than one year, you will need to convert the figure you are given to an annual volatility.
- If you are given a monthly volatility (standard deviation) the annual standard deviation is the monthly standard deviation multiplied by a factor \(\times \sqrt{12}\) ( \(\times 3.4641\) ).
- If you are given a three-monthly volatility (standard deviation) the annual standard deviation is the monthly standard deviation multiplied by a factor \(\times \sqrt{4}\) ( \(\times 2.000\) ).
- If you are given a six-monthly volatility (standard deviation) the annual standard deviation is the monthly standard deviation multiplied by a factor \(\times \sqrt{2}\) ( \(\times 1.4142\) ).

\subsection*{4.2 Initial problems with applying the formula}

Two problems have to be dealt with before the formulae can be used. These are:
- Sorting out which is the fixed currency and which is the variable currency, and the currency in which the option is priced.
- Calculating the forward rate when this is not given in the examination question.

\section*{Sorting out the base currency and variable currency}

It is important to identify which currency is the base currency and which is the variable currency. Suppose there are currency options for euros-US dollars.
- When options are options on euros, each option is for a fixed quantity of euros in exchange for US dollars at an exchange rate equal to the strike rate or exercise rate for the option. The exercise rate is a rate for the number of dollars it will cost to buy (call option) or sell (put option) one euro.
- The option pricing model should therefore use a spot rate and a forward rate where euros are the base currency and US dollars are the variable currency. In other words, we need an exchange rate of \(€ 1=\) US \(\$\) X for both the spot rate and the forward rate, to use in the option pricing model.
- The option pricing model will give a value or price for the option in US currency (cents per \(€ 1\) ).
- The risk-free interest rate to use in the pricing formula is the risk-free rate for the currency in which the option is priced.

\section*{Example}

Traded currency options for euros-US dollars are 100,000 euros per contract. The spot exchange rate is \(€ 1=\) US \(\$ 1.3250\).

Since options are for a fixed quantity of euros, the exchange rate for \(€ 1=\$ \mathrm{X}\) should be used in the option pricing formula. This is \(€ 1=\) US\$1.3250.

Using the Black-Scholes formula to calculate the value of an option will give a price in cents per \(€ 1\).

For an option expiring in six months, the option pricing model should use a riskfree six-month interest rate for the US dollar, such as the six-month US dollar LIBOR rate.

\section*{Calculating the forward rate}

An examination question might give you the spot rate for a currency and the riskfree interest rate in each currency, but might not give you the forward exchange rate.

When this happens, you need to calculate the forward rate from the spot rate and the two risk-free interest rates. A technique for doing this is as follows:
- Start with the spot rate. This will be given as 1 unit of base currency \(=X\) units of variable currency.
- The option expires in n months' time.
- Calculate how much 1 unit of base currency will be worth in \(n\) months if it is invested now for n months at the risk-free rate of interest.
- Similarly, calculate how much X units of variable currency will be worth in n months if they are invested now for n months at the risk-free rate of interest.
- The forward rate for settlement in n months' time is the value of the variable currency after n months divided by the value of the base currency after n months.

\section*{Example}

The spot exchange rate for British pounds/US dollars is \(£ 1=\) US\$1.9500.
The three-month sterling LIBOR interest rate is \(5.80 \%\).
The three-month US dollar LIBOR rate is \(4.40 \%\).

\section*{Required}

Calculate the three-month forward exchange rate.

\section*{Answer}

The three-month interest rates are annual percentages. If US dollars are invested for three months, the amount of interest earned will be approximately \(4.40 \% \times 3 / 12=\) \(1.10 \%\).
- Similarly, if sterling is invested for three months, the amount of interest earned will be approximately \(5.80 \% \times 3 / 12=1.45 \%\).
- The spot exchange rate is \(\$ 1.9500\).
- If \(£ 1\) is invested for three months, its value after three months will be \(£ 1 \times 1.0145\) \(=£ 1.0145\).
- If \(\$ 1.9500\) is invested for three months, its value after three months will be \(\$ 1.95\) \(\times 1.011=1.97145\).

The three month forward rate is \(\$ 1.97145 / £ 1.0145=1.9433\).

\section*{Alternative method of calculating a forward rate: interest rate parity model}

The same forward rate can be calculated using the interest rate parity model, although this is effectively the same as the method explained above.

The interest rate parity model is:
\[
\mathrm{F}_{0}=\mathrm{S}_{0} \times \frac{\left(1+\mathrm{i}_{\mathrm{c}}\right)}{\left(1+\mathrm{i}_{\mathrm{b}}\right)}
\]
where
\(\mathrm{F}_{0}\) is the forward rate for settlement after n months
\(\mathrm{S}_{0}\) is the current spot rate
\(\mathrm{i}_{\mathrm{c}}\) is the interest that would be earned by investing 1 unit of the variable currency for n months (so if the three-month rate of interest is \(4 \%\) at an annualised rate, \(\mathrm{i}_{\mathrm{c}}=4 \% \times\) \(3 / 12=1 \%\) or 0.01
\(\mathrm{i}_{\mathrm{b}}\) is the interest that would be earned by investing 1 unit of the base currency for n months.

\section*{Example}

In the previous example, the forward rate could be calculated as:
\[
\begin{aligned}
& \mathrm{F}_{0}=1.9500 \times \frac{1.011}{1.0145} \\
& =1.9433 .
\end{aligned}
\]

\subsection*{4.3 Using the forex variant for pricing FX options}

The forex variant of the Black-Scholes model is similar to the basic Black-Scholes model, and the following example illustrates how it should be used:

\section*{Example}

Traded currency options for sterling-euros are 100,000 euros per contract. The spot exchange rate is \(€ 0.7000=£ 1\).

The three-month euro Libor rate is \(4.25 \%\) and the three-month sterling Libor rate is 5.5\%.

Monthly volatility in the sterling-euro exchange rate is \(3.6 \%\) (measured as the monthly standard deviation of the exchange rate).

\section*{Required}
(a) Calculate the current price of a three-month at-the-money currency call option on \(€ 100,000\).
(b) Calculate the current price of a three-month at-the-money currency put option on \(€ 100,000\).

Answer

We are given the one-month volatility of the exchange rate. The annual volatility is \(3.6 \% \times \sqrt{12}=12.47 \%\).

The options are for euros, which means that euros are the variable currency and sterling is the base currency. We want the exchange rate expressed as \(£ 1=€ X\). The spot exchange rate is given as \(£ 1=€ 0.7000\).

The three-month forward rate (at-the-money strike rate) is calculated as follows:
- If euros are invested for three months, the amount of interest earned will be approximately \(4.25 \% \times 3 / 12=1.0625 \%\).
- If sterling is invested for three months, the amount of interest earned will be approximately \(5.5 \% \times 3 / 12=1.375 \%\).
- The three-month forward rate is:
\[
\begin{aligned}
\mathrm{F}_{0} & =0.7000 \times \frac{1.010625}{1.01375} \\
& =0.6978 .
\end{aligned}
\]

Using the formulae: Step 1. Calculate the value of \(d_{1}\) and \(d_{2}\)
\(\ln (0.6978 / 0.7000)=\ln 0.99686\).
Using a calculator, the value of \(\ln 0.99686=0.0031478\).
\(\mathrm{d}_{1}=\frac{0.0031478+(0.1247)^{2} 0.25 / 2}{0.1247 \sqrt{0.25}}\)
\(=0.0050916 / 0.06235\)
\(=0.08166\)
\(\mathrm{d}_{2}=0.08166-0.1247 \sqrt{ } 0.25\)
\(=0.0193\)
Using the formulae: Step 2. Calculate the value of \(N\left(d_{1}\right)\) and \(N\left(d_{2}\right)\)
From normal distribution tables, we get the following values:
■ When \(\mathrm{z}=0.08166\), probability \(=0.0326\). Therefore \(\mathrm{N}\left(\mathrm{d}_{1}\right)=0.0326+0.5000=\) 0.5326 .
- When \(\mathrm{z}=0.0193\), probability \(=0.0078\). Therefore \(\mathrm{N}\left(\mathrm{d}_{2}\right)=0.0078+0.5000=0.5078\).
(Note: \(\mathrm{d}_{1}\) and \(\mathrm{d}_{2}\) both have a positive value so add 0.5000 .)
Using the formulae: Step 3. Calculate the value of an at-the-money call option
\[
\begin{aligned}
& \mathrm{c}=\mathrm{e}^{-\mathrm{rt}}\left[\mathrm{~F}_{0} \mathrm{~N}\left(\mathrm{~d}_{1}\right)-\mathrm{XN}\left(\mathrm{~d}_{2}\right)\right] \\
& \mathrm{e}^{-\mathrm{rt}}=1 / \mathrm{e}^{(0.055 \times 0.25)}=1 / \mathrm{e}^{0.01375}=0.9863 . \\
& \mathrm{c}=0.9863 \times[(0.6978 \times 0.5326)-(0.7000 \times 0.5078)] \\
& \mathrm{c}=0.9863 \times 0.01619=0.016
\end{aligned}
\]

The value of an at-the-money call option is \(£ 0.016\) or 1.6 pence per euro.
The cost of an at-the-money call option on 100,000 euros is therefore \(£ 1,600\).
Using the formulae: Step 4. Calculate the value of an at-the-money put option
\[
\begin{aligned}
& \mathrm{p}=\mathrm{e}^{-\mathrm{rt}}\left[\mathrm{XN}\left(-\mathrm{d}_{2}\right)-\mathrm{F}_{0} \mathrm{~N}\left(-\mathrm{d}_{1}\right)\right] \\
& \mathrm{d}_{2}=0.0193, \text { so }-\mathrm{d}_{2}=-0.0193 \text { and } \mathrm{N}\left(-\mathrm{d}_{2}\right)=0.5000-0.0078=0.4922 . \\
& \mathrm{d}_{1}=0.08166, \text { so }-\mathrm{d}_{1}=-0.08166 \text { and } \mathrm{N}\left(-\mathrm{d}_{1}\right)=0.5000-0.0326=0.4674 .
\end{aligned}
\]
\(\mathrm{p}=0.9863 \times[(0.7000 \times 0.4922)-(0.6978 \times 0.4676)]\)
\(\mathrm{p}=0.9863 \times 0.01825=0.018\)
The value of an at-the-money call option is \(£ 0.018\) or 1.8 pence per euro.
The cost of an at-the-money call option on 100,000 euros is therefore \(£ 1,800\).

\section*{Delta hedging}
- Delta defined
- Constructing a delta hedge for a call options exposure
- Constructing a delta hedge for a put options exposure
- The Greeks

\section*{5 Delta hedging}

Buying call options or put options is a method of hedging financial risk. Options are a zero-sum game, and to the extent that an option holder is protected against risk, the option writer is exposed to risk. For banks and other institutions that write options, their own exposure to risk from writing options can be hedged with a delta hedge.

\subsection*{5.1 Delta defined}

The delta of an option is defined as the change in the price of an option in proportion to the change in the value of the underlying item.

Delta \(=\frac{\text { Change in value of the option }}{\text { Change in market value of underlying item }}\)
For example, in the case of an option on a company's shares, if the share price goes up by \(\$ 1\), the option price will go up if it is a call option or down if it is a put option. However, the change in the price of the option will not exceed \(\$ 1\). The value of an option delta is therefore in the range -1 to +1 .
- A call option that is deeply in-the-money has a delta close to +1
- A put option that is deeply in-the-money has a delta close to -1 .
- An option that is deeply out-of-the-money has a delta close to 0 .
- An option that is at-the-money has a delta close to +0.5 (call option) or -0.50 (put option).

The value of an option's delta can be calculated. It is related the value of \(N\left(-d_{1}\right)\) or \(\mathrm{N}\left(\mathrm{d}_{1}\right)\) in the Black-Scholes option pricing model.

\subsection*{5.2 Constructing a delta hedge for a call options exposure}

A bank that writes a large number of options has an option portfolio. It might want to create a hedge for its exposure to adverse price movements.
- An option portfolio is said to be delta neutral when its delta is 0 .
- A bank that writes call options can create an option position that is delta neutral by purchasing a quantity of the underlying item. For example, a bank that has written call options on the shares of XYZ Company can hedge the position by holding some shares in XYZ.
- If the value of the underlying shares goes up, the value of the call options will also go up. The bank will incur a loss on its options position, because it has written options. However, it makes a gain on the rise in the value of the underlying shares.
- A delta neutral position will exist when the rise in the value of the options (= benefit to the option holders and loss for the option writer) is matched by an equal rise in the value of the shares held by the option writer (bank). This will leave the bank with neither a loss nor a gain.
The number of shares that a call option writer should hold to create a delta hedge \(=\) [Quantity of the underlying item on which there are options] \(\times\) [Option delta.]

For example if the delta value for call options on 1,000,000 shares of XYZ Company at an exercise price of \(\$ 15\) is 0.45 , a delta hedge will be created by holding 450,000 of the shares \((1,000,000 \times 0.45)\).

\section*{Calculating delta for call options}

The value of delta for a position in call options is the value of \(N\left(d_{1}\right)\) in the BlackScholes model formulae:

Amount of underlying item to be hedged \(\times \mathrm{N}\left(\mathrm{d}_{1}\right)=\) Amount of underlying to hold as a hedge

\section*{Example}

A UK bank is writing currency call options on \(€ 200\) million in exchange for British pounds. The value of delta of the options is 0.5326 , measured as \(N\left(d_{1}\right)\).

In order for the bank to be certain that it will not make a loss, it could hedge its options position by holding a quantity of euros.

The quantity of euros that it would need to hold to create a hedge is \(€ 200\) million \(\times\) \(0.5326=€ 106.52\) million.

\section*{Example}

In principle, the same formula can be used to calculate how many at-the-money call options it would be necessary to hold to eliminate exchange rate risk entirely.

For example, suppose that a UK company needs to buy \(€ 100\) million in three months time and wants to use currency call options to create a hedge that would eliminate the exchange rate risk entirely. At-the-money call options on euros have a delta value of 0.5326 . Each option is for \(€ 100,000\).

Re-arranging the previous formula, we can calculate the amount of euros for which at-the-money call options should be purchased is:
\(€ 100\) million/ \(\mathrm{N}\left(\mathrm{d}_{1}\right)\)
= \(€ 100\) million \(/ 0.5326\)
\(=€ 187.758\) million.

Since each option contract is for \(€ 100,000\), this would mean buying 1,878 contracts.
Any increase in the value of the euro would mean that the cost of buying the \(€ 100\) million at the option exercise date would increase, but this 'loss' would be offset entirely by a gain on the call options position.

In practice however, creating a hedge with options that eliminates the risk entirely is very expensive, because of the cost of the options. In this example, if the company did decide to use options to create a hedge, the number of options that it would buy is likely to be \(€ 100\) million/ \(€ 100,000\) per contract \(=1,000\) contracts.

\subsection*{5.3 Constructing a delta hedge for a put options exposure}

A similar technique can be applied to calculate a hedge position for a writer of put options. The difference is that if a bank writes put options, it will incur a loss on the options position if the price of the underlying item falls.

To create a hedge, it therefore needs to be 'short' in the underlying item. It could do this by borrowing the underlying item in the case of shares, or selling the item forward if it is currency.

For example if the delta value for put options on \(1,000,000\) shares of XYZ Company at an exercise price of \(\$ 15\) is -0.27 , a delta hedge will be created by borrowing 270,000 of the shares ( \(1,000,000 \times 0.27\) ), and the settlement date for returning the borrowed shares should be the same as the expiry date for the option..

\section*{Calculating delta for put options}

The value of delta for a position in put options is the value of \(N\left(-d_{1}\right)\) in the BlackScholes model formulae:

Amount of underlying item to be hedged \(\times \mathrm{N}\left(-\mathrm{d}_{1}\right)=\) Amount of underlying to hold as a hedge

\section*{Example}

A UK bank is writing currency put options on \(€ 50\) million in exchange for British pounds. The value of delta of the options is 0.4674 , measured as \(N\left(-d_{1}\right)\).

In order for the bank to be certain that it will not make a loss, it could hedge its options position by selling forward a quantity of euros, for settlement on the same date as the expiry date for the options.

The quantity of euros that it would need to sell forward to create a hedge is \(€ 50\) million \(\times 0.4674=€ 23.37\) million.

\section*{Example}

In principle, the same formula can be used to calculate how many at-the-money put options it would be necessary to hold to eliminate exchange rate risk entirely.

For example, suppose that a UK company will receive \(€ 20\) million in three months time and wants to use currency put options to create a hedge that would eliminate the exchange rate risk entirely. At-the-money put options on euros have a delta value of \(0.4764\left(N\left(-d_{1}\right)\right.\). Each option is for \(€ 100,000\).

Re-arranging the previous formula, we can calculate the amount of euros for which at-the-money call options should be purchased is:
\(€ 20\) million/ \(\mathrm{N}\left(-\mathrm{d}_{1}\right)\)
\(=€ 20\) million \(/ 0.4764\)
= €41,981,528.
Since each option is for \(€ 100,000\) this would mean having to buy 420 put option contracts.

As stated earlier, creating a hedge with options that eliminates the risk entirely is very expensive, because of the cost of the options. In this example, if the company did decide to use options to create a hedge, the number of options that it would buy is likely to be \(€ 20\) million/ \(€ 100,000\) per contract = 200 contracts.

\subsection*{5.4 The Greeks}

You might come across the term 'the Greeks'. Organisations that deal in options calculate a variety of option ratios for establishing and monitoring their option positions. These ratios are called 'the Greeks', because they are all designated by a Greek letter.

\section*{Delta}

Delta has already been explained. It is the change in the price of an option (in money value) for a given change in the price of the underlying item.

The value of delta changes as the price of the underlying item changes. For example, if the market price of a share rises, the delta value of a call option on the shares will increase.

The delta value of an option may also be seen as a probability estimate that the option will expire in-the-money. For example, a deeply in-the-money option has a delta close to +1 (call option) or -1 (put option) and the probability estimate that it will expire in the money is therefore close to \(100 \%\).

A deeply out-of the-money option has a delta close to 0 , and the probability that it will expire in-the-money is therefore close to \(0 \%\).

For an at-the-money option, the probability that it will expire in the money is about \(50 \%\).

\section*{Gamma}

The gamma for an option is the ratio of the change in the delta of an option for a given change in the value of the underlying item. For example, it is the change in the delta of a share option for an increase or decrease of \(\$ 0.01\) in the share price.

The value of gamma for an option is very low as the certainty increases that the option will expire in-the-money or out-of-the-money. The value of gamma increases with uncertainty as to whether the option will expire in-the-money or out of-themoney.

\section*{Vega}

Vega measures the ratio of the change in the option price for a \(1 \%\) change in the volatility of the underlying item. If volatility in the underlying item increases, vega increases.
(It might help to remember that Vega measures sensitivity of the option price to changes in Volatility - two Vs.)

\section*{Theta}

Theta is a measure of the sensitivity of the option price to the remaining time to expiry of the option. Theta decreases as the remaining time to expiry falls, and is higher for options with a long time remaining to expiry.
(It might help to remember that Theta measures sensitivity of the option price to changes in Time to expiry - two Ts.)

The most important of the Greeks is the delta, and (as described earlier) you might be expected in the examination to show your understanding of what it represents and how a delta hedge might be constructed.

\section*{Paper P4}

Advanced Financial Management

\section*{17}

\section*{Financial management and multinationals}

\section*{Contents}

1 Problems for multinational companies
2 Transfer pricing and taxation
3 Free trade and protectionism
4 International and financial markets

\section*{Problems for multinational companies}
- Global competition
- MNCs and national governments: political risk and regulatory risk
- MNCs and management co-ordination

\section*{1 Problems for multinational companies}

\subsection*{1.1 Global competition}

In many industries, the market place and the competition is global. Global competition exists largely for products, but there is also global competition in many service industries, such as banking and financial services and telecommunications. When an industry becomes global, there is usually a trend towards the creation of a number of large multinational companies or corporations (MNCs). Powerful MNCs make important economic decisions about:
- where to locate production facilities and who to employ
- pricing goods (and services).

Some large MNCs control very large amounts of resources, and have more economic power than many of the countries in which they invest.

\subsection*{1.2 MNCs and national governments: political risk and regulatory risk}

The main objective of a MNC is presumably to maximise the wealth of its shareholders. The main objective of many national governments is to provide for the well being of their population. The objectives of MNCs and national governments therefore differ, particularly when the MNC is a 'foreign company' with its head office in another country.

National governments might expect MNCs to:
- invest in their country, and provide employment
- through investment, add to the economic wealth of the country and help to develop the country's economy
- pay taxes on their profits, that the government can spend on the well-being of its population.

MNCs might therefore be exposed to large political and regulatory risks when they invest in another country, particularly when the local government and population suspect that the multinational is more concerned about the interests of its shareholders than in the welfare of the country. Potential risks include:
- the risk of confiscation of their assets by the government
- the risk of new regulations of the industry, such as regulations requiring minimum ownership in all businesses by 'nationals' of the country
- the risks of political unrest and violence.

Some MNCs are aware of their corporate social responsibility to the countries in which they invest, and are aware of the need to educate the local population, preserve the local environment and create a sustainable industry within that country. On the other hand, some MNCs might show little awareness of corporate social responsibility, and seek to avoid paying taxes if possible. As a consequence, in many countries, MNCs are treated with dislike and suspicion.

Investment decisions by multinationals might be affected by the perceived political and regulatory risks of investing in particular countries.

\subsection*{1.3 MNCs and management co-ordination}

Corporate governance issues were described in an earlier chapter. Corporate governance is a particular problem for MNCs. A part of the problem is that MNCs need to recognise the interests of local governments and populations as stakeholders in their foreign subsidiaries.

Another issue is the agency problem, and the requirement for management to act in the bests interests of the company's shareholders. In large multinational companies, a considerable amount of authority is delegated to 'local' management in divisions and subsidiaries around the world. Delegation of authority is essential for the effective management of large global businesses.

On the other hand, when authority is delegated to local managers, head office management need to co-ordinate the worldwide activities of the MNC, and try to ensure that the strategic objectives of the MNC are met and the interests of its shareholders are satisfied.

Decisions by head office management might be resisted by local management in some countries, where head office managers might be trying to act in the interests of the shareholders, whereas local managers have more concern for their own position or for the interests of employees, customers and economic conditions in their own country.

\section*{Transfer pricing and taxation}
- Corporate objectives and the relevance of tax and transfer pricing
- MNCs: transfer pricing and tax rules
- Multinationals and tax planning

\section*{2 Transfer pricing and taxation}

\subsection*{2.1 Corporate objectives and the relevance of tax and transfer pricing}

The corporate objective of a multinational company, as for any other company, is linked to profitability and returns for shareholders. Multinational companies operate in different countries, and are therefore subject to differing tax regulations and systems. MNCs will try to improve earnings for shareholders by minimising their worldwide tax liabilities.
- Multinational companies will seek to invest in countries where the taxation system is more favourable. For example, companies might seek relief from taxation for making capital investments in a country, and will choose one country in preference to another on the basis of the tax advantages offered
- Some MNCs have been accused of using transfer pricing to minimise their tax liabilities. Transfer prices are used to charge for goods and services transferred between companies in the group. Transfer prices can be charged in such as way that:
- transfers from a subsidiary in a high-tax country to a subsidiary in a low-tax country are fixed at high levels, and
- low transfer prices are set for transfers from a low-tax to a high-tax country.

In this way, profits in the high-tax subsidiaries are reduced, and more profits are earned in the low-tax countries.

\subsection*{2.2 MNCs: transfer pricing and tax rules}

Transfer prices are prices charged internally within a group, and have no effect on the total pre-tax profit of the group. Transfer prices are simply a way of sharing the profit between the two companies concerned.

Transfer prices between a company and a foreign subsidiary have implications for taxation, and the aim of a group of companies should be to minimise total tax payable on the group profits. Issues to consider are:
- the rate of taxation on profits in the two countries
- the existence of a double taxation agreement
- the existence of any withholding tax in the country of the foreign subsidiary
- tax rules on transfer pricing.

\section*{Tax rates}

The rate of taxation on profits will vary between the two countries and two companies concerned. As a general principle, the aim of a group should be to set a transfer price that shares the profits in a way that reduces the profits of the company in the higher-tax country.

For example, suppose that a parent company in country A sells goods to its \(100 \%\) owned subsidiary in country B, and the rate of tax is \(20 \%\) in country A and \(30 \%\) in country B. The transfer price should be set as high as the tax rules permit, because this will increase profits of the parent company in lower-tax country A and reduce the profits of the subsidiary in higher-tax country B.

\section*{Double taxation agreement}

There should usually be a double taxation agreement between the two countries. From the point of view of the parent company, this means that:
- If the rate of tax on profits is higher in the country of the foreign subsidiary than in the parent company's country, the subsidiary pays tax in its own country on its profits at the rate applicable in that country. No further tax is payable in the parent company's country on the profits of the foreign subsidiary.
- If the rate of tax on profits is lower in the country of the foreign subsidiary than in the parent company's country, the subsidiary pays tax in its own country on its profits at the rate applicable in that country. However, tax is also payable by the parent company in its own country. The tax payable is the difference between tax on profits at the rate in the parent company and the tax payable by the subsidiary in its own country.

\section*{Withholding tax}

Withholding tax is additional tax that is 'withheld' when a company pays interest or dividends to a foreign investor. It affects international groups where foreign subsidiaries are located in a country where withholding tax is charged. Payments of interest or dividends by a subsidiary to the parent company will be subject to the withholding tax.

\section*{Tax rules on transfer prices}

The tax rules in a country are likely to make it difficult for an international group to charge inter-company transfer prices that are significantly different from market prices (where these exist). This is because the tax authorities recognise that transfer prices might be set artificially so as to minimise the total tax burden for the group.

These various tax aspects of transfer pricing are illustrated by the following example.

\section*{Example}

A UK company has recently acquired a foreign subsidiary in another country, Outland. A part of the operating arrangements of the new group is that the parent company will export a component to the subsidiary, and the subsidiary will use this component to make a product that it will sell in Outland. The management of the parent company have to decide what the transfer price for the component should be.

The following information is available:
\begin{tabular}{llll}
\begin{tabular}{lll} 
UK (annual) \\
Components transferred to \\
Outland
\end{tabular} & 100,000 units & \begin{tabular}{l} 
Outland (annual) \\
Units of product sold
\end{tabular} & 100,000 units \\
Variable cost per unit & \(£ 15\) & Sales price per unit & \(\$ 150\) \\
Fixed costs & \(£ 900,000\) & \begin{tabular}{l} 
Local variable cost per unit
\end{tabular} & \(\$ 40\) \\
Tax rate & \(30 \%\) & Local fixed costs & \(\$ 600,000\) \\
& Tax rate & \(25 \%\)
\end{tabular}

The exchange rate is \(£ 1=\$ 4\).
Withholding tax of \(10 \%\) is charged on all remittances of interest and dividends from Outland. The UK parent company intends to remit all available profits from Outland to the UK.

There is a double taxation agreement between the UK and Outland. Tax on income and distributions in one country may be credited against a tax liability on the same income in the other country.

\section*{Required}

Calculate the group profits if the transfer price for the component is set at
(a) its market price, which is \(£ 26\) per unit
(b) total UK cost per unit plus \(25 \%\).

\section*{Answer}

Tax on profits in Outland will be \(25 \%\), but there will be an additional withholding tax of \(10 \%\) since all available profits will be remitted to the UK. The total tax payable on income in Outland is therefore \(35 \%\) which is higher than the rate of UK tax. This means that profits earned in Outland will not be taxed at all in the UK.
(a) Transfer price \(=\) market price
\begin{tabular}{lrr} 
Parent company, UK & \(£\) & \(£\) \\
Component sales (£26 per unit) & & \(2,600,000\) \\
Variable costs (£15 per unit) & \(1,500,000\) & \\
\cline { 2 - 2 } Fixed costs & 900,000 & \\
& & \(2,400,000\) \\
\hline Profit before tax & & 200,000 \\
Tax at 30\% & & \((60,000)\) \\
Profit after tax & & 140,000 \\
\hline
\end{tabular}

The transfer price in Outland dollars is \(£ 26 \times \$ 4=\$ 104\).
Subsidiary company, Outland
Product sales (\$150 per unit)
Transfer costs (\$104 per unit)
Local variable costs (\$40 per unit)
Local fixed costs

Profit before tax
Tax at 25\%
Profit after tax
Withholding tax at 10\%
Profit in the UK after tax
\begin{tabular}{|c|c|c|}
\hline \multirow[t]{2}{*}{\$} & \$ & \\
\hline & \multicolumn{2}{|l|}{15,000,000} \\
\hline \multicolumn{3}{|l|}{10,400,000} \\
\hline \multicolumn{3}{|l|}{1,500,000} \\
\hline \multicolumn{3}{|l|}{600,000} \\
\hline \multicolumn{3}{|c|}{12,500,000} \\
\hline \multicolumn{3}{|c|}{2,500,000} \\
\hline \multicolumn{3}{|c|}{\((625,000)\)} \\
\hline \multicolumn{3}{|c|}{1,875,000} \\
\hline \multicolumn{3}{|c|}{\((187,500)\)} \\
\hline & 1,687,500 & \(=£ 421,875\) \\
\hline
\end{tabular}

The total group profit will be \(£ 140,000+£ 421,875=£ 561,875\).
(b) Transfer price \(=\) cost plus \(\mathbf{2 5 \%}\)

Total UK costs are \(£ 2,400,000\) or \(£ 24\) per unit. Cost plus \(25 \%=£ 30\) per unit. The transfer price will therefore be \(£ 30\), which is \(\$ 120\) per unit.
\begin{tabular}{lr} 
Parent company, UK & \(£\) \\
Component sales (£30 per unit) & \(3,000,000\) \\
Total costs (as above) & \(2,400,000\) \\
\hline Profit before tax & 600,000 \\
Tax at 30\% & \((180,000)\) \\
Profit after tax & 420,000 \\
\hline
\end{tabular}
\begin{tabular}{lrr} 
Subsidiary company, Outland & \(\$\) & \(\$\) \\
Product sales (\$150 per unit) & & \(15,000,000\) \\
Transfer costs (\$120 per unit) & \(12,000,000\) & \\
Local variable costs (\$40 per unit) & \(1,500,000\) & \\
Local fixed costs & 600,000 & \\
& & \(14,100,000\) \\
\hline Profit before tax & 900,000 & \\
Tax at 25\% & \((225,000)\) & \\
Profit after tax & 675,000 & \\
Withholding tax at 10\% & \((67,500)\) & \\
Profit in the UK after tax & & 607,500 \\
\hline
\end{tabular}

The total group profit will be \(£ 420,000+£ 151,875=£ 571,875\).
The higher transfer price will provide a bigger after-tax profit for the group. However, the tax authorities in Outland might not agree to accept a transfer price of \(\$ 20\), because this is \(\$ 4\) per unit above the market price for the component.

\subsection*{2.3 Multinationals and tax planning}

Multinationals are subject to tax on their profits in all the countries in which they operate. They are also subject to tax on their world-wide profits in their country of registration. As explained earlier, the tax burden is reduced by the existence of double taxation agreements between countries, and a multinational is able to set off tax paid in foreign countries against tax payable in the country of registration.

For example, a UK multinational may be liable to tax in Country \(X\) on the profits of a subsidiary in Country \(X\). Suppose that the amount of this tax liability is the equivalent of \(£ 500,000\). The UK parent company will also be liable to tax in the UK for the profits (or dividends) of the foreign subsidiary. If a double taxation agreement exists between the UK and Country \(X\), the following tax positions might arise.
- If the UK tax liability for the foreign subsidiary is more than \(£ 500,000\), tax payable in the UK will be reduced by the tax payable in Country \(X(£ 500,000)\).
- If the UK tax liability for the foreign subsidiary is less than \(£ 500,000\), no tax is payable in the UK.

\section*{Tax planning to reduce a tax liability}

A multinational may be able to reduce its world-wide tax liability through tax planning. One measure might be to establish a subsidiary company in a tax haven, and by channelling its profits and capital gains in other countries through the tax haven, its total tax liability might be reduced.

An example may help to illustrate this point.

\section*{Example}

A UK multinational company has three wholly-owned foreign subsidiaries, in Country A, Country B and Country C. The profits of each subsidiary for the year are the equivalent of \(£ 100,000\). The rate of corporate tax in the UK is \(30 \%\).

The tax rates in the other countries are set out below.
\begin{tabular}{lrrr} 
& \begin{tabular}{r} 
Tax on \\
profits
\end{tabular} & \begin{tabular}{r} 
Withholding tax on \\
dividends
\end{tabular} & \begin{tabular}{r} 
\% of after-tax profit \\
remitted to the UK as \\
dividends
\end{tabular} \\
Country & \(20 \%\) & \(5 \%\) & 90 \\
\hline Country A & - & 90 \\
Country B & \(25 \%\) & \(10 \%\) & 80
\end{tabular}

Double taxation agreements exist between the UK and each of the three countries.

\section*{Required}

Calculate the world-wide tax liability of the UK multinational in each of the three following circumstances:
(a) The UK government charges companies tax on the dividends remitted from foreign subsidiaries. The amount of the dividend is grossed up at the local tax rate, before deduction of any withholding tax, and UK tax is calculated on this grossed-up dividend.
(b) The UK government charges multinationals tax on the gross income of their foreign subsidiaries. A tax credit is available for all corporate taxes paid in foreign countries, including withholding tax.
(c) The multinational sets up a subsidiary in a tax haven (Country D). All income from the foreign subsidiaries would be transferred to the UK via this tax haven
company. The UK government would then treat all the foreign income as coming through a single source, the tax haven company in Country D.
Calculate the total tax payable on the foreign profits of the multinational if the tax rules are:
(i) the same as in (a)
(ii) the same as in (b).

\section*{Answer}
(a)
\begin{tabular}{|c|c|c|c|c|c|c|}
\hline & \multicolumn{2}{|r|}{Country A} & \multicolumn{2}{|r|}{Country B} & \multicolumn{2}{|r|}{Country C} \\
\hline & & £000 & & £000 & & £000 \\
\hline Taxable income & & 100.0 & & 100.0 & & 100.0 \\
\hline Local tax on profits & (20\%) & 20.0 & (25\%) & 25.0 & (40\%) & 40.0 \\
\hline & & 80.0 & & 75.0 & & 60.0 \\
\hline Dividend payment before withholding tax & (90\%) & 72.0 & (90\%) & 67.5 & (80\%) & 48.0 \\
\hline Withholding tax & (5\%) & 3.6 & & & (10\%) & 4.8 \\
\hline Dividends to UK before deducting withholding tax & & 72.0 & & 67.5 & & 48.0 \\
\hline Gross up for local tax & ( \(\times 100 / 80\) ) & 90.0 & ( \(\times 100 / 75\) ) & 90.0 & ( \(\times 100 / 60\) ) & 80.0 \\
\hline UK tax liability at 30\% & & 27.0 & & 27.0 & & 24.0 \\
\hline Foreign tax credit (see note below) & \((20+3.6)\) & 23.6 & & 25.0 & & 24.0 \\
\hline UK tax payable & & 3.4 & & 2.0 & & 0.0 \\
\hline
\end{tabular}

Note: The foreign tax credit is the lower amount of tax payable in the foreign country and tax payable in the UK.
\begin{tabular}{lr} 
Summary & \(£ 000\) \\
\hline Tax payable in other countries \((20+25+40+3.6+4.8)\) & 93.4 \\
Tax payable in the UK \((3.4+2.0)\) & 5.4 \\
Worldwide tax liability & 98.8 \\
\cline { 2 - 2 }
\end{tabular}
(b)
\begin{tabular}{|c|c|c|c|c|c|c|}
\hline & \multicolumn{2}{|r|}{Country A} & \multicolumn{2}{|r|}{Country B} & \multicolumn{2}{|r|}{Country C} \\
\hline & & £000 & & £000 & & £000 \\
\hline Taxable income & & 100.0 & & 100.0 & & 100.0 \\
\hline Local tax on profits & (20\%) & 20.0 & (25\%) & 25.0 & (40\%) & 40.0 \\
\hline & & 80.0 & & 75.0 & & 60.0 \\
\hline Dividend payment before withholding tax & (90\%) & 72.0 & (90\%) & 67.5 & (80\%) & 48.0 \\
\hline Withholding tax & (5\%) & 3.6 & & & (10\%) & 4.8 \\
\hline Income for UK tax purposes & & 100.0 & & 100.0 & & 100.0 \\
\hline UK tax liability at 30\% & & 30.0 & & 30.0 & & 30.0 \\
\hline Foreign tax credit (see note above) & & 23.6 & & 25.0 & & 30.0 \\
\hline UK tax payable & & 6.4 & & 5.0 & & 0.0 \\
\hline
\end{tabular}
\begin{tabular}{lr} 
Summary & \(\mathbf{£ 0 0 0}\) \\
\hline Tax payable in other countries \((20+25+40+3.6+4.8)\) & 93.4 \\
Tax payable in the UK \((6.4+5.0)\) & 11.4 \\
\cline { 2 - 2 } Worldwide tax liability & 104.8 \\
\hline
\end{tabular}

Under these tax rules, more tax is payable in the UK than with the tax system in (a).
(c)
\begin{tabular}{lrrr} 
& \begin{tabular}{r} 
Tax system \\
in (a)
\end{tabular} & \begin{tabular}{r} 
Tax system \\
in (b)
\end{tabular} \\
\hline Tax payable in other countries & \(£ 000\) & \(£ 000\) \\
& & 93.4 & 93.4 \\
UK tax liability & & & \\
& \((90+90\) & 260.0 & 300.0 \\
UK tax at 30\% & & \\
Foreign tax credit & & 78.0 & 90.0 \\
Tax payable in the UK & & 78.0 & 90.0 \\
\hline
\end{tabular}

The worldwide tax payable is just \(£ 93,400\) because no tax is now payable in the UK (because a tax credit of up to \(£ 93,400\) would be available). Setting up the tax haven therefore reduces the word-wide tax burden for the multinational.

\section*{Free trade and protectionism}
- Protectionism
- Free trade
- The arguments for free trade and potectionism
- Common markets
- Restrictions on movements of capital or currency
- World Trade Organisation (WTO)

\section*{3 Free trade and protectionism}

You need to be aware of the arguments for and against free trade, and for and against protectionism.

\subsection*{3.1 Protectionism}

Protectionism involves measures by a national government to protect its domestic industries and companies against competition from imports by foreign suppliers. Protectionism takes several forms.
- Taxes on imports of particular types of goods. These are import tariffs, which are charged on goods entering the country.
- Import quotas. These are limits on the quantities of particular items that can be imported.
- Import ban. There might be a ban on the import of certain items.
- Procedural regulations that make it time-consuming and expensive to import goods (for example, documentation requirements, slow procedures by the customs authorities).
- Imposing regulations relating to the required minimum standard of products, which imported goods may not meet; for example a country may impose that require particular products (such as cars) to meet minimum environmental regulations.
- Regulations requiring a proportion of any new business to be owned by 'local' companies or individuals.
- Providing government subsidies to national companies, so that they can compete more successfully against foreign companies. For example, the European Union is criticised in discussions on world trade for the large agricultural subsidies paid to its farmers.
- Protectionist measures may also take the form of government restrictions or legal restrictions on the purchase of domestic companies by foreign investors or foreign companies.

Protectionist measures are used by governments to preserve jobs and businesses in their own country. They exist not only in developing and under-developed countries, but also in countries with advanced economies.

\subsection*{3.2 Free trade}

Free trade is the conduct of international trade without protectionist measures. Free trade involves the absence of import tariffs (or the taxation of imported goods at the same rate as the tax on domestically-produced goods), and the absence of import quotas and government subsidies.

\subsection*{3.3 The arguments for free trade and protectionism}

The governments of many countries have accepted the principle that free trade will increase the total volume of international trade. Total output in the world economies will grow, there will be more employment. Economic growth creates additional wealth. The supporters of free trade argue that this additional wealth will benefit all the countries involved and will be shared between them.

However, although there is agreement on the principle that free trade leads to growth in the world economy, it might be argued that it increases the wealth of some countries but reduces the wealth of others. For example, free trade might help the economies of the advanced industrialised nations to expand, but might prevent the economies of under-developed countries from growing. It might therefore be argued that protectionist measures are necessary to give domestic industries an opportunity to develop, and to protect jobs in those industries.

There are also difficulties raised by the fact that protectionist measures exist and are used extensively by governments. Some countries refuse to abandon their measures for protection as long as another country still has different measures in place relating to different industries and products. For example, one country night refuse to remove tariffs on imports of agricultural produce when another country charges tariffs on imports of electronic goods.

There are also some difficulties relating to intellectual property, such as patents, trade marks and copyright. Although there are international agreements for the protection of intellectual property, not all countries have subscribed to them and intellectual property laws are not applied worldwide. This creates disputes between countries that have such laws and those that do not.

The World Trade Organisation seeks to achieve free trade internationally, but needs to win the agreement of all the countries affected. Progress towards the abolition of protectionism has therefore been slow (and continues to be slow).

\subsection*{3.4 Common markets}

Some countries have joined together to create common markets in which there is free trade between the member countries. The most notable example is probably the European Community, particularly the eurozone of countries sharing the euro as their common currency.

Free trade and either a common currency or exchange rate stability create an environment in which international trade can grow.

\subsection*{3.5 Restrictions on movements of capital or currency}

Some developing countries seek to restrict the movement of capital or foreign currency out of the country, by imposing foreign exchange restrictions or restrictions on the movement of capital.

You might be given an examination question involving the appraisal of a proposed capital investment in a foreign country where the government applies restrictions on the payment of dividends to foreign shareholders. Companies investing in the country (by setting up a local subsidiary) are therefore unable or unwilling to 'repatriate' the profits from their foreign investment, because:
- there is a ban on payments of dividends to foreign investors, so that companies they cannot pay themselves dividends on the profits of their foreign subsidiary, or
- (more commonly) there is a withholding tax on the payment of dividends and interest to foreign investors, so that companies have to pay an additional tax if they want to repatriate profits of their foreign subsidiary.

\subsection*{3.6 World Trade Organisation (WTO)}

The World Trade Organisation is an organisation to which countries become members on a voluntary basis. Most countries of the world are members of the WTO.

The aim of the WTO is to encourage countries to:
- adopt more free trade measures
- remove protectionist barriers against imports of goods or services from other countries
- ban the practice of 'dumping' surplus goods at a low price on the markets of other countries. (Dumping can have a serious adverse effect on the domestic industry of the countries where the goods are 'dumped'.)

The WTO promotes the view that reducing barriers to world trade (protectionist measures) will promote economic growth and prosperity.

There are three key agreements that have been negotiated and signed by most of the world's governments:
- The General Agreement on Trade and Tariffs (GATT). This is an agreement relating to trade in goods.
- A General Agreement on Trade in Services
- The Agreement on Trade-Related Aspects of Intellectual Property.

These agreements contribute towards free trade and the removal of protectionist barriers, but they are not 'perfect' and many protectionist measures still exist. Some of these measures can lead to serious disputes between governments, which sometimes escalate. One country's government might demand the removal of protectionist measures against its exports by another country, and make threats of protectionist counter-measures in retaliation.

It is generally accepted that there is still a long way to go in achieving the objective of free trade. Member countries meet regularly and try to come to agreements on trade. The WTO encourages direct discussions between particular countries or blocs of countries, such as the US and the European Union, and offers an arbitration service for the settlement of trade disputes.

All member countries also meet regularly to try to agree a world-wide series of new measures to enhance free trade and remove trade barriers. The so-called Uruguay Round of discussions began in 1986 and agreement was eventually reached on some new measures in 1994.Starting in 2001, the WTO tried to obtain agreement to a variety of new measures. The discussions between governments have been called the 'Doha Development Agenda' or the 'Doha round' of talks. So far, the talks have failed to make any significant progress, and the discussions were formally suspended in July 2006 without any agreement having been reached.

\section*{International and financial markets}
- International lending
- International bond markets
- International money markets
- International equity
- The World Bank
- International Monetary Fund (IMF)
- Systemic risk in the global financial system
- Bank for International Settlements (BIS)
- Raising capital in the world's financial markets

\section*{4 International financial markets}

\subsection*{4.1 International lending}

The capital markets, as well as many markets for goods and services, are international. Capital can be transferred freely between many countries, usually in one of the major international currencies (particularly the US dollar, euro and Japanese yen).

Large companies are able to obtain large syndicated bank loans in a major currency, from syndicates of international banks.

\subsection*{4.2 International bond markets}

Companies can raise debt finance by issuing bonds. There are domestic bond markets in many countries, but these markets are often dominated by domestic government bonds. For example, in the UK the domestic bond markets are dominated by the market for UK government bonds (gilts). Some countries, notably the US, have large domestic bond markets for corporate bonds.

International companies are able to obtain bond finance by issuing bonds:
- in the international bond markets, or
- in a domestic bond market such as the US.

The international bond markets are still often called the eurobond markets.
The main features of the international bond markets are as follows:
- Governments, companies and other organisations are able to issue bonds in the market, with the advice and assistance of financial institutions in several countries.
- Bonds can be denominated in any major currency, but the most commonly-used currencies for the denomination of international bonds are US dollars and euros.
- The bonds are sold by banks in different countries to investment institutions in different countries.
- A second-hand market in the bonds is provided by some of the banks after the issue has been made.

\section*{How do companies raise capital in the international bond markets?}

The ability of companies to raise debt capital by issuing bonds varies between countries as well as between different types of company.
- It is much easier to issue bonds in the domestic US bond market than in other countries. The UK, for example, has only a small sterling corporate bond market.
- When there is a large domestic bond market, it is easier for smaller companies to issue bonds, because investors in those markets are more willing to consider investing in smaller companies. In the European bond markets, in contrast, investors generally prefer to invest in bonds of large companies.
- It is possible for foreign companies to issue bonds in a domestic bond market. For example, foreign companies might be able to issue bonds in the US market. However, foreign issuers need to be aware of the regulations in the bond market, and they need to persuade investors in the market to buy their bonds. Investors might be reluctant to buy bonds of foreign companies that they do not recognise.

If you are asked in your examination about raising debt capital for a foreign investment, you should consider the possibility of using a foreign subsidiary to issue bonds in the country concerned. For example, it might be possible for a company wanting to invest in China to issue bonds in the Chinese bond market. Such a strategy might be possible, although this would depend on the size and nature of the domestic bond market concerned and the amount of debt finance that the company needs to obtain. You cannot be expected to know the current status of bond markets in every financial centre in the world, but you should be aware in general terms that a possibility of borrowing in other financial centres might exist.
(Bond issues are normally for large amounts of capital. Companies wanting smaller amounts of debt capital are much more likely to try to obtain a bank loan.)

\subsection*{4.3 International money markets}

The international money markets can be used to borrow short-term from banks in foreign currencies. For example, companies can borrow US dollars short-term (up to 12 months or possibly longer) in London. There are several major international money market centres around the world. The largest is London.

\subsection*{4.4 International equity}

Investment institutions buy equity shares in countries throughout the world. Some countries have larger and more liquid stock markets than others. Investment institutions prefer to invest in stock markets that are properly regulated, and where the market is 'liquid'. (A liquid market for shares is a market in which shares can be readily bought or sold at a price that is a fair market price. The fair price is usually at or close to the most recent traded price for the shares).

However, although investment institutions might invest internationally, most stock exchanges are national. A company wishing to issue new shares (or to seek a flotation for its shares, when it is coming to the stock market for the first time) will issue them in a specific stock market, usually a domestic stock market in its own country.

There have been some movements towards the creation of international stock markets, but progress is slow. For example, the stock exchanges of Paris, Brussels and Amsterdam and also the London-based LIFFE futures exchange, are now combined within a single exchange organisation, Euronext.

\subsection*{4.5 The World Bank}

The international capital and money markets are accessible for governments of many countries and large companies. However, they are not accessible to governments and companies in many developing countries or to governments of under-developed countries.

However, unless under-developed countries and developing countries can improve their infrastructure of transport and communication systems and standards of education, foreign companies might be reluctant to invest in new capital projects. Before companies invest large amounts of capital, governments therefore need to invest heavily in their economic infrastructure

The difficulties of developing countries in gaining access to capital have been recognised, and a role of the World Bank is to provide loan finance to countries in need of capital. In practice, this means lending to developing countries, subject to certain conditions (such as the requirement for the borrowing country to maintain strict discipline over its economic policies).

The objective of the World Bank is to promote long-term economic development and reduce poverty, by providing technical and financial support to help countries reform particular sectors of their economy and implement specific projects such as building schools, providing water and electricity and protecting the environment.

Financial support provided to countries by the World Bank is generally long-term, and the World Bank obtains funding to provide this financial assistance:
- from contributions by governments of wealthy countries, and
- by issuing bonds in the international bond markets.

Even so, the funds provided by the World Bank are insufficient to meet the total needs of developing and under-developed countries.

The World Bank consists mainly of two institutions:
- the International Bank for Reconstruction and Development (IBRD)
- the International Development Association (IDA).

\section*{International Bank for Reconstruction and Development (IBRD)}

The IBRD provides financial assistance to 'middle income' countries and creditworthy poorer countries. This financial assistance is in the form of loans to pay for projects that:
- will reduce poverty and promote sustainable economic growth, and
- private sector companies will not finance.

The IBRD obtains most of the finance for its loans from money raised through issues of bonds by the World Bank. The loans are repayable.

\section*{International Development Association (IDA)}

The IDA provides assistance to poorer countries in the form of interest-free loans and grants (known as 'credits'). Interest-free loans are repayable over a period of about \(35-40\) years, often with a grace period of ten years during which no repayments at all are made. Grants and loans are provided for projects relating to health, education, infrastructure development and similar purposes.

Some examples of IDA projects in recent years are as follows:
- In Nicaragua, finance was provided to restore a section of the Pan American Highway, and other roads, after severe devastation by Hurricane Mitch.
- In Bangladesh the IDA has provided finance for the building of a number of secondary schools for girls.
- In Vietnam, finance was provided to improve electricity supply.

\subsection*{4.6 The International Monetary Fund (IMF)}

The International Monetary Fund (IMF) was established in 1944 as an international organisation with member countries, whose purpose was to:
■ improve international monetary co-operation
- promote exchange rate stability
- foster economic growth, and

■ provide temporary financial assistance to countries with balance of payments difficulties.

It has three main functions.
■ Surveillance. The IMF carries out regular in-depths appraisals of the economic situation in each member country, and issues a report on its findings.
- Providing temporary financial assistance to some countries.

■ Providing technical assistance to countries, for example on fiscal policy issues, monetary policy, exchange rate policy and supervision and regulation of a country's financial system and markets.
Finance for the IMF comes from quota subscriptions by member countries, which are denominated in Special Drawing Rights or SDRs. The maximum amount of temporary financing that a company can obtain from the IMF is related to the size of its quota subscriptions.

\subsection*{4.7 Systemic risk in the global financial system}

Systemic risk in the financial system refers to the risk that financial collapse in one part of the system, such as the financial collapse of a large international bank or a sharp fall in share prices in a major stock market, might have severe repercussions throughout the world's financial systems and lead to the collapse of other institutions and markets.

Systemic risk has grown as the global economy has developed, because financial institutions and markets are often very closely linked. International banks operate in many countries of the world, and banks in different countries make financial transactions with each other. Large investment institutions hold securities in many different countries, and might therefore be vulnerable to a major collapse in share prices in a foreign stock market.

The potential threat from systemic risk is recognised, and efforts have been made to reduce risks globally in financial systems. The regulation of financial markets is largely conducted at national level (or through EU Directives in the case of the European Union). In banking, the prudential regulation of international banks is based on rules developed by the Bank for International Settlements (BIS).

\subsection*{4.8 Bank for International Settlements (BIS)}

The BIS is based in Basle in Switzerland and was established in 1930. Its aim is to foster international monetary and financial co-operation, and its membership consists of the central banks of many of the countries of the world. It also provides financial services to assist central banks.

A committee of the BIS is the Basle Committee on Banking Supervision, which is a forum for achieving co-operation on matters relating to the supervision of banks. In 1988, the Basle Committee produced a set of rules and guidelines for international banks, known as the Basel Accord. ('Basel' is the German spelling of Basle, which is a French spelling.)

The main purposes of the Basel Accord were to:
- reduce systemic risk in the international banking system by requiring banks to maintain a minimum amount of capital in relation to their credit risks
■ achieve a 'level playing field' in international banking, by preventing some banks from competing over-aggressively by lending too much money with too little capital.

The requirement for a minimum amount of capital in relation to credit risks meant that banks would be much more secure financially. If a bank suffers large bad debt losses on its lending, its capital should be sufficient to absorb the losses without threatening the bank with financial collapse.

The Basel Accord did not have the force of law, but it was adopted by many countries. Countries adopting the Basel rules monitor the financial position of banks. In the UK for example the 'prudential supervision' of banks is the responsibility of the financial markets regulator, the Financial Services Authority.

The original Basel Accord required banks to have a minimum amount of capital to cover credit risk. The rules were extended in 1996 to cover 'market risks' as well. These are risks of losses by banks on their trading positions in the financial markets - losses due to adverse movements in share prices, bond prices, interest rates, currency exchange rates and commodity prices.

The Basel Accord is now being replaced in many countries by an 'improved' set of capital adequacy rules for banks, known as Basel 2. Banks are required by the new rules to maintain a minimum amount of capital to cover credit risks, market risks and also more general operational risks. However, banks are allowed to use their own internal computer models to calculate how much their minimum capital requirement should be. (Another 'pillar' of the Basel 2 system is that banks should be subject to supervision by a financial markets regulator or central bank.)

The Basel rules have reduced financial risks in the international banking system. This also reduces risks for international companies that deal with banks in different countries.

\subsection*{4.9 Raising capital in the world's financial markets}

Multinational companies might be able to raise capital in different financial markets around the world, should they wish to do so. To be able to raise capital in any financial market, a company must meet the legal, regulatory and compliance requirements that apply in that particular market.

\section*{Legal requirements}

There are certain legal requirements that must be met before a company can raise capital from investors on a stock market. In the UK for example, companies cannot raise capital from the general investing public unless they have the status of a public limited company.

\section*{Regulatory requirements}

There are also regulatory requirements that have to be met. These include regulations relating to obtaining official permission to raise capital in the form of shares or bonds, and issuing a prospectus to potential investors. Having obtained approval from the regulatory authorities, the shares or bonds must be accepted for trading by a stock exchange.

\section*{Compliance requirements}

Having issued shares or bonds, the company must then continue to comply with continuing obligations (rules) of the regulatory authorities and the stock market. These include rules relating to the regular provision of information about the company to the stock market (such as profit warnings and trading updates).
To meet the regulatory and compliance requirements for issuing shares or bonds on a stock market, a company might have to commit considerable time and money. The burden is greater for shares than for bonds. This is because bonds are often traded 'privately' between professional and institutional investors whereas shares are traded much more on public stock markets.

\section*{The process of raising capital by issuing shares in a stock market}

Although the detailed rules vary between countries, the broad nature of the rules for issuing shares in a stock market are fairly similar in most countries with a large stock market. There is a two-stage process of acceptance before the shares can be issued.
- The company and its shares must be approved by a regulatory authority for the country's stock markets. In the UK, this involves getting the shares accepted on to an 'Official List' by the Listing Authority, which is a department of the financial markets regulator (the Financial Services Authority), or by a similar listing authority in any other EEA country of Europe. Obtaining a listing for the shares includes an approval process for a draft prospectus for the share issue. Companies whose shares have been accepted on to the Official List are called 'listed companies'. In the US, approval must be obtained from the US stock markets regulator, the Securities and Exchange Commission or SEC.
- The company must also apply to the country's stock exchange for the shares to be accepted for trading on the stock market operated by the exchange. In the UK, for example, companies apply to the London Stock Exchange for the new shares to be admitted to trading on the exchange. Similarly, in the US, a company might apply for its shares to be traded on the New York Stock Exchange or NASDAQ.

When the shares have been issued and are trading on the stock market, the company must comply with continuing regulations of both the financial markets regulator and the stock exchange. In the UK for example, listed companies must comply with the Listing Rules and the Disclosure and Transparency Rules of the financial markets regulator, and must also comply with the Admission and Disclosure Standards of the London Stock Exchange which require for example that securities must be eligible for electronic settlement.

Some multinational companies obtain listings for their shares in two or more countries. For example a company in Germany might obtain a listing in Germany and its shares might be traded on the German stock market. In addition, the company might obtain a listing for its shares in the USA or in the UK.

International companies might obtain a listing for their shares in a major international stock market such as London, in addition to their own country. In recent years, several major companies in countries such as Russia and Kazakhstan have sought a listing in the UK. The advantage of a London listing for these companies is that they gain access to global investment capital: it is much easier to raise capital by issuing shares in the UK than in countries with financial markets that are much less developed.

\section*{Raising capital by issuing bonds}

In recent years, multinational companies have often preferred to raise capital by issuing bonds rather than shares. Debt capital has been much cheaper than equity, and institutional investors have been willing to invest in large quantities of bonds.

If a multinational company wants to raise capital in the international markets, it might prefer to consider the possibility of issuing bonds in a national bond market rather than consider the possibility of a share issue.

\section*{Paper P4}

Advanced Financial Management

\section*{\(\mathbf{Q}_{\star} \mathrm{A}\)}

\section*{Practice questions}
\begin{tabular}{|lll|}
\hline \multicolumn{2}{c|}{ Contents } \\
\hline \multicolumn{2}{|c|}{\begin{tabular}{l} 
The role and responsibilities of financial \\
managers \\
1
\end{tabular}} & Corporate social responsibility \\
2 & Corporate governance & \\
3 & Dividends and retentions & 446 \\
4 & Financial plan & 446 \\
& Capital investment appraisal & 446 \\
5 & DCF exercises & 446 \\
6 & DCF and tax & 448 \\
7 & Free cash flow & 448 \\
& & 449 \\
Investing: portfolio theory and the CAPM & \\
8 & Two-asset portfolio & 449 \\
9 & Risk and return & 450 \\
10 & Coefficient of variation & 450 \\
11 & Capital market line & 451 \\
12 & Obtaining a beta factor & 451 \\
& & \\
Cost of capital \\
13 & WACC & 452 \\
14 & Optimal WACC & 452 \\
15 & Change in gearing & 453 \\
\hline
\end{tabular}
\begin{tabular}{|c|c|c|}
\hline 16 & Geared beta & 453 \\
\hline \multicolumn{3}{|l|}{Other aspects of capital expenditure appraisal} \\
\hline 17 & APV method & 453 \\
\hline 18 & Adjusted present value & 454 \\
\hline 29 & More APV & 455 \\
\hline \multicolumn{3}{|l|}{International investment and financing decisions} \\
\hline 20 & Cash flows from a foreign project & 456 \\
\hline \multirow[t]{2}{*}{21} & Foreign investment & 456 \\
\hline & \multicolumn{2}{|l|}{Valuations. Mergers and acquisitions} \\
\hline 22 & Valuation & 457 \\
\hline 23 & Valuation of bonds & 457 \\
\hline 24 & Annuities and bond prices & 458 \\
\hline 25 & EVA & 458 \\
\hline 26 & MM, gearing and company valuation & 458 \\
\hline 27 & Acquisition & 459 \\
\hline 28 & Takeover & 459 \\
\hline \multicolumn{3}{|l|}{Foreign exchange risk and currency risk management} \\
\hline 29 & Interest rate parity & 461 \\
\hline 30 & Foreign exchange & 459 \\
\hline 31 & Money market hedge & 462 \\
\hline \multicolumn{3}{|l|}{Interest rate risk. Hedging with FRAs and swaps} \\
\hline 32 & FRA & 462 \\
\hline 33 & Swap & 462 \\
\hline 34 & Credit arbitrage (1) & 463 \\
\hline 35 & Credit arbitrage (2) & 463 \\
\hline 36 & Currency swap & 463 \\
\hline \multicolumn{3}{|l|}{Futures and hedging with futures} \\
\hline 37 & Currency futures & 464 \\
\hline 38 & More currency futures & 465 \\
\hline 39 & Basis & 465 \\
\hline
\end{tabular}
\begin{tabular}{|lll|}
\hline 40 & Imperfect hedge and basis & 465 \\
41 & Currency hedge & 466 \\
42 & Hedging with STIRs & 466 \\
43 & More hedging with STIRs & 467 \\
44 & FRAs and futures & 468 \\
& & \\
Options and hedging with options & 469 \\
45 & Traded equity options & 469 \\
46 & Currency options & 470 \\
47 & Interest rate hedge & \\
& & 470 \\
Option pricing and delta hedging & 471 \\
48 & Black-Scholes & 471 \\
49 & Put-call parity & 471 \\
50 & Delta hedge & 472 \\
51 & Bonus scheme & \\
52 & Delta neutral &
\end{tabular}

\section*{1 Corporate social responsibility}
(a) What are the main issues that might be covered by a company's policy on corporate social responsibility?
(b) What types of company are most likely to face CSR risks?
(c) What is the nature of CSR risk?

\section*{2 Corporate governance}
(a) What is the main purpose of statutory or voluntary codes of corporate governance?
(b) What are the main problems or issues that a code of corporate governance might be expected to cover?

\section*{3 Dividends and retentions}

The directors of an all-equity company are considering the company's policy on dividends and retentions. The cost of capital is \(9 \%\) and the company is able to invest in new capital projects that will earn this return. The company's shares are quoted and traded on a major stock market.

In the year just ended, the earnings per share were \(\$ 2.00\) per share. The company pays a dividend annually, and is about to pay a dividend for the year just ended on the basis of its selected dividend and retentions policy.

\section*{Required}

Suggest what the company's share price might be if the directors select a policy of paying annual dividends that are equal to:
(a) \(25 \%\) of earnings
(b) \(50 \%\) of earnings
(c) \(70 \%\) of earnings.

\section*{4 Financial plan}

The board of directors of NNW have asked for a four-year financial plan to be prepared for Year 5 to Year 8. They have approved the following assumptions for the plan:
(1) Sales growth will be at the rate of \(6 \%\) each year into the foreseeable future.
(2) Cash operating costs will be \(70 \%\) of sales.
(3) Investment in new plant and equipment is expected to grow in line with the growth in sales, and the net book value of plant and equipment will grow at the same rate.
(4) Tax-allowable depreciation will grow in line with the growth in sales.
(5) Inventory, receivables, cash and trade payables will also increase at the same rate as the growth in sales.
(6) There will be no change in long-term borrowing. Interest on the bank overdraft will be payable at \(7 \%\). The interest charge for bank overdraft in the income statement each year should be calculated on the opening bank overdraft at the beginning of the year.
(7) Tax on company profits will be \(30 \%\).
(8) The company policy is to pay dividends as a constant percentage amount of earnings. This policy will not change.
(9) The cost of equity capital has been estimated as \(12 \%\).

The income statement of NNW for the year to 31st December Year 4 is as follows:
\begin{tabular}{lr} 
& \begin{tabular}{r} 
\$ million \\
Sales \\
1,800 \\
Cash operating costs \\
EBITDA \\
Tax allowable depreciation \\
Earnings before interest \\
Interest \\
Profit before tax \\
Tax at 30\% \\
Profit after tax \\
Dividends \\
Retained profit
\end{tabular}\(\quad 380\) \\
\hline
\end{tabular}

The balance sheet of NNW as at the end of Year 4 is as follows:
\begin{tabular}{lrr} 
& \(\$ \mathrm{~m}\) & \(\$ \mathrm{~m}\) \\
Plant and equipment & & 2,020 \\
Current assets & 520 & \\
Inventory & 640 & \\
Receivables & 30 & 1,190 \\
Cash & - & 3,210 \\
\cline { 2 - 3 } & & 450 \\
Total assets & & 1,200 \\
\hline Share capital (shares of \$0.05 each) & 1,650 \\
Reserves & 800 \\
Long term loan at 8\% & 450 \\
Trade payables & 310 \\
Bank overdraft & & 3,210 \\
\hline
\end{tabular}

\section*{Required}
(a) Prepare a financial plan for Years 5 to 8, showing the profit after tax, dividends, retained profits for each year and a summary balance sheet as at the end of each year.
(b) Calculate the expected free cash flow in each year of the financial plan.
(c) Comment briefly on the financial plan.
(d) Use the dividend growth model to estimate a market value per share as at the end of Year 8 (the end of the financial planning period). State any assumptions that you make in your estimate.

\section*{5 DCF exercises}
(a) Calculate the NPV of an investment with the following estimated cash flows, assuming a cost of capital of \(8 \%\) :
\begin{tabular}{lr} 
Years & Annual cash flow \\
\hline & \(\$\) \\
0 & \((3,000,000)\) \\
\(1-4\) & 500,000 \\
\(5-8\) & 400,000 \\
\(9-10\) & 300,000 \\
11 onwards in perpetuity (per year) & 100,000
\end{tabular}
(b) The cash flows for an investment project have been estimated at current prices, as follows:
\begin{tabular}{lrrr} 
Year & Equipment & Revenue & Running costs \\
\hline & \(\$\) & \(\$\) & \(\$\) \\
0 & \((900,000)\) & & \\
1 & & 800,000 & \((400,000)\) \\
2 & & 800,000 & \((400,000)\) \\
3 & 200,000 & 400,000 & \((350,000)\) \\
4 & & & \((300,000)\)
\end{tabular}

It is expected that the cash flows will differ because of inflation. The annual rates of inflation are expected to be:
- Equipment value: \(4 \%\) per year
- Revenue: \(3 \%\) per year
- Running costs: \(5 \%\) per year.

The cost of capital is \(12 \%\)

\section*{Required}
(i) Calculate the NPV of the project ignoring inflation.
(ii) Calculate the NPV of the project allowing for inflation.

\section*{6 DCF and tax}

A company is considering whether or not to invest in a four-year investment project. The project will require the purchase of equipment costing \(\$ 800,000\). This will have an estimated residual value of \(\$ 200,000\) at the end of Year 4. The equipment will be depreciated by the straight-line method.

The profits before interest and tax from the project are expected to be \(\$ 250,000\) each year. Tax is payable at \(30 \%\) one year in arrears.

The equipment will qualify for capital allowances (tax depreciation allowances) of \(25 \%\) each year, using the reducing balance method. The first claim for an allowance would be made against Year 0 profits.

The after-tax cost of capital is \(15 \%\).

\section*{Required}

Calculate the NPV of the project.

\section*{\(7 \quad\) Free cash flow}

A company expects to make profits before interest and tax next year of \(\$ 3\) million.
Other budgeted information is as follows:
\begin{tabular}{lr} 
& \(\$\) \\
\hline Interest charges & 400,000 \\
Taxation & 600,000 \\
Dividend payments & \(1,200,000\) \\
Depreciation charges & 550,000 \\
Increase in working capital & 150,000 \\
Capital expenditure: & \\
Asset replacement expenditure & \(1,000,000\) \\
Discretionary expenditure & 700,000
\end{tabular}

\section*{Required}

Calculated the expected amount of free cash flow next year.

\section*{8 Two-asset portfolio}

An investor is planning to invest in two securities, Security X and Security Y. The expected return from each security will depend on the state of the economy, as follows:
\begin{tabular}{lrrr}
\begin{tabular}{l} 
State of the \\
economy
\end{tabular} & Probability & \begin{tabular}{r} 
Return from \\
Security \(\mathbf{X}\)
\end{tabular} & \begin{tabular}{r} 
Return from \\
Security Y
\end{tabular} \\
\hline & & \(\%\) & \(\%\) \\
Strong & 0.25 & 15 & 20 \\
Fair & 0.60 & 10 & 8 \\
Weak & 0.15 & 2 & \((6)\)
\end{tabular}

\section*{Required}
(a) Calculate the mean and standard deviation of the expected return from Security X.
(b) Calculate the mean and standard deviation of the expected return from Security Y.
(c) Calculate the covariance of the returns from Security X and Security Y. The formula for a covariance is:
\[
\operatorname{Cov}_{x, y}=\Sigma r(x-\bar{x})(y-\bar{y})
\]
(d) Calculate the correlation coefficient for returns from Security X and Security Y, for a portfolio consisting of \(50 \%\) of the funds invested in Security \(X\) and \(50 \%\) of the funds invested in Security Y. The formula for correlation coefficient is:
\[
\frac{\rho_{x, y}=\operatorname{Cov}_{x, y}}{\sigma_{x} \times \sigma_{y}}
\]
where:
\(\sigma_{\mathrm{x}}=\) the standard deviation of returns from Security \(X\)
\(\sigma_{y}=\) the standard deviation of returns from Security \(Y\)
Comment on the correlation coefficient.
(e) Calculate expected return, the variance and standard deviation of a portfolio consisting of \(50 \%\) of the funds invested in Security X and \(50 \%\) of the funds invested in Security Y. The formula for correlation coefficient is:
\(\mathrm{a}^{2}(\text { Variance } \mathrm{X})^{2}+(1-\mathrm{a})^{2}(\text { Variance } \mathrm{Y})^{2}+2 \mathrm{a}(1-\mathrm{a}) \operatorname{Cov}_{\mathrm{x}, \mathrm{y}}\)
where:
a = the proportion of the portfolio invested in Security X
\((1-a)=\) the proportion of the portfolio invested in Security Y
Variance \(X=\) the variance of the returns from Security \(X\)
Variance \(\mathrm{Y}=\) the variance of the returns from Security Y
(f) Calculate expected return, the variance and standard deviation of a portfolio consisting of \(80 \%\) of the funds invested in Security X and \(20 \%\) of the funds invested in Security Y.

\section*{\(9 \quad\) Risk and return}

A divisional manager's attitude to investing in new projects is affected by his attitude to risk. He is prepared to invest in a project that is more risky, provided that it offers a higher expected return.

He is currently considering four mutually exclusive projects, for which the estimated returns and risk are as follows:
\begin{tabular}{llr} 
Project & Estimated project NPV & Risk ( \(\boldsymbol{\sigma}\) ) \\
\hline Project 1 & \(80 \%\) chance of \(+\$ 4\) million, \(20 \%\) chance of \(+\$ 2\) million & 0.80 \\
Project 2 & \(70 \%\) chance of \(+\$ 5\) million, \(30 \%\) chance of \(+\$ 1.5\) million & 1.60 \\
Project 3 & \(60 \%\) chance of \(+\$ 6\) million, \(40 \%\) chance of \(+\$ 1\) million & Not yet calculated \\
Project 4 & \(50 \%\) chance of \(+\$ 8\) million, \(50 \%\) chance of \(-\$ 1\) million & Not yet calculated
\end{tabular}

\section*{Required}
(a) Calculate the risk with Project 3 and Project 4.
(b) Suggest which of the four projects the divisional manager will select.

\section*{10 Coefficient of variation}

A multinational company is planning to invest in two developing countries, and it will invest equal amounts of capital in each country. It is looking at returns and risk in each of three possible countries that might be selected for investment.

The company is particularly concerned about the political risk in each country, and the threat of political risk to its expected returns. A firm of management consultants has produced the following statistical estimates of expected returns ad political risk in each of the countries.
\begin{tabular}{lrr} 
Country & \begin{tabular}{r} 
Expected investment \\
return (\%)
\end{tabular} & \begin{tabular}{r} 
Political \\
risk (\%)
\end{tabular} \\
\hline Country A & 16 & 25 \\
Country B & 22 & 36 \\
Country C & 30 & 45
\end{tabular}

The expected return from investing in any of the three countries is independent of the returns that would be obtained from the other countries.

\section*{Required}
(a) Calculate the risk, return and coefficient of variation of the following three investment portfolios:
(i) \(50 \%\) in Country A, \(50 \%\) in Country B
(ii) \(50 \%\) in Country A, \(50 \%\) in Country C
(iii) \(50 \%\) in Country B, \(50 \%\) in Country C
(b) Comment on the results.

\section*{11 Capital market line}

The risk-free rate of return is \(3 \%\). The return on the market portfolio is \(9 \%\).
Suggest how an investor should create a portfolio to provide a return of \(7 \%\) with minimum risk.

\section*{12 Obtaining a beta factor}

A beta factor will be estimated for Security Y from the following data.
\begin{tabular}{lrr} 
Month & \begin{tabular}{r} 
Returns from the \\
market portfolio
\end{tabular} & \begin{tabular}{r} 
Returns from \\
Security \(\mathbf{Y}\)
\end{tabular} \\
\hline & \(\%\) & \(\%\) \\
1 & +2 & +3 \\
2 & -1 & -2 \\
3 & -2 & -2 \\
4 & +3 & +5 \\
EV of monthly return & +0.5 & +1.0
\end{tabular}

\section*{Required}
(a) Use this data to calculate:
(i) the standard deviation of the monthly return from the market portfolio and
(ii) the standard deviation of the monthly return from Security Y.
(b) Calculate the correlation coefficient for the market returns and the returns from Security Y. This is calculated as:
\[
\frac{\rho_{\mathrm{m}, \mathrm{y}}=\operatorname{Cov}_{\mathrm{m}, \mathrm{y}}}{\sigma_{\mathrm{m}} \times \sigma_{\mathrm{y}}}
\]
where:
\(\sigma_{m}=\) the standard deviation of returns from the market portfolio
\(\sigma_{y}=\) the standard deviation of returns from Security \(Y\)
The formula for the covariance is:
\[
\operatorname{Cov}_{x, y}=\Sigma(x-\bar{x})(y-\bar{y})
\]
(c) Use this data to calculate the beta factor for Security Y. You can use either of the following formulas.
\[
\beta=\frac{\operatorname{Cov}_{m, y}}{\operatorname{Var}_{\mathrm{m}}}
\]

Alternatively
\(\beta=\frac{\rho_{\mathrm{m}, \mathrm{y}} \times \sigma_{\mathrm{y}}}{\sigma_{\mathrm{m}}}\)

\section*{13 WACC}

A company has just paid an annual dividend of \(\$ 0.18\). Investors expect the annual dividend to grow by \(3 \%\) each year in perpetuity, The current share price is \(\$ 1.55\) and the total market value of the company's shares is \(\$ 1,200,000\).

The company has debt capital on which the yield is \(7.8 \%\) before tax. The rate of tax is \(30 \%\). The total value of the company's debt is \(\$ 350,000\).

Calculate the weighted average cost of capital. Use the dividend growth model to estimate the cost of equity.

\section*{14 Optimal WACC}

A company has estimated that its cost of debt capital varies according to the level of gearing, as follows:
\begin{tabular}{lr} 
Gearing & Cost of debt \\
\hline & \(\%\) \\
20 & 5.0 \\
30 & 5.4 \\
40 & 5.8 \\
50 & 6.5 \\
60 & 7.2
\end{tabular}

Gearing is measured as the market value of the company's debt as a proportion of the total market value of its equity plus debt.

The rate of tax is \(30 \%\). The ungeared equity beta factor for the company is 0.90 .
The risk-free rate of return is \(4 \%\) and the return on the market portfolio is \(9 \%\)

\section*{Required}

Identify the optimal gearing level and WACC.

\section*{15 Change in gearing}

A company is all-equity financed, with 60 million shares. The current market value of the shares is \(\$ 5\) each. The company currently makes an annual profit before interest and tax of \(\$ 40\) million. There are no expectations of growth in annual profits.

The company is considering whether to raise \(\$ 100\) million in debt. The pre-tax interest rate would be \(8 \%\) and the tax rate is \(25 \%\). The debt capital would be used to repurchase and cancel some equity shares.

\section*{Required}

Use appropriate Modigliani-Miller formulas to estimate the effect the issue of debt and repurchase of shares would have on the weighted average cost of capital (WACC).

\section*{16 Geared beta}

A company has \(\$ 1,500,000\) in equity capital and \(\$ 500,000\) in debt capital (at market values). The beta value of the equity is 1.126 and the beta of the debt capital is 0 .

The risk-free cost of capital is \(5 \%\) and the market portfolio return is \(11 \%\). The tax rate is \(30 \%\).

\section*{Required}
(a) Calculate the current weighted average cost of capital (WACC).
(b) Calculate what the WACC would be if the company consisted of \(60 \%\) equity and \(40 \%\) debt.

\section*{17 APV method}

A company in the engineering industry is considering making an investment in a telecommunications project. The investment will cost \(\$ 2,000,000\), and will be financed by a new issue of \(\$ 1,000,000\) in equity and a new issue of \(\$ 1,000,000\) debt capital.

The company's current gearing level is 30\% debt and 70\% equity.
The telecommunications industry has an average industry equity beta of 1.30625 . The average gearing ratio in the industry is \(20 \%\) debt and \(80 \%\) equity.

The rate of taxation is \(25 \%\).
The risk-free rate of return is \(4 \%\) and the average market return is \(9 \%\). The company's debt is risk-free.

The cash flows from the project before taxation are expected to be:
\begin{tabular}{lr} 
Year & Cash flow \\
\hline & \(\$\) \\
1 & 100,000 \\
2 & 140,000 \\
3 & 120,000
\end{tabular}

Tax is payable one year in arrears.
You should ignore tax depreciation (capital allowances) on the initial investment.
The costs of raising the equity capital will be \(4 \%\) of the amount raised. The costs of raising the debt capital will be \(3 \%\) of the amount raised. The debt will be in the form of a three-year loan, and the principal will be repaid in full at the end of Year 3.

\section*{Required}

Calculate:
(a) the NPV of the project, using the Modigliani and Miller formulas to derive a cost of capital for the project
(b) calculate the adjusted present value (APV) of the project.

\section*{18 Adjusted Present Value}

Harvey is an aluminium engineering company. It now wishes to diversify its operations into the plastics business. The proposed investment project will require the purchase of a machine costing \(\$ 450,000\). This will produce cash flows of \(\$ 220,000\) for each of the three years of its life, and it will have no residual value at the end of that time.

It is proposed to finance the purchase of the machine with a mixture of debt and equity capital. \(40 \%\) of the cost will be financed by a three-year loan that will be repaid in three equal instalments. The remaining \(60 \%\) of the cost will be financed by a placing of new equity.

Issue costs, which are tax-allowable, will be \(5 \%\) for the equity and \(2 \%\) for the debt, measured as a percentage of the net finance raised.

The plastics industry has an average equity beta of 1.356 and an average debt: equity ratio of \(1: 5\), at market values. Harvey's current equity beta is 1.8 and \(20 \%\) of its capital (at market value) consists of long-term debt which is regarded as risk-free.

The risk-free rate is \(10 \%\) per annum and the expected return on an average market portfolio is \(15 \%\). Corporation tax is at \(35 \%\), payable one year in arrears. The machine will attract a \(70 \%\) capital allowance in the first year, and the balance will be allowable against tax over the next three years, at an equal amount in each year.

\section*{Required}

Carry out an appraisal of the investment using each of the three following methods:
(a) NPV of the project, using the company's current weighted average cost of capital (WACC)
(b) NPV of the project, using a WACC adjusted for business risk and financial risk
(c) the adjusted present value (APV) of the project.

\section*{19 More APV}

Pobol Company specialises in business consultancy, but its directors are considering an investment in software development, which would represent a major diversification of the company's business activities. The following draft financial proposal has been prepared:
\begin{tabular}{|c|c|c|c|c|c|}
\hline Year & 0 & 1 & 2 & 3 & 4 \\
\hline & \$000 & \$000 & \$000 & \$000 & \$000 \\
\hline Revenue & & 6,800 & 7,800 & 8,800 & 9,200 \\
\hline Cash operating costs & & 5,500 & 6,600 & 7,100 & 7,500 \\
\hline Allocated head office costs & & 100 & 150 & 150 & 200 \\
\hline Royalty payments & 600 & 500 & 400 & 300 & 200 \\
\hline Market research costs & 120 & - & - & - & - \\
\hline & 720 & 6,100 & 7,150 & 7,550 & 7,900 \\
\hline Expenditure on equipment & 3,000 & & & & \\
\hline Working capital & 400 & & & & \\
\hline
\end{tabular}

The following information is also available:
(1) The project will have a six-year life.
(2) All prices are calculated in money terms, allowing for inflation. After Year 4, it is expected that revenues and cash operating costs will remain unchanged in real terms, but will increase at the rate of inflation which is expected to be \(3 \%\) per year. Royalty payments are expected to be \(\$ 200,000\) per year in Years 5 and 6.
(3) Head office cash flows wil increase as a consequence of the investment by \(\$ 50,000\) per year in Years \(1-3\) and by \(\$ 60,000\) per year in Years 4-6.
(4) The market research costs in Year 0 have already been incurred.
(5) Highly-skilled consultancy staff will have to be switched to managing the project, resulting in lost contribution of \(\$ 100,000\) per year in Years 1 and 2.
(6) The working capital investment will remain unchanged. The investment in equipment and working capital will be financed by a new six-year loan at \(6 \%\) interest. Issue costs for the loan will be \(2 \%\) and are not tax-allowable.
(7) The cash for the royalty payments and market research in Year 0 come from internally-generated cash flows.
(8) Tax is payable at the rate of \(25 \%\), and is payable in the same year that the tax liability arises.
(9) Tax-allowable depreciation will be \(20 \%\) in Year 1 and will then be a constant amount for the next five years.
(10) The average equity beta of companies in the software sector that Pobol Company is considering is 1.39 . TRhe market return is \(10 \%\) and the risk-free interest rate is \(6 \%\).
(11) The average gearing of companies in the software sector that Pobol Company is considering is \(80 \%\) equity and \(20 \%\) debt.

\section*{Required}

Calculate the adjusted present value (APV) of this project.

\section*{20 Cash flows from a foreign project}

A UK company intends to invest in a foreign country, Frankland. The cost of the investment will be 45 million francs, which is \(£ 9\) million in sterling at the current exchange rate. The entire cost of the investment will be paid at the beginning of the project.

A DCF analysis has been carried out on the project's expected cash flows in Frankland, and the NPV is positive.

The project is expected to generate the following dividend payments to the company in the UK:
\begin{tabular}{lr} 
Year & Francs \\
\hline 1 & 10 million \\
2 & 20 million \\
3 & 25 million \\
4 & 10 million
\end{tabular}

The current exchange rate is \(£ 1=5\) francs. The expected annual rate of inflation in the UK for the next four years is \(3 \%\) and in Frankland it is \(5 \%\).

Tax in the UK is \(30 \%\), and will be payable one year in arrears of dividend receipts. The company's weighted average cost of capital is \(9 \%\).

\section*{Required}

Calculate the NPV of the company's expected sterling cash flows, to decide whether the project should be undertaken.

\section*{21 Foreign investment}

Entity Green is a company whose domestic currency is dollars. It is considering an investment in a country, Francia, where the domestic currency is Francs (FR).

The investment will involve buying equipment in the foreign country at a cost of \(1,000,000\) Francs. The currency to make the purchase will be bought spot in the FX market.

The equipment and the project will have a four-year life. At the end of this time, the equipment will have no residual value. The equipment will attract an allowance for tax purposes of \(25 \%\) of its cost each year. The first capital allowance will be claimed against profits in Year 1.

The cash profits from the project will be \(\$ 500,000\) in each of the four years. Tax is payable at \(40 \%\) and is paid one year in arrears of the profits to which they relate. In addition, Entity Green will incur expenditure of \(\$ 25,000\) at the beginning of the project in its own currency.

There are foreign exchange restrictions in the country, and only \(50 \%\) of the profits after tax each year can be paid to any shareholder in another country. The balance of the profits from the project can be paid out as a dividend to Entity Green at the end of Year 5.

Entity Green has a cost of capital of \(10 \%\), but a cost of capital of \(16 \%\) is considered appropriate for evaluating the investment cash flows.

The current exchange rate is \(\$ 1=\) FR3.00. However, the rate of inflation is expected to be \(10 \%\) in each year in the Francia and \(4 \%\) each year in Entity Green's country.

\section*{Required}
(a) Calculate the NPV of the project in the currency of the investment, using a discount rate appropriate to the investment.
(b) Calculate the expected annual dividend payments, in Francs.
(c) Calculate the dollar value of the expected annual dividend payments.
(d) Evaluate the NPV of the investment in dollars, using an appropriate discount rate.

Valuation
A company has just paid an annual dividend of 38 . The board of directors has a target of increasing the share price to 800, and is considering policies for investment and growth.

Shareholders expect a return on their investment of \(10 \%\) per year.

\section*{Required}

Calculate the annual expected growth rate in dividends that would be required to raise the share price to 800 . Use the dividend growth model to make your estimate.

\section*{23 Valuation of bonds}

Assume that bond investors require a return of \(9 \%\) per year on their investments.

\section*{Required}

Estimate the market value of the following bonds:
(a) Irredeemable \(7.5 \%\) bonds that pay interest annually.
(b) Bonds paying coupon interest of 6\% per year annually, that are redeemable at par in four years' time.
(c) Bonds paying coupon interest of \(10 \%\), redeemable at par after three years, where interest is payable every six months.

\section*{Notes:}

An annual cost of capital of \(9 \%\) is equal to a six-monthly cost of capital of 4.4\%.

DCF factor at \(4.4 \%\), periods \(1-7=5.914\)
DCF factor at \(4.4 \%\), periods \(1-8=6.623\)
(d) A convertible bond with a coupon of \(5 \%\) and interest payable annually: these bonds are convertible after three years into equity shares at the rate of 20 shares for every \(\$ 100\) nominal value of bonds. The expected share price in three years' time is \(\$ 7\).

\section*{24 Annuities and bond prices}
(a) Calculate the value of the following bonds:
(i) a zero coupon bond redeemable at par in ten years' time
(ii) a bond with an \(8 \%\) coupon, with interest payable half-yearly, and redeemable at par after ten years.
Assume that the yield required by investors is 5\%, and that this is \(2.5 \%\) each half year for the purpose of valuing the \(8 \%\) coupon bond.
(b) Calculate the value of both bonds in part (a) of the question if the yield required by investors goes up by \(1 \%\), to \(6 \%\) for the zero coupon bond and \(3 \%\) each half year for the \(8 \%\) coupon bond.

\section*{25 EVA}

A group of companies has \(\$ 8,000,000\) of equity capital and \(\$ 4,000,000\) of long-term debt capital.

Its assets include:
(a) development expenditure of \(\$ 500,000\)
(b) goodwill \(\$ 400,000\), which is \(\$ 1,200,000\) at cost less cumulative impairment of \(\$ 800,000\).

During a financial year, net profit before interest and tax is \$2,100,000. Interest charges are \(\$ 300,000\) and taxation is \(\$ 450,000\).

The company's weighted average cost of capital is \(12 \%\).

\section*{Required}

On the basis of this data, calculate the economic value added (EVA) for the year.

\section*{26 MM, gearing and company valuation}

A company has \(4,000,000\) equity shares in issue. The shares have a current market value of \(\$ 10\) each. The company is considering whether to issue \(\$ 15,000,000\) of debt finance and use the cash to buy back and cancel some equity shares. The tax rate is \(30 \%\).

According to Modigliani and Miller, if the company decided to issue the debt capital and repurchase shares, what would be:
(a) the total value of the geared company, and
(b) the value of equity in the company?

\section*{27 Acquisition}

Big Entity is considering a takeover bid for Little Entity, another company in the same industry. Little is expected to have earnings next year of \(\$ 86,000\).

If Big acquires Little, the expected results from Little will be as follows:
\begin{tabular}{lrrr} 
& \multicolumn{3}{c}{ Year after the acquisition } \\
& Year 1 & Year 2 & Year 3 \\
\hline & \(\$\) & \(\$\) & \(\$\) \\
Sales & 200,000 & 280,000 & 320,000 \\
Cash costs/expenses & 120,000 & 160,000 & 180,000 \\
Capital allowances & 20,000 & 30,000 & 40,000 \\
Interest charges & 10,000 & 10,000 & 10,000 \\
Cash flows to replace assets and finance growth & 25,000 & 30,000 & 35,000
\end{tabular}

From Year 4 onwards, it is expected that the annual cash flows from Little will increase by \(4 \%\) each year in perpetuity.

Tax is payable at the rate of \(30 \%\), and the tax is paid in the same year as the profits to which the tax relates.

If Big acquires Little, it estimates that its gearing after the acquisition will be \(35 \%\) (measured as the value of its debt capital as a proportion of its total equity plus debt). Its cost of debt is \(7.4 \%\) before tax. Big has an equity beta of 1.60.

The risk-free rate of return is \(6 \%\) and the return on the market portfolio is \(11 \%\).

\section*{Required}
(a) Suggest what the offer price for Little should be if Big chooses to value Little on a forward P/E multiple of 8.0 times.
(b) Calculate a cost of capital for Big.
(c) Suggest what the offer price for Little might be using a DCF-based valuation.

\section*{28 \\ Takeover}

Flat Company intends to make a takeover bid for Slope Company, a company in the same industry. The initial offer will be to exchange every 3 shares in Slope for 2 new shares in Flat.

The most recent annual data for the two companies is shown below.
\begin{tabular}{|c|c|c|}
\hline & Flat & Slope \\
\hline & \$000 & \$000 \\
\hline Sales revenue & 7,619 & 6,000 \\
\hline Operating costs & 4,962 & 3,480 \\
\hline Tax allowable depreciation & 830 & 700 \\
\hline Earnings before interest and taxation & 1,827 & 1,820 \\
\hline Interest & 410 & 860 \\
\hline Profit before tax & 1,417 & 960 \\
\hline Tax at 30\% & 425 & 288 \\
\hline & 992 & 673 \\
\hline Dividends & 500 & 410 \\
\hline Retained earnings & 492 & 263 \\
\hline Annual replacement capital expenditure & 920 & 790 \\
\hline
\end{tabular}

\section*{Other information}
\begin{tabular}{lrr} 
& Flat & Slope \\
\begin{tabular}{l} 
Expected annual growth in sales, operating costs including depreciation, \\
replcement capital expenditure and dividends for the next 4 years
\end{tabular} & \(5 \%\) & \(4 \%\) \\
& & \(3 \%\) \\
Expected annual growth in these items from year 5 onwards & \(25 \%\) & \(40 \%\) \\
Gearing, measured as the ratio of debt to debt plus equity, (both debt and & \(25 \%\) & \\
equity measured at market value) & & \\
& 320 & 154 \\
Market price per share (cents) & 6 & 9 \\
Number of shares in issue (millions) & \(7 \%\) & \(8 \%\) \\
Market cost of fixed interest debt & 1.20 & 1.35
\end{tabular}

The risk-free rate of return is \(5 \%\) and the market return is \(11 \%\).

The takeover will result in some cost savings in operations so that the earnings before interest and taxation of the combined group would be \(\$ 4,100,000\) in Year 1 after the takeover, and growth in sales, costs, depreciation and replacement capital expenditure would by \(5 \%\) per year for the following three years and then \(4 \%\) per year from Year 5 onwards.

The senior financial manager of Flat Company has been assessing the value of the takeover bid for the shareholders ofboth companies, and has decided to use free cash flow analysis as a basis for valuing the companies before and after the takeover. He believes that the total equity value of the group after the takeover will be significantly higher than the sum of the current equity values of the two separate companies.

The weighted average cost of the combined company should be calculated as the weighted average of the current cost of capital of the individual companies, weighted by the current market value of their debt and equity.

\section*{Required}
(a) Using free cash flow analysis, and making any assumptions you consider necessary, calculate a value for:
(i) the current equity in Flat Company
(ii) the current equity in Slope Company
(iii) the equity in Flat Company after the takeover.
(b) Explain the limitations of your estimates in (a).
(c) Give your views as to whether the takeover bid is likely to have the support of the shareholders in (1) Flat Company and (2) Slope Company.

\section*{29 Interest rate parity}

The following are spot exchange rates:
■ US\$/£1 (GBP/USD): 1.8000
■ \(€ / £ 1\) (GBP/EUR): 1.5000
- US\$ / €1 (EUR/USD): 1.2000

The rates of interest for the next three years are \(2.5 \%\) on the euro, \(3.5 \%\) on the US dollar and \(5 \%\) on sterling.

\section*{Required}

If the interest rate parity theory applies, what will the spot exchange rates:
(a) after one year
(b) after three years?

\section*{30 Foreign exchange}
(a) A UK company expects to pay \(\$ 750,000\) to a supplier in three months' time. The following exchange rates are available for the dollar against sterling (GBP/USD):
\begin{tabular}{llll} 
Spot & 1.8570 & - & 1.8580 \\
3 months forward & 1.8535 & - & 1.8543
\end{tabular}

The company is concerned about a possible increase in the value of the dollar during the next three months, and would like to hedge its FX risk.

\section*{Required}

Explain how the exposure to currency risk might be hedged, and the amount that the UK company will have to pay in sterling in three months' time to settle its liability.
(b) A German company expects to receive US\$450,000 from a customer in two months' time. It is concerned about the risk of a fall in the value of the dollar in the next two months, and would like to hedge the currency risk using a forward contract.

The following rates are available for the dollar against the euro (EUR/USD):
```

Spot 1.3015 - 1.3025
2 months forward 25c - 18c premium

```

\section*{Required}

Calculate the company's income in euros from settlement of the forward contract in two months' time.

\section*{31 Money market hedge}

A UK company expects to receive \(\$ 600,000\) in six months' time from a customer. It intends to convert these dollars into sterling.

The current spot rate for the dollar against sterling (GBP/USD) is 1.8800 . The sixmonth interest rates are \(5 \%\) per year for sterling and \(3.5 \%\) per year for the US dollar.

\section*{Required}
(a) Show how the company can create a money market hedge for its exposure to a fall in the value of the dollar.
(b) Estimate what the exchange rate should be for a six-month forward contract, GBP/USD.

\section*{32 \\ FRA}

A company will need to borrow \(\$ 5\) million for six months in three months' time. It can borrow at LIBOR \(+0.50 \%\). It expects interest rates to rise before it borrows the money, and so has decided to use an FRA to hedge the risk.

The following FRA rates are available:
\begin{tabular}{llll} 
& \(2 v 5\) & 3.82 & - \\
3.77 \\
3v6 & 3.85 & - & 3.80 \\
3v9 & 3.97 & - & 3.91 \\
6v9 & 3.92 & - & 3.87
\end{tabular}

\section*{Required}
(a) How would the company use an FRA to hedge its interest rate risk, and what effective interest rate would be obtained by the hedge.
(b) What is the difference between an FRA and an interest rate coupon swap?

\section*{33 Swap}

A company has a bank loan of \(\$ 8,000,000\) on which it pays a floating rate of US LIBOR plus \(1.25 \%\). The company believes that interest rates will soon increase and remain high for the foreseeable future, and it would therefore like to switch its debt liabilities from floating rate to fixed rate.

The loan has four years remaining to maturity. A bank has quoted the following rates for four-year interest rate swaps in dollars:
\(5.20 \%-5.25 \%\)

\section*{Required}

Show how an interest rate swap can be used to switch from floating rate to fixed rate liabilities, and calculate what the effective fixed rate would be.

\section*{34 Credit arbitrage (1)}

Company X can borrow for six years at a fixed rate of \(7.25 \%\) or a variable rate of LIBOR plus \(1.25 \%\). Company Y can borrow for six years at a fixed rate of \(8.00 \%\) or a variable rate of LIBOR plus \(1.50 \%\).

Company X wants to borrow at a floating rate and company Y wants to borrow at a fixed rate.

The rates available on six-year swaps are \(6.27-6.30\).

\section*{Required}

Show how an interest rate swap can be used by both companies to reduce their borrowing costs.

\section*{\(35 \quad\) Credit arbitrage (2)}

Entity A has an AA credit rating and Entity B has a BBB- credit rating. Both companies want to raise the same amount of long-term debt capital. Entity A wants to borrow at a floating rate of interest and Entity B wants to borrow at a fixed rate.

They are able to borrow at the following rates:
\begin{tabular}{llr} 
& Fixed rate & Floating rate \\
\hline Entity A & \(6.35 \%\) & LIBOR \(+0.75 \%\) \\
Entity B & \(7.25 \%\) & LIBOR \(+1.25 \%\)
\end{tabular}

A bank has identified an opportunity to arrange interest rate swaps with the companies. It would expect to receive a profit margin on the arrangement of \(0.10 \%\) of the notional principal amount in the swap. The remaining benefits of the credit arbitrage should be shared equally between the two entities.

\section*{Required}

Explain how the interest rate swaps might be arranged, and show the effective interest rate that will be paid by each entity as a result of the swap.

\section*{36 Currency swap}

Small Company, a UK company, has an opportunity to invest in Zantland for three years, by setting up and operating an operations centre on behalf of the Zantland government. The cost of establishing the centre will be 3 million zants. At the end of the three years, the Zantland government will pay 6 million zants to purchase the centre from Small Company and take over the operations. During the three years that Small Company will operate the centre, the Zantland government will pay an annual fee of 200,000 zants. The entire operation will be free from tax.
The current exchange rate is \(£ 1=9.00\) zants spot. There is no forward market in zants. Economic conditions in Zantland are unstable, and the expected inflation rate in the country over the next three years could be anywhere between \(10 \%\) and \(50 \%\). Inflation is expected to be negligible in the UK.

A bank in Zantland has identified a Zantland company that would be interested in entering a currency swap with Small Company. The swap would involve the exchange of 3 million zants at the current spot rate, at the beginning and the end of the swap. An opportunity for credit arbitrage exists, because the rates at which Small company and the Zantland swap counterparty can borrow directly for three years are as follows.
\begin{tabular}{lcc} 
& Sterling & Zants \\
Small Company & \(6.5 \%\) & ZIBOR \(+2 \%\) \\
Zantland counterparty & \(8.5 \%\) & ZIBOR \(+1.5 \%\)
\end{tabular}

ZIBOR is the Zantland inter-bank offered rate, which is usually set very close to the inflation rate in Zantland.

The bank would take an annual fee of \(0.5 \%\) in sterling for arranging the swap, and Small Company would receive \(75 \%\) of the net arbitrage benefit from the swap.

\section*{Required}
(a) Suggest how a currency swap might be arranged between the counterparties, and indicate whether Small Company would arrange the swap if it decides to invest in the project.
(b) Making whatever assumptions you consider necessary and using a discount rate of \(15 \%\), recommend whether Small Company should undertake the project.

\section*{37 \\ Currency futures}

The euro/US dollar currency future is a contract for \(€ 125,000\). It is priced in US dollars, and the tick size is \(\$ 0.0001\).

Currency futures are not normally used by companies to hedge currency risks. However, assume that a French company intends to use currency futures to hedge the following currency exposure.

It is now February. The French company has to make a payment of US\$640,000 in May to a supplier.

The price of June euro/US dollar futures is currently 1.2800.
The company is concerned that the value of the dollar will increase in the next few months, and it therefore decides to use futures to hedge the exposure to currency risk.

\section*{Required}
(a) How should the company hedge its currency risk with futures?
(b) Suppose that in May when the company must make the payment in dollars, the June futures price is 1.2690 and the spot rate (US\$/ \(€ 1\) ) is 1.2710 .
Show what will happen when the futures position is closed, and calculate the effective exchange rate that the company has obtained for the US\$640,000.

\section*{38 More currency futures}

The sterling/US dollar currency future is a contract for \(£ 62,500\). It is priced in US dollars, and the tick size is \(\$ 0.0001\).

Currency futures are not normally used by companies to hedge currency risks. However, assume that a US company intends to use currency futures to hedge the following currency exposure.

It is now October. The US company expects to receive \(£ 400,000\) in January from a customer.

The price of March sterling/US dollar futures is currently 1.8600 .
The company is concerned that the value of sterling will fall in the next few months, and it therefore decides to use futures to hedge the exposure to currency risk.

\section*{Required}
(a) How should the company hedge its currency risk with futures?
(b) Suppose that in January when the company receives the sterling payment, the March futures price is 1.8420 and the spot rate (US\$/ \(£ 1\) ) is 1.8450 .

Show what will happen when the futures position is closed, and calculate the effective exchange rate that the company has obtained for the \(£ 400,000\).

Basis
It is 1st March. The current spot exchange rate for dollars against sterling (US\$/£1) is 1.8540 . The exchange rate is volatile, and the June futures price for sterling/US dollar futures is 1.8760 .
Assume that the settlement date for the June futures contract is 30th June.
A company has used sterling/US dollar futures to hedge two currency exposures, one relating to a cash payment on 1st May and the other relating to a cash payment in mid-June.

\section*{Required}

Calculate the expected futures price for June futures:
(a) at the end of the day's trading on 30th April, if the spot sterling/dollar rate is 1.8610
(b) at the end of the day's trading on 15th June, if the spot sterling/dollar rate is 1.8690 .

\section*{40 Imperfect hedge and basis}

It is 20th April. A US company expects to receive \(£ 625,000\) in three months' time, in July and it wants to hedge its exposure to the risk of a fall in the value of the dollar by hedging with US dollar/sterling futures.

A dollar/sterling futures contract is for \(£ 62,500\) and the value of a tick is \(£ 6.25\).

On 20th April, the spot exchange rate is \(\$ 1.8050 / £ 1\). The company deals in the September futures contracts at a price of 1.7800 . Settlement date for the September futures is in five months' time exactly.

The US company receives the \(£ 625,000\) on 20 th July and immediately closes its futures position. The spot rate on 20th July is 1.7700 and the futures price is 1.7600 .

\section*{Required}
(a) To what extent does the futures position provide a hedge for the company against currency risk, between 20th April and 20th July? To do this, compare the gain or loss on the underlying currency exposure with the gain or loss on the futures position.
(b) Explain why the hedge is imperfect.

\section*{41 Currency hedge}

It is now the end of July. A UK company expects the following receipts and payments in euros at the end of the month in three months' time (at the end of October):

Receipts €540,000
Payments €2,650,000

The company is concerned about the exposure to a risk of a movement in the sterling/euro exchange rate, and it has decided to hedge the exposure.

It is considering three methods of hedging the exposure:
(a) with a forward exchange contract
(b) using a money market hedge
(c) using currency futures.

Relevant data is as follows:

FX rates, €/£1
\begin{tabular}{llll} 
Spot & 1.4537 & -1.4542 \\
3 months forward & 1.4443 & -1.4448
\end{tabular}
\begin{tabular}{lrr} 
3-month interest rates & Borrow & Invest \\
\hline Sterling (UK) & \(6.2 \%\) & \(5.6 \%\) \\
Euro & \(3.8 \%\) & \(3.4 \%\)
\end{tabular}

\section*{Currency futures}

Currency futures for sterling/euro are each for \(€ 100,000\) and are priced in sterling.
Assume that the futures contracts mature at the end of the month.

Assume for the purpose of this question that when the futures position is closed at the end of October, the basis is 0 .

Futures prices as at end of July
September futures 0.6890
December futures 0.6929

\section*{Required}

Calculate the net cost in sterling of hedging the currency risk:
(a) with a forward exchange contract
(b) using a money market hedge
(c) using currency futures.

\section*{42 Hedging with STIRS}

It is now December.
A UK company wants to borrow \(£ 4.5\) million in two months for a period of five months. The loan period will be from a date in February to a date in July.

It wants to use short-term interest rate futures to create a hedge against a rise in short-term interest rates within the next two months.

Short sterling futures are for notional three-month deposits of \(£ 500,000\).

\section*{Required}

State how futures should be use as a hedge for the exposure to interest rate risk.

\section*{43 More hedging with STIRs}

It is now 31st October.

A company must borrow US \(\$ 12\) million in three months' time, on the first day of February, for a period of four months. It can borrow at US dollar LIBOR \(+1 \%\).

The company is concerned about the risk of an increase in short-term interest rates before February, and has decided to hedge the risk with short-term interest rate futures.

Eurodollar futures are for three-month notional deposits of \(\$ 1,000,000\).
The current three-month LIBOR rate at the end of October is \(5.5 \%\).
The following futures prices are available at the end of October:
Futures prices as at end of July
\begin{tabular}{ll}
\hline December futures & 94.20 \\
March futures & 93.70
\end{tabular}

Assume that the settlement date for futures is the last day of the relevant month.

\section*{Required}
(a) State how a hedge would be created using eurodollar futures.
(b) Suppose that at the beginning of February, three-month interest rates for the dollar (spot) have risen by \(2 \%\) to \(7.5 \%\).
Allowing for basis risk, state what the effective interest rate for borrowing should be when the futures position is closed.

FRAs and futures
It is now 1st April. Your company will receive \(£ 8.2\) million from a customer in four months' time, and it will invest this money for five months until the end of December, when it will be needed for spending on a planned capital project. The company treasurer intends to put the money on deposit for five months when it is received, and expects to be able to invest short term to earn LIBOR plus \(0.40 \%\).

The treasurer is worried about the risk of a fall in interest rates and wants to secure an effective interest rate for the investment of the \(£ 8.2\) million for the five-month period.

The following information is available:

\section*{LIFFE \(£ 500,0003\) month sterling futures}

Tick size (0.0001) \(£ 12.50\)
September: 95.35
December: 95.70

Futures contracts mature at the end of the relevant month.
The current three-month LIBOR rate is \(5 \%\).

\section*{FRA prices}

4v5: 4.75 - 4.70
4v9: \(4.57-4.52\)
5v9: \(4.49-4.44\)

\section*{Required}
(a) Explain how you would lock in an effective interest rate for the income from investing the \(£ 8.2\) million, using:
(1) FRAs
(2) Interest rate futures
(b) Show what will happen at the end of July if the three-month LIBOR rate is \(4.25 \%\) and the interest rate exposure had been hedged as indicated in part (a) of the answer, using:
(1) FRAs
(2) Interest rate futures

\section*{45 Traded equity options}

It is mid-February. A UK investor believes that in the next few weeks, the share price of company TBA will fall by a substantial amount. The share price is currently 982.

The investor has decided to speculate on a fall in the share price using equity options, and is prepared to invest up to \(£ 12,000\) in an options transaction.

On the LIFFE exchange, traded options are for 2,000 shares in a company, and the following option prices (in pence) are currently available for TBA shares:
\begin{tabular}{lrr} 
Strike price & \multicolumn{2}{c}{ March } \\
\hline pence & Calls & Puts \\
950 & 40 & 15 \\
1,000 & 10 & 50
\end{tabular}

\section*{Required}
(a) Explain how this investor might use options to speculate on a fall in the TBA.
(b) Show what would happen when the options expire if the TBA share price is 910.

\section*{46 Currency options}

A UK company will receive US\$2 million in six months' time. It is now 20th March. The company is not sure whether the US dollar will rise or fall in value against sterling over the next few months, and it has decided to hedge its exposure to currency risk using traded currency options.

On the Philadelphia Stock Exchange, traded currency options are available in a contract size of \(£ 31,250\). Options are priced in cents per \(£ 1\). Assume that option contracts expire on 20th of each month.

The following option prices are currently available:
\begin{tabular}{lrrrr} 
Exercise price & \multicolumn{2}{c}{ Calls } & \multicolumn{2}{c}{ Puts } \\
\hline & June & September & June & September \\
1.8500 & 1.4 & 1.9 & 4.0 & 5.1
\end{tabular}

The current spot exchange rate (US\$/£1) is \(1.8325-1.8375\).

\section*{Required}
(a) Explain how the company's currency exposure could be hedged using traded currency options.
(b) Show what would happen if the options are still held by the company at expiry and the spot exchange rate is \(\$ 1.9150-1.9200\).

\section*{47 Interest rate hedge}

A UK company will need to borrow \(£ 21\) million for two months, starting in three months' time. It is now mid-March. The current LIBOR rate is 5\% and the company can borrow at LIBOR \(+0.75 \%\).

The company is concerned about the possibility of an increase in short-term interest rates during the next two months, and it is looking at methods of hedging its exposure to the risk. The three methods it is considering are interest rate futures, options on interest rate futures and an FRA.

Current prices for futures, options and FRAs are as follows. (Note: Assume that all exchange-traded derivatives reach settlement on the last day of the relevant month).
\begin{tabular}{lr} 
Interest rate futures \\
\hline Notional three-month deposit \(£ 500,000\) \\
Value of 1 tick = \(£ 12.50\) \\
March & 94.740 \\
June & 94.610 \\
September & 94.500
\end{tabular}

Options on interest rate futures
Premium cost expressed as an annual interest rate \%
\begin{tabular}{lrrrrrr} 
Strike price & & Calls & & \multicolumn{3}{c}{ Puts } \\
\hline & March & June & September & March & June & September \\
94750 & 0.140 & 0.200 & 0.280 & 0.320 & 0.390 & 0.500 \\
95000 & 0.124 & 0.080 & 0.120 & 0.470 & 0.560 & 0.850
\end{tabular}
\begin{tabular}{lr}
\multicolumn{2}{l}{ Forward rate agreements (FRAs) } \\
\hline 3 v 5 & \(5.38-5.32\) \\
3v6 & \(5.43-5.46\) \\
3v8 & \(5.52-5.44\)
\end{tabular}

\section*{Required}

Show how the exposure to interest rate risk would be hedged using each of the three types of derivative.

Show what will happen in mid-June when the company borrows \(\$ 21\) million, assuming that the LIBOR rate has risen to \(6.0 \%\).

\section*{48 Black-Scholes}

The current price of shares in FAME is 870.

\section*{Required}

Use the Black-Scholes option pricing model to calculate the value of a call option maturing in six months' time and with an exercise price of 900.
The following data should be used in your calculation
(1) Risk-free interest rate \(=5 \%\) per year
(2) Variance of returns on FAME shares \(=9 \%\)

\section*{\(49 \quad\) Put-call parity}

The price of a call option on PWW shares, with a strike price of 600 and an expiry date in six months, is 33 . The current share price is 582 . The cost of capital is \(4 \%\) per year.

\section*{Required}

Use the put-call parity equation to calculate the price of a put option on PWW shares with a strike price of 600 and an expiry date in six months.

\section*{50 Delta hedge}
(a) The delta value of shares in TER is 0.125 .

\section*{Required}

How might a bank use this information to construct a delta hedge for its option trading operations in TER shares?
(b) The share price of BVZ is currently 258. A bank is writing call options on BVZ shares at a strike price of 260 , with an expiry date in six months' time. The risk-free rate of return is \(4 \%\) per year and the variance of returns on BVZ shares is \(16 \%\).

\section*{Required}

How might the bank use this information to construct a delta hedge for its option trading operations in BVZ shares?

\section*{51 Bonus scheme}

The remuneration committee of Grand plc is considering a remuneration package for a new executive director. The committee want to include an attractive incentive arrangement in the remuneration package, and are considering two possible schemes.
(a) Scheme 1. The annual cash bonus of the executive director would be \(0.5 \%\) of the economic value added (EVA®) of the company.
(b) Scheme 2. The executive should be given share options on 2 million of the company's shares, exercisable in one year's time. The share options would have an exercise price of the company's current market price, which is 140 cents.
The income statement of the company for the financial year just ended is as follows:
\begin{tabular}{lr} 
& \$ million \\
Sales & 683 \\
Cost of sales & \((421)\) \\
Depreciation & \((65)\) \\
Research and development & \((14)\) \\
Net interest & \((35)\) \\
Profit before tax & 148 \\
Tax & \((44)\) \\
Profit after tax & 104 \\
\hline
\end{tabular}

The following information is available:
(1) The capital employed of the company is reported as \(\$ 525\) million as at the beginning of the financial year just ended, and \(\$ 570\) million as at the end of the year.
(2) The research and development expenditure programme has involved spending of \(\$ 14,000\) for each of the past four years.
(3) Accounting depreciation and economic depreciation are about the same.
(4) The company's weighted average cost of capital is \(9 \%\).
(5) The rate of tax on profits is \(30 \%\).
(6) The risk-free rate is \(5 \%\).
(7) The members of the remuneration committee do not know the value of share options in the company, but they are aware that the current premium for an at-the-money put option on Grand plc shares, exercisable in one year, is 30 cents.

\section*{Required}

Calculate the potential value of each of the incentive schemes to the executive director (and the cost to the company). State any assumptions that you make.

\section*{52 \\ Delta neutral}

Barn plc has an investment in 4 million shares of a rival UK company, Door plc. The investment has been acquired with a view to possibly making a takeover bid for Door plc, although a decision will not be taken until at least the end of November. It is now 1st July, and Door plc shares are currently trading at 635 pence

The treasurer of Barn plc is concerned about the risk of a fall in the market price of Door plc shares, and is considering the use of traded options in Door shares to hedge the risk.

Traded options: ( 1,000 shares)
\begin{tabular}{lcccccc} 
\\
& August & CALLS & November & February & August & \begin{tabular}{c} 
PUTS \\
November
\end{tabular} \\
600 & 34.0 & 47.5 & 54.5 & 2.0 & 22.0 & 31.5 \\
650 & 6.0 & 21.5 & 30.5 & 19.0 & 47.5 & 56.0
\end{tabular}

\section*{Required}
(a) Show how Barn plc might use traded options to create a hedge against the risk of a fall in the price of Door plc shares. Assuming that at the end of November the price of Door plc shares is 580, show the outcome of your hedge.
(b) Assume that the delta of the relevant Door plc option is 0.46 . Illustrate how a delta neutral hedge might be created to protect against a fall in the price of Door plc shares. Explain any practical difficulties there might be with using a delta hedge for this purpose.

\section*{Paper P4}

Advanced Financial Management

\section*{\(Q_{\&} A\)}

Answers
\begin{tabular}{|lll|}
\hline \multicolumn{2}{|c|}{ Contents } \\
\hline \multicolumn{2}{|c|}{ The role and responsibilities of financial } \\
managers \\
1 & Corporate social responsibility & Page \\
2 & Corporate governance & 476 \\
3 & Dividends and retentions & 476 \\
4 & Financial plan & 477 \\
& Capital investment appraisal & 478 \\
5 & DCF exercises & \\
6 & DCF and tax & 479 \\
7 & Free cash flow & 480 \\
& & 481 \\
Investing: portfolio theory and the CAPM & \\
8 & Two-asset portfolio & 481 \\
9 & Risk and return & 483 \\
10 & Coefficient of variation & 483 \\
11 & Capital market line & 484 \\
12 & Obtaining a beta factor & 484 \\
& & \\
Cost of capital \\
13 & WACC & 485 \\
14 & Optimal WACC & 485 \\
15 & Change in gearing & 486 \\
\hline
\end{tabular}
\begin{tabular}{|c|c|c|}
\hline 16 & Geared beta & 487 \\
\hline \multicolumn{3}{|l|}{Other aspects of capital expenditure appraisal} \\
\hline 17 & APV method & 488 \\
\hline 18 & Adjusted present value & 489 \\
\hline 29 & More APV & 492 \\
\hline \multicolumn{3}{|l|}{International investment and financing decisions} \\
\hline 20 & Cash flows from a foreign project & 493 \\
\hline \multirow[t]{2}{*}{21} & Foreign investment & 493 \\
\hline & \multicolumn{2}{|l|}{Valuations.Mergers and acquisitions} \\
\hline 22 & Valuation & 495 \\
\hline 23 & Valuation of bonds & 496 \\
\hline 24 & Annuities and bond prices & 497 \\
\hline 25 & EVA & 497 \\
\hline 26 & MM, gearing and company valuation & 497 \\
\hline 27 & Acquisition & 497 \\
\hline 28 & Takeover & 498 \\
\hline \multicolumn{3}{|l|}{Foreign exchange risk and currency risk management} \\
\hline 29 & Interest rate parity & 501 \\
\hline 30 & Foreign exchange & 501 \\
\hline 31 & Money market hedge & 501 \\
\hline \multicolumn{3}{|l|}{Interest rate risk. Hedging with FRAs and swaps} \\
\hline 32 & FRA & 502 \\
\hline 33 & Swap & 503 \\
\hline 34 & Credit arbitrage (1) & 503 \\
\hline 35 & Credit arbitrage (2) & 503 \\
\hline 36 & Currency swap & 504 \\
\hline \multicolumn{3}{|l|}{Futures and hedging with futures} \\
\hline 37 & Currency futures & 505 \\
\hline 38 & More currency futures & 506 \\
\hline 39 & Basis & 507 \\
\hline
\end{tabular}
\begin{tabular}{|lll|}
\hline 40 & Imperfect hedge and basis & 507 \\
41 & Currency hedge & 508 \\
42 & Hedging with STIRs & 509 \\
43 & More hedging with STIRs & 510 \\
44 & FRAs and futures & 511 \\
& & \\
Options and hedging with options \\
45 & Traded equity options \\
46 & Currency options & 512 \\
47 & Interest rate hedge & 513 \\
& & 514 \\
Option pricing and delta hedging & \\
48 & Black-Scholes & 515 \\
49 & Put-call parity & 516 \\
50 & Delta hedge & 516 \\
51 & Bonus scheme & 517 \\
52 & Delta neutral & 518 \\
\hline
\end{tabular}

\section*{1 Corporate social responsibility}
(a) (i) Ethics and behaving in an ethical manner in business (and in some cases, bribery and corruption)
(ii) Concern for employees
(iii) Concern for the community
(iv) Concern for human rights
(v) Concern for the environment: protecting the environment, creating a sustainable business.
(b) CSR issues vary between different types of company. Companies most likely to be affected include:
(i) companies that enjoy some form of monopoly, such as energy supply companies
(ii) companies that deal directly with consumers, such as banks and retail organisations
(iii) companies manufacturing food, drink and medicines
(iv) companies that extract resources from the environment: manufacturing companies whose products or manufacturing systems pollute the environment
(v) companies with a supply chain in developing countries (such as clothing manufacturers and oil exploration and extraction companies).
(c) CSR risk is the risk that a CSR issue might affect a company in any of the following ways:
(i) The company might suffer a loss of reputation (reputation risk). This could lead to a loss of customers who switch to the products of rival companies. A poor reputation could also lead to the threat of legislation.
(ii) The company might be exposed to the risk of legal action by customers and others (litigation risk). A notable example has been the litigation faced by tobacco companies, particularly in the US.
(iii) Another aspect of litigation risk is the possibility that a government will introduce legislation against a company's products (e.g. tobacco products, food products, medicines and drugs) or its production methods (e.g. anti-pollution laws).
(iv) Some investors choose to invest only in 'ethical companies'. A company with a poor CSR record might therefore attract fewer investors.
(v) Research appears to show that companies with positive CSR policies benefit financially, in terms of profits and share price.

\section*{2 Corporate governance}
(a) The main aim of a code of corporate governance should be to ensure that the directors or managers of a company should run the company in the interests of its shareholders.
(b) The main issues in corporate governance, for both statutory and voluntary codes, are likely to be:
(i) the composition of the board of directors: achieving a balance of skills and experience, preventing the board from being dominated by one individual or a small group of individuals, the role of non-executive directors (and in particular independent non-executive director)
(ii) the decision-making responsibilities of the board (and how much decision-making responsibility should be delegated to management)
(iii) the process of nominating new directors for appointment to the board
(iv) the process of deciding the remuneration of directors, and trying to ensure that remuneration systems reward directors for success and do not reward them for failure
(v) the reliability of financial reporting, the independence of the external auditors, and the role of the audit committee
(vi) communications between a company and its shareholders
(vii) risk management and internal control, and the review of internal control and risk management systems
(viii) restricting the scope for insider trading or market abuse.

Corporate social responsibility and protection for whistle-blowers could also be added to this list.

\section*{3 Dividends and retentions}

The dividend growth model will be used to estimate what share price might be expected.

It is assumed that the growth rate in earnings and dividends will be br, where \(b\) is the proportion of earnings that is retained and \(r\) is the return on new investment.
(a) Dividends are \(25 \%\) of earnings and \(75 \%\) of earnings are retained:

Growth rate \(=0.75 \% \times 0.09=0.0675\).
Expected share price \(=\frac{0.50(1.0675)}{(0.09-0.0675)}=\$ 23.72\)
(b) Dividends are \(50 \%\) of earnings and \(50 \%\) of earnings are retained:

Growth rate \(=0.50 \% \times 0.09=0.045\).
Expected share price \(=\frac{1.00(1.045)}{(0.09-0.045)}=\$ 23.22\)
(c) Dividends are \(70 \%\) of earnings and \(30 \%\) of earnings are retained:

Growth rate \(=0.30 \% \times 0.09=0.027\).
Expected share price \(=\frac{1.40(1.027)}{(0.09-0.027)}=\$ 22.82\)

\section*{4 Financial plan}

Tutorial note. Many of the figures for the financial plan can be calculated by increasing the amount by \(8 \%\) each year, in line with sales growth. The bank overdraft interest each year is calculated by taking the bank overdraft at the end of the previous year. The bank overdraft is a balancing figure in the balance sheet, that makes the equity and liabilities add up to the total assets.

\section*{Income statements}
\begin{tabular}{|c|c|c|c|c|c|}
\hline \multicolumn{2}{|l|}{\multirow[t]{2}{*}{}} & Year 5 & Year 6 & Year 7 & Year 8 \\
\hline & & \$m & \$m & \$m & \$m \\
\hline EBITDA & (+8\% per year) & 583 & 630 & 680 & 735 \\
\hline Depreciation & (+ 8\% per year) & (173) & (187) & (202) & (218) \\
\hline Earnings before interest & & 410 & 443 & 478 & 517 \\
\hline Interest & (see workings) & (86) & (95) & (106) & (118) \\
\hline Profit before tax & & 324 & 348 & 372 & 399 \\
\hline Tax (30\%) & & (97) & (104) & (112) & (120) \\
\hline Profit after tax & & 227 & 244 & 260 & 279 \\
\hline Dividends & (64\%) & (145) & (159) & (166) & (179) \\
\hline Retained earnings & & 82 & 85 & 94 & 100 \\
\hline
\end{tabular}

\section*{Balance sheets}
\begin{tabular}{|c|c|c|c|c|c|}
\hline \multirow[b]{2}{*}{Plant and equipment} & \multirow[b]{2}{*}{\[
\begin{aligned}
& \text { (+8\% per } \\
& \text { year) }
\end{aligned}
\]} & \[
\text { Year } 5
\]
\$m & \[
\begin{gathered}
\text { Year } 6 \\
\$ m
\end{gathered}
\] & \[
\text { Year } 7
\]
\[
\$ \mathrm{~m}
\] & Year 8 \$m \\
\hline & & 2,182 & 2,356 & 2,545 & 2,748 \\
\hline \multirow[t]{3}{*}{Inventory + receivables trade payables Cash} & (+ 8\% per
year) & 767 & 828 & 894 & 966 \\
\hline & \[
\begin{aligned}
& \text { (+ 8\% per } \\
& \text { year) }
\end{aligned}
\] & 32 & 35 & 38 & 41 \\
\hline & & 2,981 & 3,219 & 3,477 & 3,755 \\
\hline \multirow[t]{3}{*}{Share capital Reserves} & & 450 & 450 & 450 & 450 \\
\hline & (add retained profit) & 1,283 & 1,368 & 1,462 & 1,562 \\
\hline & & 1,733 & 1,818 & 1,912 & 2,012 \\
\hline \multirow[t]{2}{*}{Long-term loan} & & 800 & 800 & 800 & 800 \\
\hline & & 2,533 & 2,618 & 2,712 & 2,812 \\
\hline \multirow[t]{2}{*}{Bank overdraft} & (balancing figure) & 448 & 601 & 765 & 943 \\
\hline & & 2,981 & 3,219 & 3,477 & 3,755 \\
\hline
\end{tabular}

\section*{Workings}
(1) At the end of year 4, inventory + receivables - trade payables \(=710\) (in \$ million). This amount will increase by \(8 \%\) each year.
(2) Interest charges
\begin{tabular}{|c|c|c|c|c|c|}
\hline & - & Year 5 & Year 6 & Year 7 & Year 8 \\
\hline & & \$m & \$m & \$m & \$m \\
\hline Long-term loan & (8\% \(\times 800\) ) & 64 & 64 & 64 & 64 \\
\hline Bank overdraft & ( \(7 \% \times\) previous year) & 22 & 31 & 42 & 54 \\
\hline & & 86 & 95 & 106 & 118 \\
\hline
\end{tabular}
(b)

There are several definitions of free cash flow. Other definitions are acceptable for your answer.
\begin{tabular}{|c|c|c|c|c|}
\hline & Year 5 & Year 6 & Year 7 & Year 8 \\
\hline & \$m & \$m & \$m & \$m \\
\hline EBIT ( \(1-\mathrm{t}\) ) \(\begin{aligned} & \text { Earnings before interest } \\ & \text { less tax at } 30 \%\end{aligned}\) & 287 & 310 & 335 & 362 \\
\hline Depreciation & 173 & 187 & 202 & 218 \\
\hline Increase in plant and equipment & (162) & (174) & (189) & (203) \\
\hline Increase in inventory + receivables payables & (57) & (61) & (66) & (72) \\
\hline Free cash flow & 241 & 262 & 282 & 305 \\
\hline
\end{tabular}
(c)

A feature of the financial plan that might need review is the cash position of the company. The bank overdraft is forecast to increase from \(\$ 310,000\) to \(\$ 943,000\), although the company expects to make a profit each year. The free cash flow each year, as measured, is not much more than the interest payments and dividend payments.

This suggests that the company might need to reconsider its dividend policy, and pay lower dividends. In addition, the company might possible consider alternative sources of finance, so that it does not have to rely so much on an overdraft facility. More long-term debt might be appropriate, if this can be obtained at a suitable interest rate.
(d)

A possible value of the company's shares at the end of the financial planning period can be estimated using the dividend growth model, assuming that dividends will grow by about \(8 \%\) per year (in line with sales growth) and the cost of equity will remain at \(12 \%\).

Expected equity value in \(\$\) millions \(=\frac{179(1.08)}{(0.12-0.08)}\)
\(=\$ 4,833\) million .
There are 9,000,000 shares of \(\$ 0.05\) each . This gives a valuation of \(\$ 537\) per share.

\section*{5 DCF exercises}
(a)
\begin{tabular}{lr} 
Annuity factor at 8\%, years 1-8 & 5.747 \\
Annuity factor at 8\%, years 1-4 & 3.312 \\
\hline Annuity factor at 8\%, years 5-8 & 2.435 \\
\hline
\end{tabular}

The Year 10 value of a perpetuity of \(\$ 100,000\) each year in perpetuity from Year 11 onwards:
\[
\begin{aligned}
& =\$ 100,000 \times 1 / 0.08 \\
& =\$ 1,250,000
\end{aligned}
\]
\begin{tabular}{lrrr} 
Years & Annual cash flow & Discount factor at \(8 \%\) & PV \\
\hline & \(\$\) & & \(\$\) \\
0 & \((3,000,000)\) & 1.000 & \((3,000,000)\) \\
\(1-4\) & 500,000 & 3.312 & \(1,656,000\) \\
\(5-8\) & 400,000 & 2.435 & 974,000 \\
\(9-10\) & 300,000 & 0.963 & 288,900 \\
10 & \(1,250,000\) & 0.463 & 578,750 \\
NPV & & & \(+497,650\) \\
\hline
\end{tabular}
(b) (i) Ignoring inflation
\begin{tabular}{lrrr} 
Year & Annual cash flow & Discount factor at 12\% & PV \\
\hline & \(\$\) & & \(\$\) \\
0 & \((900,000)\) & 1.000 & \((900,000)\) \\
1 & 400,000 & 0.893 & 357,200 \\
2 & 400,000 & 0.797 & 318,800 \\
3 & 250,000 & 0.636 & 159,000 \\
4 & 300,000 & 0.567 & 170,100 \\
& & & \(+105,100\) \\
& & &
\end{tabular}
(ii) Allowing for inflation
\begin{tabular}{|c|c|c|c|c|c|}
\hline Year & 0 & 1 & 2 & 3 & 4 \\
\hline & \$ & \$ & \$ & \$ & \$ \\
\hline Equipment & \((900,000)\) & & & & 243,101 \\
\hline Revenue & & 824,000 & 848,720 & 655,636 & 450,204 \\
\hline Running costs & & \((420,000)\) & \((441,000)\) & \((405,169)\) & \((364,652)\) \\
\hline Net cash flow & \((900,000)\) & 404,000 & 407,720 & 250,467 & 328,653 \\
\hline Discount factor at \(12 \%\) & 1.000 & 0.893 & 0.797 & 0.636 & 0.567 \\
\hline Present value & \((900,000)\) & 360,772 & 324,953 & 159,297 & 186,346 \\
\hline NPV = + 131,368 & & & & & \\
\hline
\end{tabular}

\section*{6 DCF and tax}

Annual depreciation \(=\$(800,000-200,000) / 4\) years \(=\$ 150,000\) per year.
Annual cash profits \(=\) Accounting profit + Deprecation \(=\$ 250,000+\$ 150,000=\) \$400,000.

Tax depreciation allowances (capital allowances)
\begin{tabular}{|c|c|c|c|c|}
\hline Year of claim & & Written down value & Tax saved at 30\% & Year of cash flow \\
\hline \multirow{4}{*}{0} & \multirow{4}{*}{Allowance (25\%)} & \$ & \$ & \\
\hline & & 800,000 & & \\
\hline & & \((200,000)\) & 60,000 & 1 \\
\hline & & 600,000 & & \\
\hline \multirow[t]{2}{*}{1} & \multirow[t]{2}{*}{Allowance (25\%)} & \((150,000)\) & 45,000 & 2 \\
\hline & & 450,000 & & \\
\hline \multirow[t]{2}{*}{2} & Allowance (25\%) & \((112,500)\) & 33,750 & 3 \\
\hline & & 337,500 & & \\
\hline
\end{tabular}


\section*{\(7 \quad\) Free cash flow}
\begin{tabular}{lr} 
& \$ million \\
\hline Profit before interest and tax & 3.00 \\
Interest & \((0.40)\) \\
Taxation & \((0.60)\) \\
Depreciation charges & 0.55 \\
Essential capital expenditure & \((1.00)\) \\
\cline { 2 - 2 } & 1.55 \\
cash flow &
\end{tabular}

\section*{8 Two-asset portfolio}
(a) Security \(\mathbf{X}\)

EV of return ( \(\bar{x}\) )
\(=(0.25 \times 15)+(0.60 \times 10)+(0.15 \times 2)=10.05\).
\begin{tabular}{lrrr}
\cline { 3 - 4 } Probability & Return & \(\mathbf{x}-\bar{x}\) & \(\mathbf{p}(\mathbf{x}-\bar{x})^{\mathbf{2}}\) \\
\hline p & \(\mathbf{x}\) & & \\
0.25 & 15 & 4.95 & 6.1256 \\
0.60 & 10 & \((0.05)\) & 0.0015 \\
0.15 & 2 & \((8.05)\) & 9.7204 \\
& & Variance \(\sigma^{2}\) & 15.8475 \\
& & &
\end{tabular}

Standard deviation of return \(\sigma_{x}=\sqrt{15.8475}=3.98\).
(b) Security Y

EV of return ( \(\bar{x}\) )
\(=(0.25 \times 20)+(0.60 \times 8)+(0.15 \times(6))=8.90\)
\begin{tabular}{lrrr} 
Probability & Return & \(\mathbf{y}-\bar{y}\) & \(\mathbf{p}(\mathbf{y}-\bar{y})^{\mathbf{2}}\) \\
\hline p & y & & \\
0.25 & 20 & 11.10 & 30.8025 \\
0.60 & 8 & \((0.90)\) & 0.4860 \\
0.15 & \((6)\) & \((14.90)\) & 33.3015 \\
& & Variance \(\sigma^{2}\) & 64.5900 \\
& &
\end{tabular}

Standard deviation of return \(\sigma_{y}=\sqrt{64.59}=8.04\).
(c)
\begin{tabular}{lrrr} 
Probability & \(\mathbf{x}-\bar{x}\) & \(\mathbf{y}-\bar{y}\) & \(\mathbf{p}(\mathbf{x}-\bar{x})^{\mathbf{2}}\) \\
\hline p & & & \\
0.25 & 4.95 & 11.10 & 13.7363 \\
0.60 & \((0.05)\) & \((0.90)\) & 0.0270 \\
0.15 & \((8.05)\) & \((14.90)\) & 17.9917 \\
& & \(\operatorname{Cov}_{\mathrm{x}, \mathrm{y}}\) & \begin{tabular}{ll}
31.7550 \\
&
\end{tabular}
\end{tabular}
(d) Correlation coefficient
\(\rho_{x, y}=\frac{31.7550}{3.98 \times 8.04}\)
\(=+0.992\).
This shows a high level of positive correlation between the returns from Security X and the returns from Security Y.
(e) The EV of the return from a portfolio consisting of \(50 \%\) Security X and \(50 \%\) Security Y
\(=(0.50 \times 10.05)+(0.50 \times 8.90)=9.475 \%\).
The variance of the returns from this portfolio would be:
\(\left[(0.50)^{2} \times 15.8475\right]+\left[(0.50)^{2} \times 64.5900\right]+[2 \times 0.50 \times 0.50 \times 31.7550]\)
\(=3.9619+16.1475+15.8775=35.9869\).
The standard deviation of the portfolio returns \(=\sqrt{35.9869}=6.0 \%\).
(f) For a portfolio consisting of \(80 \%\) Security X and \(20 \%\) Security Y :

The EV of the return
\(=(0.80 \times 10.05)+(0.20 \times 8.90)=9.82 \%\).
The variance of the returns from this portfolio would be:
\(\left[(0.80)^{2} \times 15.8475\right]+\left[(0.20)^{2} \times 64.5900\right]+[2 \times 0.80 \times 0.20 \times 31.7550]\)
\(=10.1424+2.5836+10.1616=22.8876\).
The standard deviation of the portfolio returns \(=\sqrt{22.8876}=4.78 \%\).
Note: In this example, since Security Y has a lower expected return than Security X and a higher standard deviation, expected returns will be highest and risk lowest with a 'portfolio' consisting of Security X only, and none of Security Y.

\section*{\(9 \quad\) Risk and return}
(a)

Project 3
Expected return
\((0.6 \times 6)+(0.4 \times 1)=+4.0\)
\begin{tabular}{|c|c|c|c|c|c|c|}
\hline & Return r & Probability p & \[
(\mathbf{r}-\bar{r})^{2}
\] & Return r & Probability p & \((\mathrm{r}-\overline{\mathrm{r}})^{2}\) \\
\hline & 6.0 & 0.6 & 2.40 & 8.0 & 0.5 & 10.125 \\
\hline & 1.0 & 0.4 & 3.60 & -1.0 & 0.5 & 10.125 \\
\hline Variance ( \(\sigma)^{2}\) & & & 6.00 & & & 20.250 \\
\hline \(\sigma\) & & & 2.45 & & & 4.5 \\
\hline
\end{tabular}
(b) The divisional manager will invest in projects that are more risky provided that they offer a higher return.
The manager will not invest in Project 4 because it offers a lower expected return than Project 3 but higher risk.
The expected return from Project 1 is \((0.8 \times 4)+(0.2 \times 2)=+3.6\).
The expected return from Project 2 is \((0.7 \times 5)+(0.3 \times 1.5)=+3.95\).
The highest expected return is offered by Project 3, which has a higher risk than Project 1 and Project 2. It would seem that the divisional manager will invest in Project 3 because he is prepared to take the higher risk for a higher expected return. However, Project 2 might seem more attractive: its expected return is almost as high as for Project 3 and the risk is much less.

\section*{10 Coefficient of variation}
\begin{tabular}{llr} 
Portfolio & Expected return \\
\hline \(50 \%\) Country A, \(50 \%\) Country B & \((0.5 \times 16)+(0.5 \times 22)\) & 19.0 \\
\(50 \%\) Country A, \(50 \%\) Country C & \((0.5 \times 16)+(0.5 \times 30)\) & 23.0 \\
\(50 \%\) Country B, \(50 \%\) Country C & \((0.5 \times 22)+(0.5 \times 30)\) & 26.0
\end{tabular}

The standard deviation of a portfolio is:
\(\sigma \rho={\sqrt{\sigma_{\mathrm{A}}{ }^{2} x^{2}+\sigma_{\mathrm{B}}{ }^{2}(1-x)+2 x(1-x) \rho_{\mathrm{A}, \mathrm{B}} \sigma_{\mathrm{A}} \sigma_{\mathrm{B}}}}^{2}\)
However, since the returns from each country are independent of each other, the covariance of returns ( \(\rho_{\mathrm{A}, \mathrm{B}}\) ) is 0 ; therefore the second half of the formula can be ignored because its value is zero.
\begin{tabular}{llr} 
Portfolio & & \begin{tabular}{r} 
Standard deviation \\
of returns
\end{tabular} \\
\hline \(50 \%\) Country A, \(50 \%\) Country B & {\(\left[\left(22^{2} \times 0.5^{2}\right)+\left(36^{2} \times 0.5^{2}\right)\right]^{1 / 2}\)} & 21.9 \\
\(50 \%\) Country A, 50\% Country C & {\(\left[\left(22^{2} \times 0.5^{2}\right)+\left(45^{2} \times 0.5^{2}\right)\right]^{1 / 2}\)} & 25.7 \\
\(50 \%\) Country B, 50\% Country C & {\(\left[\left(36^{2} \times 0.5^{2}\right)+\left(45^{2} \times 0.5^{2}\right)\right]^{1 / 2}\)} & 28.8
\end{tabular}
(Tutorial note: 'To the power of \(1 / 2\) ' is the same as 'the square root'.)

\section*{Coefficient of variation}

The coefficient of variation is the ratio of the risk (standard deviation of returns) to the expected return.
\begin{tabular}{lrr} 
Portfolio & \begin{tabular}{r} 
Coefficient \\
of variation
\end{tabular} \\
\hline \(50 \%\) Country A, \(50 \%\) Country B & \(21.9 / 19.0=\) & 1.15 \\
\(50 \%\) Country A, \(50 \%\) Country C & \(25.7 / 23.0=\) & 1.12 \\
\(50 \%\) Country B, \(50 \%\) Country C & \(28.8 / 26.0=\) & 1.11
\end{tabular}

The ratio of risk to expected returns is roughly the same for all three portfolios.

\section*{11 Capital market line}

The investor wanting a return of \(7 \%\) can invest \(\mathrm{x} \%\) of his funds in risk-free securities earning \(3 \%\) and \((1-x) \%\) in the market portfolio earning \(9 \%\).
\(3 x+9(1-x)=7\)
\(6 x=2\)
\(x=1 / 3\) or \(33.33 \%\).

The investor should invest one-third of his funds in risk-free securities and twothirds in the market portfolio.

\section*{12 Obtaining a beta factor}
(a)
\begin{tabular}{lrrrr} 
Month & \multicolumn{2}{c}{ Market portfolio } & \multicolumn{2}{c}{ Security \(\mathbf{Y}\)} \\
\hline & \(\mathbf{x}-\bar{x}\) & \((\mathbf{x}-\bar{x})^{\mathbf{2}}\) & \(\mathbf{y}-\bar{y}\) & \((\mathbf{y}-\bar{y})^{\mathbf{2}}\) \\
1 & +1.5 & 2.25 & +2 & 4 \\
2 & \((1.5)\) & 2.25 & \((3)\) & 9 \\
3 & \((2.5)\) & 6.25 & \((3)\) & 9 \\
4 & +2.5 & 6.25 & +4 & 16 \\
& & 17.00 & & 38 \\
& & & &
\end{tabular}

Standard deviation of market returns \(=\sqrt{17.00}=4.123 \%\).
Standard deviation of Security Y returns \(=\sqrt{38}=6.164 \%\).
(b)
\begin{tabular}{lrrr} 
Month & \(\mathbf{x}-\bar{x}\) & \(\mathbf{y}-\bar{y}\) & \begin{tabular}{r}
\((\mathbf{x}-\bar{x})\) \\
\((\mathrm{y}-\bar{y})\)
\end{tabular} \\
\hline 1 & +1.5 & +2 & +3.0 \\
2 & \((1.5)\) & \((3)\) & +4.5 \\
3 & \((2.5)\) & \((3)\) & +7.5 \\
4 & +2.5 & +4 & +10.0 \\
\hline
\end{tabular}

Covariance of returns \(=25.0\)
(c) Correlation coefficient \(\rho_{\mathrm{m}, \mathrm{y}}\)
\[
\begin{aligned}
\rho_{x, y} & =\frac{25.0}{4.123 \times 6.164} \\
& =+0.984 .
\end{aligned}
\]
(d) Beta factor for Security \(\mathbf{Y}\)
\[
\beta=\frac{\operatorname{Cov}_{m, y}}{\operatorname{Var}_{\mathrm{m}}}=\frac{25}{17}=1.47
\]

Alternatively
\[
\beta=\frac{\rho_{\mathrm{m}, \mathrm{y}} \mathrm{x} \sigma_{\mathrm{y}}}{\sigma_{\mathrm{m}}}=\frac{0.984 \times 6.164}{4.123}=1.47
\]

\section*{13 WACC}
\[
\begin{aligned}
& \text { Cost of equity }=\frac{18(1.03)}{155}+0.03 \\
& =0.1496 \text { or } 14.96 \% \\
& \text { WACC }=\left[\frac{350}{(1,200+350)} \times 7.8(1-0.30)\right]+\left[\frac{1,200}{(1,200+350)} \times 14.96\right] \\
& =1.23+11.58 \\
& =12.81 \%
\end{aligned}
\]

\section*{14 Optimal WACC}

The optimal WACC is the lowest WACC, because this will maximise the value of the company and the wealth of shareholders.

\section*{Step 1}

Calculate the geared beta for equity at each level of gearing.
\begin{tabular}{ll} 
Gearing & Geared beta \\
\hline \(20 \%\) & \(0.90 \times \frac{80+20(1-0.30)}{80}=1.0575\) \\
\(30 \%\) & \(0.90 \times \frac{70+30(1-0.30)}{70}=1.170\) \\
\(40 \%\) & \(0.90 \times \frac{60+40(1-0.30)}{60}=1.320\) \\
\(50 \%\) & \(0.90 \times \frac{50+50(1-0.30)}{50}=1.530\) \\
\(60 \%\) & \(0.90 \times \frac{40+60(1-0.30)}{40}=1.845\)
\end{tabular}

\section*{Step 2}

Use the geared beta value and the CAPM to calculate a cost of equity at each gearing level.
\begin{tabular}{lclr} 
Gearing & \multicolumn{2}{c}{ Cost of equity \((\mathbf{4 \%}+\boldsymbol{\beta} \mathbf{( 9 - 4 ) \%}\)} \\
\hline \(20 \%\) & \(4+1.0575 \times 5\) & \(=\) & \(7.17 \%\) \\
\(30 \%\) & \(4+1.170 \times 5\) & \(=\) & \(7.51 \%\) \\
\(40 \%\) & \(4+1.320 \times 5\) & \(=\) & \(7.96 \%\) \\
\(50 \%\) & \(4+1.530 \times 5\) & \(=\) & \(8.59 \%\) \\
\(60 \%\) & \(4+1.845 \times 5\) & \(=\) & \(9.54 \%\)
\end{tabular}

\section*{Step 3}

Calculate the WACC at each level of gearing, and identify the gearing level with the lowest WACC.
\begin{tabular}{llllr} 
Gearing & & & WACC \\
\hline \(20 \%\) & {\([20 \% \times 5.0(1-0.30)]\)} & \(+[80 \% \times 7.17]\) & \(=6.44 \%\) \\
\(30 \%\) & {\([30 \% \times 5.4(1-0.30)]\)} & \(+[70 \% \times 7.51]\) & \(=6.39 \%\) \\
\(40 \%\) & {\([40 \% \times 5.8(1-0.30)]\)} & \(+[60 \% \times 7.96]\) & \(=6.40 \%\) \\
\(50 \%\) & {\([50 \% \times 6.5(1-0.30)]\)} & \(+[50 \% \times 8.59]\) & \(=6.58 \%\) \\
\(60 \%\) & {\([60 \% \times 7.2(1-0.30)]\)} & \(+[40 \% \times 9.54]\) & \(=6.84 \%\)
\end{tabular}

\section*{Conclusion}

The optimal gearing level is \(30 \%\), because the WACC is lowest at this gearing level. However, the WACC is almost as low at a gearing level of \(40 \%\).

\section*{15 Change in gearing}
\begin{tabular}{lr} 
Annual earnings if the debt is issued & \$ million \\
\hline Profit before interest and tax & 40 \\
Interest (\$100 million x 8\%) & \((8)\) \\
Profit before tax & 32 \\
Tax (25\%) & \((8)\) \\
Profit after tax = earnings & 24 \\
\cline { 2 - 2 }
\end{tabular}

Value of geared company \(=\) Value of company ungeared + (Value of debt \(\times\) Tax rate)
\(\mathrm{V}_{\mathrm{g}}=\mathrm{V}_{\mathrm{u}}+\mathrm{Dt}\)
\(\mathrm{V}_{\mathrm{g}}=(60\) million \(\times \$ 5)+(\$ 100\) million \(\times 25 \%)\)
= \(\$ 325\) million
\begin{tabular}{lr} 
& \$ million \\
\hline Total value of geared company (equity + debt) & 325 \\
Value of debt & \((100)\) \\
\cline { 2 - 2 } Therefore value of equity in geared company & 225 \\
\cline { 2 - 2 }
\end{tabular}

Therefore cost of equity in geared company \(=24 / 225=0.1067\) or \(10.67 \%\).

WACC \(=\left[\frac{100}{325} \times 8.0(1-0.25)\right]+\left[\frac{225}{325} \times 10.67\right]=9.23 \%\)
(The WACC of the all-equity company is \(10 \%\). The increase in gearing reduces the WACC).

\section*{16 Geared beta}
(a) The current proportion of equity in the capital structure is \(1,500 /(1,500+500)\) \(=0.75\) or \(75 \%\).
The current proportion of debt in the capital structure is \(500 /(1,500+500)=\) 0.25 or \(25 \%\).

Cost of equity \(=5 \%+1.126(11-5) \%=11.756 \%\).
Since the beta factor of debt is 0 , the debt must be risk-free, with a pre-tax cost of \(5 \%\).
WACC \(=[0.25 \times 5.0(1-0.30)]+[0.75 \times 11.756]=9.692 \%\),say \(9.7 \%\)
(b) Step 1

Convert the current geared beta into an ungeared beta:
\(B_{\text {ungeared }}=B_{\text {geared }} \times \frac{E}{E+D(1-t)}\)
\(B_{\text {ungeared }}=1.126 \times \frac{75}{75+75(1-0.30)}\)
\(=0.913\)

\section*{Step 2}

Convert the ungeared beta into a geared beta for the new level of gearing
\(0.913=\mathrm{B}_{\text {geared }} \times \frac{60}{60+40(1-0.30)}\)
\(B_{\text {geared }}=\frac{0.913}{0.6818}=1.339\)

\section*{Step 3}

Use this geared beta factor to calculate the cost of equity at this gearing level.
Cost of equity \(=5 \%+1.339(11-5) \%=13.03 \%\).

\section*{Step 4}

Calculate the WACC at this gearing level. It is assumed that the cost of debt remains risk-free.
\(W A C C=\lfloor 0.40 \times 5.0(1-0.30)\rfloor+\lfloor 0.60 \times 13.03\rfloor=9.218 \%\), say \(9.2 \%\)

\section*{17 APV method}
(a) Modigliani-Miller formula approach

Ungeared beta for the telecommunications industry:
\(=1.30625 \times \frac{80}{80+20(1-0.25)}\)
\(=1.10\)
The company's gearing is \(70 \%\) equity and \(30 \%\) debt; therefore we need to regear the equity beta for the company.
\(1.10=\) Beta \(_{\text {geared }} \times \frac{70}{70+30(1-0.25)}\)
Beta \(_{\text {geared }}=1.45\)
The cost of equity for the project is therefore \(4 \%+1.45(9 \%-4 \%)=11.25 \%\).
\(\mathrm{WACC}=(0.70 \times 11.25 \%)+(0.3 \times 4 \%(1-0.25))\)
\(=7.875 \%+0.9 \%\)
\(=8.775 \%\), say \(8.8 \%\)
\begin{tabular}{|c|c|c|c|c|c|}
\hline Year & 0 & 1 & 2 & 3 & 4 \\
\hline & \$ & \$ & \$ & \$ & \$ \\
\hline Capital & \((200,000)\) & & & & \\
\hline expenditure & & & & & \\
\hline Cash profits & & 100,000 & 165,000 & 120,000 & \\
\hline Tax at 25\% & & & \((25,000)\) & \((41,250)\) & \((30,000)\) \\
\hline Net cash flow & \((200,000)\) & 100,000 & 140,000 & 78,750 & \((30,000)\) \\
\hline DCF factor at 8.8\% & 1.000 & 1/(1.088) & 1/(1.088) & 1/(1.088) & 1/(1.088) \({ }^{4}\) \\
\hline Present value & \((200,000)\) & 91,912 & 118,269 & 61,145 & \((21,409)\) \\
\hline NPV = \$49,917 & & & & & \\
\hline
\end{tabular}
(b) APV method

The ungeared beta for the telecommunications industry is 1.10 (see above)
The cost of ungeared equity in the industry is \(4 \%+1.10(9 \%-4 \%)=9.5 \%\).
The cash flows of he project are discounted at this cost of capital, to obtain the base case NPV.
\begin{tabular}{|c|c|c|c|c|c|}
\hline Year & 0 & 1 & 2 & 3 & 4 \\
\hline & \$ & \$ & \$ & \$ & \$ \\
\hline Capital & \((200,000)\) & & & & \\
\hline expenditure & & & & & \\
\hline Cash profits & & 100,000 & 165,000 & 120,000 & \\
\hline Tax at 25\% & & & \((25,000)\) & \((41,250)\) & \((30,000)\) \\
\hline Net cash flow & \((200,000)\) & 100,000 & 140,000 & 78,750 & \((30,000)\) \\
\hline DCF factor at 8.8\% & 1.000 & 1/(1.095) & 1/(1.095) & 1/(1.095) & 1/(1.095) \({ }^{4}\) \\
\hline Present value & \((200,000)\) & 91,324 & 116,762 & 59,980 & \((20,867)\) \\
\hline \multicolumn{6}{|l|}{Base case NPV = \$47,199} \\
\hline
\end{tabular}

PV of issue costs
\begin{tabular}{lrr} 
Issue costs before tax: & & \(\$\) \\
\hline Equity & \(\$ 1,000,000 \times 4 / 96\) & 41,667 \\
Debt & \(\$ 1,000,000 \times 3 / 97\) & 30,928 \\
\hline Total issue costs & & 72,595 \\
\hline
\end{tabular}

The PV of issue costs is calculated using the risk-free rate of \(4 \%\) as the discount rate.
\begin{tabular}{llrrr} 
Year & Item & Cash flow & \begin{tabular}{r} 
Discount \\
factor at 4\%
\end{tabular} & PV \\
\hline & & \(\$ \$\) & & \(\$\) \\
0 & Issue costs & \((72,595)\) & 1.000 & \((72,595)\) \\
1 & Tax saved at \(25 \%\) & 18,149 & 0.962 & 17,459 \\
\cline { 2 - 3 } & PV of issue costs & & & \((55,136)\) \\
\end{tabular}

\section*{PV of tax shield}

The amount borrowed will be \(\$ 1,000,000+\$ 30,928=\$ 1,030,928\).
The interest rate will be \(4 \%\).
The annual interest cost will be \(\$ 1,030,928 \times 4 \%=\$ 41,237\) each year, years \(1-3\).
The reduction in tax due to the interest payments \(=\$ 10,309(=25 \% \times \$ 41,237)\) each year, years 2-4.

Discount factor at 4\%, years 1-4
3.630

Discount factor at 4\%, year 1
\(\begin{array}{r}0.962 \\ \hline 2.668\end{array}\)
Discount factor at 4\%, years 2-4
\[
2.668=\$ 27,504 .
\]
\(\mathbf{P V}\) of tax shield \(=\$ 10,309 \times 2.668=\$ 27,504\).

Adjusted present value
\begin{tabular}{lr} 
& \(\$\) \\
\hline Base case NPV & 47,199 \\
PV of issue costs & \((55,136)\) \\
PV of tax shield & 27,504 \\
& \(+19,567\) \\
\hline
\end{tabular}

\section*{18 Adjusted present value}

Capital allowances: Workings
\begin{tabular}{lrrr}
\begin{tabular}{l} 
Year of \\
claim
\end{tabular} & \begin{tabular}{r} 
Tax saving \\
at \(35 \%\)
\end{tabular} & \begin{tabular}{r} 
Year of \\
cash flow
\end{tabular} \\
\hline & \begin{tabular}{r} 
\$
\end{tabular} \\
0 & \begin{tabular}{r}
450,000 \\
\((315,000)\)
\end{tabular} & 110,250 & \\
1 & \begin{tabular}{r}
135,000 \\
\((45,000)\)
\end{tabular} & 15,750 & 1 \\
2 & \begin{tabular}{r}
90,000 \\
\((45,000)\) \\
3
\end{tabular} & 15,750 & 2 \\
& \begin{tabular}{r}
45,000 \\
\((45,000)\)
\end{tabular} & 15,750 & 3 \\
& 0 & & 4
\end{tabular}

\section*{(a) Current WACC}

Cost of equity \(=10 \%+1.8(15-10) \%=19 \%\).
\(\mathrm{WACC}=(0.80 \times 19 \%)+[0.20 \times 10 \%(1-0.65)]\)
= \(16.5 \%\)
\begin{tabular}{|c|c|c|c|c|c|}
\hline Year & 0 & 1 & 2 & 3 & 4 \\
\hline & \$000 & \$000 & \$000 & \$000 & \$000 \\
\hline Machine & (450) & & & & \\
\hline Tax saved, tax allowances & & 110.25 & 15.75 & 15.75 & 15.75 \\
\hline Cash profits & & 220.00 & 220.00 & 220.00 & \\
\hline Tax on cash profits (35\%) & & & (77.0) & (77.0) & (77.00) \\
\hline Net cash flow & (450) & 330.25 & 158.75 & 158.75 & (61.25) \\
\hline Discount factor at 16.5\% & 1.000 & 0.858 & 0.737 & 0.632 & 0.543 \\
\hline Present value & (450) & 283.35 & 117.00 & 100.33 & (33.26) \\
\hline NPV = + \$17,420 & & & & & \\
\hline
\end{tabular}
(b) WACC adjusted for business risk and financial risk

\section*{Step 1}

Calculate an ungeared beta for the plastics industry.
\(=1.356 \times \frac{5}{5+1(1-0.35)}\)
\(=1.20\)

The company's gearing is \(60 \%\) equity and \(40 \%\) debt; therefore we need to regear the equity beta for the company.
\(1.20=\) Beta \(_{\text {geared }} \times \frac{60}{60+40(1-0.35)}\)
Betageared \(=1.72\)

The cost of equity for the project is therefore \(10 \%+1.72(15 \%-10 \%)=18.6 \%\).
WACC \(=(0.60 \times 18.6 \%)+(0.40 \times 10 \%(1-0.35))\)
\(=13.76 \%\), say \(14 \%\).
\begin{tabular}{lrrrrr} 
Year & \(\mathbf{0}\) & \(\mathbf{1}\) & \(\mathbf{2}\) & \(\mathbf{3}\) & \(\mathbf{4}\) \\
\hline & \(\$ 000\) & \(\$ 000\) & \(\$ 000\) & \(\$ 000\) & \(\$ 000\) \\
Net cash flows (as in (a)) & \((450.00)\) & 330.25 & 158.75 & 158.75 & \((61.25)\) \\
DCF factor at 14\% & 1.000 & 0.877 & 0.769 & 0.675 & 0.592 \\
Present value & \((450.00)\) & 289.63 & 122.08 & 107.16 & \((36.26)\) \\
NPV = \$32,610 & & & & &
\end{tabular}
(c) APV method

\section*{Step 1}

The ungeared beta for the plastics industry is 1.20 (see above)
The cost of ungeared equity in the industry is \(10 \%+1.20(15 \%-10 \%)=16 \%\).

The cash flows of he project are discounted at this cost of capital, to obtain the base case NPV.
\begin{tabular}{lrrrrr} 
Year & \(\mathbf{0}\) & \(\mathbf{1}\) & \(\mathbf{2}\) & \(\mathbf{3}\) & \(\mathbf{4}\) \\
\hline & \(\$ 000\) & \(\$ 000\) & \(\$ 000\) & \(\$ 000\) & \(\$ 000\) \\
Net cash flow & \((450.00)\) & 330.25 & 158.75 & 158.75 & \((61.25)\) \\
DCF factor at 16\% & 1.000 & 0.862 & 0.743 & 0.641 & 0.552 \\
Present value & & & & & \\
Base case NPV = \$20,580 & \((450.00)\) & 284.68 & 117.95 & 101.76 & \((33.81)\)
\end{tabular}

Step 2: PV of issue costs
\begin{tabular}{|c|c|c|c|}
\hline Issue costs before tax & \begin{tabular}{l}
Net \\
finance obtained
\end{tabular} & & Issue costs \\
\hline & \$ & & \$ \\
\hline Debt: \((40 \% \times 450,000)\) & 180,000 & 2\% & 3,600 \\
\hline Equity: \((60 \% \times 450,000)\) & 270,000 & 5\% & 13,500 \\
\hline Total issue costs & & & 17,100 \\
\hline
\end{tabular}

The PV of issue costs is calculated using the risk-free rate of \(10 \%\) as the discount rate.
\begin{tabular}{llrrr} 
Year & Item & Cash flow & \begin{tabular}{r} 
Discount factor \\
at 10\%
\end{tabular} & PV \\
\hline & & \(\$\) & & \(\$\) \\
0 & Issue costs & \((17,100)\) & 1.000 & \((17,100)\) \\
1 & Tax saved at \(35 \%\) & 5,985 & 0.909 & 5,440 \\
\hline PV of issue costs & & & \((\mathbf{1 1 , 6 6 0 )}\) \\
\hline
\end{tabular}

\section*{Step 3: PV of tax shield}

The amount borrowed will be \(\$ 180,000+\$ 3,600=\$ 183,600\).
The interest rate will be \(10 \%\).
If the loan is repaid in three equal annual instalments, the annual repayments will be:
\[
\frac{\text { Loan }}{\text { PV factor, years } 1-3 \text { at } 10 \%}=\frac{\$ 183,600}{2,487}=\$ 73,824
\]
\begin{tabular}{|c|c|c|c|c|}
\hline Year & Balance at beginning of year & Loan payment & Interest at 10\% & Loan repayment \\
\hline & \$ & \$ & \$ & \$ \\
\hline \multirow[t]{2}{*}{1} & \((183,600)\) & \multirow[t]{2}{*}{73,824} & \multirow[t]{2}{*}{18,360} & \multirow[t]{2}{*}{55,464} \\
\hline & \((55,464)\) & & & \\
\hline 2 & \begin{tabular}{l}
\[
128,136
\] \\
\((61,010)\)
\end{tabular} & 73,824 & 12,814 & 61,010 \\
\hline 3 & \[
\begin{array}{r}
67,126 \\
(67,111) \\
\hline
\end{array}
\] & 73,824 & 6,713 & 67,111 \\
\hline Balance & 15 & unding er & & \\
\hline
\end{tabular}
\begin{tabular}{lrrrrr}
\begin{tabular}{l} 
Year of interest \\
cost
\end{tabular} & \begin{tabular}{r} 
Interest
\end{tabular} & \begin{tabular}{r} 
Year of tax \\
saving
\end{tabular} & \begin{tabular}{r} 
Tax saving \\
at 35\%
\end{tabular} & \begin{tabular}{r} 
DCF factor at \\
\(\mathbf{1 0 \%}\)
\end{tabular} & \begin{tabular}{r} 
PV of tax \\
saving
\end{tabular} \\
\hline & \(\$\) & & \(\$\) & & \begin{tabular}{l}
\(\$\) \\
1
\end{tabular} \\
\hline 18,360 & 2 & 6,426 & 0.826 & 5,308 \\
2 & 1,814 & 3 & 4,485 & 0.751 & 3,368 \\
3 & 6,713 & 4 & 2,350 & 0.683 & 1,605 \\
PV of tax shield & & & & & 10,281 \\
\hline
\end{tabular}

\section*{Adjusted present value}
\begin{tabular}{lr} 
& \(\$\) \\
\hline Base case NPV & 20,580 \\
PV of issue costs & \((11,660)\) \\
PV of tax shield & 10,281 \\
\cline { 2 - 2 } APV & \(+19,201\) \\
\hline
\end{tabular}

\section*{19 More APV}

It is assumed that the company's debt capital will be risk-free.
The asset beta for the industry is \(1.39 \times 80 /[80+20(1-0.25)]=1.17\)

The cost of ungeared equity in the industry is \(6 \%+1.17(10-6) \%=10.68 \%\).
This will be rounded up to \(11 \%\).

Only relevant cash flows should be included in the DCF analysis. Non-relevant costs are the market research cost (already incurred, so a sunk cost) and head office allocated charges (a non-cash cost) - although increases in head office spending are relevant costs.
\begin{tabular}{|c|c|c|c|c|c|c|c|}
\hline Year & 0 & 1 & 2 & 3 & 4 & 5 & 6 \\
\hline & \$000 & \$000 & \$000 & \$000 & \$000 & \$000 & \$000 \\
\hline Revenue & & 6,800 & 7,800 & 8,800 & 9,200 & 9,476 & 9,760 \\
\hline Operating costs & & 5,500 & 6,600 & 7,100 & 7,500 & 7,725 & 7,957 \\
\hline Head office & & 50 & 50 & 50 & 60 & 60 & 60 \\
\hline Royalty payments & 600 & 500 & 400 & 300 & 200 & 200 & 200 \\
\hline Lost contribution & & 100 & 100 & & & & \\
\hline Tax-allowable dep'n & & 600 & 480 & 480 & 480 & 480 & 480 \\
\hline & 600 & 6,750 & 7,630 & 7,930 & 8,240 & 8,465 & 8,697 \\
\hline Taxable profit & (600) & 50 & 270 & 870 & 960 & 1,011 & 1,063 \\
\hline Tax at 25\% & 150 & (13) & (68) & (218) & (240) & (253) & (266) \\
\hline & (450) & 37 & 202 & 652 & 720 & 758 & 797 \\
\hline Add back dep'n & - & 600 & 480 & 480 & 480 & 480 & 480 \\
\hline Equipment & \((3,000)\) & & & & & & \\
\hline Working capital & (400) & & & & & & 400 \\
\hline Net cash flow & \((3,850)\) & 637 & 682 & 1,132 & 1,200 & 1,238 & 1,277 \\
\hline DCF factor \(11 \%\) & 1.000 & 0.901 & 0.812 & 0.731 & 0.659 & 0.593 & 0.535 \\
\hline Present value & \((3,850)\) & 574 & 554 & 827 & 791 & 734 & 897 \\
\hline
\end{tabular}

The base case NPV, discounting the cash flows at the ungeared cost of equity, is (in \(\$ 000)+527\).

\section*{Issue costs}

Issue costs will be \(2 \%\). The net borrowing after issue costs needs to be \(\$ 3,400,000\); therefore the gross amount borrowed will need to be \(\$ 3\) million \(/ 0.98=\$ 3,469,400\). Issue costs will be ( \(2 \%\) ) \(\$ 69,000\). It is assumed that this is a Year 0 cost.

There is no tax relief on issue costs

\section*{Tax shield}

The annual interest cost will be \(\$ 3,469,400 \times 6 \%=\$ 208,164\).
Tax relief each year will be ( \(25 \%\) ) \$52,041
Annuity factor at \(6 \%\) (the risk-free cost of capital), Years \(1-6=4.917\).
Present value of tax shield \(=\$ 255,886\), say \(\$ 256,000\).

Base case NPV
PV of issue costs
PV of tax shield
Adjusted present value
\(+714\)

\section*{20 Cash flows from a foreign project}
\begin{tabular}{lcrrr} 
Year & Expected exchange rate & \begin{tabular}{r} 
Cash flows in \\
francs
\end{tabular} & Cash flows in £ \\
\hline 0 & & 5.00 & \((45,000,000)\) & \((9,000,000)\) \\
1 & \(5.00 \times(1.05 / 1.03)^{1}\) & 5.10 & \(10,000,000\) & \(1,960,784\) \\
2 & \(5.00 \times(1.05 / 1.03)^{2}\) & 5.20 & \(20,000,000\) & \(3,846,154\) \\
3 & \(5.00 \times(1.05 / 1.03)^{3}\) & 5.30 & \(25,000,000\) & \(4,716,981\) \\
4 & \(5.00 \times(1.05 / 1.03)^{4}\) & 5.40 & \(10,000,000\) & \(1,851,852\)
\end{tabular}

Tutorial note: You may have calculated the exchange rate to three or more decimal places. Here, the exchange rate has been estimated to just two decimal places.

These cash flows in sterling should be discounted at the WACC.
\begin{tabular}{lrrr} 
Year & Cash flow & Discount factor & PV \\
\hline & \(£\) & \(9 \%\) & \(£\) \\
0 & \((9,000,000)\) & 1.000 & \((9,000,000)\) \\
1 & \(1,960,784\) & 0.917 & \(1,798,039\) \\
2 & \(3,846,154\) & 0.842 & \(3,238,462\) \\
3 & \(4,716,981\) & 0.772 & \(3,641,509\) \\
4 & \(1,851,852\) & 0.708 & \(1,311,111\) \\
& & & \(+989,121\) \\
\cline { 3 - 4 } & & &
\end{tabular}

The NPV in sterling is positive. The project is financially viable and should be undertaken.

\section*{21 Foreign investment}
(a) Calculate the NPV of the project in the currency of the investment, using a discount rate appropriate to the investment.
The annual tax allowance on the cost of the equipment is \(25 \%\) of \(\$ 1,000,000=\) \(\$ 250,000\) each year for 4 years. This will result in tax savings of \$100,000 (40\% \(\times \$ 250,000)\) each year in years \(2-5\).
\begin{tabular}{lrrrrrrr} 
Year & \(\mathbf{0}\) & \(\mathbf{1}\) & \(\mathbf{2}\) & \(\mathbf{3}\) & \(\mathbf{4}\) & \(\mathbf{5}\) \\
\hline & FR & FR & FR & FR & FR & FR \\
Equipment & \((1,000,000)\) & & 100,000 & 100,000 & 100,000 & 100,000 \\
\begin{tabular}{l} 
Capital \\
allowance
\end{tabular} & & & & & & \\
Cash profit & & 500,000 & 500,000 & 500,000 & 500,000 & \\
\begin{tabular}{l} 
Tax on cash \\
profit
\end{tabular} & & & \((200,000)\) & \((200,000)\) & \((200,000)\) & \((200,000)\) \\
Net cash flow & \((1,000,000)\) & 500,000 & 400,000 & 400,000 & 400,000 & \((100,000)\) \\
DCF factor at & 1.000 & 0.862 & 0.743 & 0.641 & 0.552 & 0.476 \\
\(16 \%\)
\end{tabular}
(b) Dividend payments
\begin{tabular}{lrrrrrr} 
Year & \(\mathbf{1}\) & \(\mathbf{2}\) & \(\mathbf{3}\) & \(\mathbf{4}\) & \(\mathbf{5}\) \\
\hline & FR & FR & FR & FR & FR \\
Cash profit & 500,000 & 500,000 & 500,000 & 500,000 & \\
Tax on profit & \((200,000)\) & \((200,000)\) & \((200,000)\) & \((200,000)\) & \\
\begin{tabular}{l} 
Tax saving from \\
capital allowance \\
Profit after tax
\end{tabular} & 100,000 & & 100,000 & & 100,000 & 100,000 \\
& 400,000 & & 400,000 & & 400,000 & \\
\cline { 2 - 2 } & & & & 400,000 & \\
Dividend (50\%) & 200,000 & 200,000 & 200,000 & 200,000 & 800,000 \\
Retained & 200,000 & 200,000 & 200,000 & 200,000 &
\end{tabular}
(c)
\begin{tabular}{lrrrrr} 
Year & \(\mathbf{1}\) & \(\mathbf{2}\) & \(\mathbf{3}\) & \(\mathbf{4}\) & \(\mathbf{5}\) \\
\hline & FR & FR & FR & FR & FR \\
Dividend in FR & 200,000 & 200,000 & 200,000 & 200,000 & 800,000 \\
& & \(3 \times\) & \(3 \times\) & \(3 \times\) & \(3 \times\) \\
Exchange rate & \(3 \times 1.10 / 1.04)\) & \((1.10 / 1.04)^{2}\) & \((1.10 / 1.04)^{3}\) & \((1.10 / 1.04)^{4}\) & \((1.10 / 1.04)^{5}\) \\
& \((1.1731\) & \(=3.3561\) & \(=3.5498\) & \(=3.7546\) & \(=3.9712\) \\
& & & & & \\
Dividend in \$ & 63,030 & 59,593 & 56,341 & 53,268 & 201,450
\end{tabular}
(d) The cost of buying the equipment in Year \(0=F R 1,000,000 / 3.00=\$ 333,333\)
\begin{tabular}{llrrr} 
Year & Cash flow & \begin{tabular}{r} 
Discount \\
factor at \(\mathbf{1 0 \%}\)
\end{tabular} & PV \\
\hline & & \(\$\) & & \(\$\) \\
0 & Equipment & \((333,333)\) & 1.000 & \((333,333)\) \\
0 & Other costs & \((25,000)\) & 1.000 & \((25,000)\) \\
1 & Dividend & 63,030 & 0.909 & 57,294 \\
2 & Dividend & 59,593 & 0.826 & 49,224 \\
3 & Dividend & 56,341 & 0.751 & 42,312 \\
4 & Dividend & 53,268 & 0.683 & 36,382 \\
5 & Dividend & 201,450 & 0.621 & 125,100 \\
\hline NPV & & & & \(\mathbf{( 4 8 , 0 2 1 )}\) \\
\hline
\end{tabular}

The project is not worthwhile because it has a negative NPV in dollars, even though it has a positive NPV in Francs. This is because:
- the restriction on dividend payments delays returns to the parent company
- the Franc is expected to fall in value against the dollar over the next five years
- there is some additional expenditure in dollars in Year 0, that is not taken into account in the calculation of the NPV in Francs.

\section*{22 Valuation}

The dividend growth model:
\[
\begin{aligned}
& 800=\frac{38(1+\mathrm{g})}{(0.10-\mathrm{g})} \\
& 800(0.10-\mathrm{g})=38(1+\mathrm{g}) \\
& 80-800 \mathrm{~g} \\
& 838 \mathrm{~g} \\
& \mathrm{~g}=0.05 \text { or } 5 \% .
\end{aligned}
\]

An expected dividend growth rate of \(5 \%\) per year is required to achieve a share price of 800 .

\section*{23 Valuation of bonds}
(a) \((7.5 / 9.0) \times 100=83.33\). \((\$ 83.33\) market value for each \(\$ 100\) nominal value of bonds.)
(b)
\begin{tabular}{llrrr} 
Year & Item & Cash flow & \begin{tabular}{r} 
Discount factor \\
at 9\%
\end{tabular} & PV \\
\hline \(1-3\) & Interest & 6 & 2.531 & 15.19 \\
4 & Interest plus capital & 106 & 0.708 & 75.05 \\
& & & & 90.24 \\
& &
\end{tabular}

The market value of the bonds should be 90.24 .
(c)
\begin{tabular}{llrrr} 
Period & Item & Cash flow & \begin{tabular}{r} 
Discount factor \\
at 4.4\%
\end{tabular} & PV \\
\hline \(1-7\) & Interest & 5 & 5.914 & 29.57 \\
8 & Interest plus capital & 105 & \(1 /(1.044)^{8}\) & 74.40 \\
& & & & 103.97 \\
& & & &
\end{tabular}

The market value of the bonds should be 103.97.
(d)
\begin{tabular}{llrrr} 
Year & Item & Cash flow & \multicolumn{3}{c}{ Discount factor } \\
\hline \(1-3\) & Interest & 5 & at 9\% & PV \\
\cline { 2 - 5 } & Value of shares acquired & 140 & 2.531 & 12.66 \\
& & & 0.708 & 99.12 \\
& & & & \\
\hline
\end{tabular}

The market value of the bonds should be 111.78.

\section*{24 Annuities and bond prices}

\section*{Tutorial note}

You might be required in the examination to remember and use the formula for the present value of an annuity. This is:

PVofannuity \(=\) Annuity \(\times \frac{1}{r}\left[1-\frac{1}{(1+r)^{n}}\right]\)
(a) (i) Valueofzerocouponbond \(=100 \times \frac{1}{(1.05)^{10}}\)
\[
=100 \times 0.6139
\]
\[
=61.39 \text {. }
\]
(ii) PV of interest payments to maturity of the bond: interest \(=4\) every six months for ten years.
PVofannuity \(=4 \times \frac{1}{0.025}\left[1-\frac{1}{(1.025)^{20}}\right]\)
\(=160 \times[0.3897]\)
\(=62.35\)
\begin{tabular}{llrrr} 
Period & Cash flow & \begin{tabular}{r} 
Discount factor \\
at 2.5\%
\end{tabular} & PV \\
\hline \(\mathbf{1 - 2 0}\) & Interest & 4 & See above & 62.35 \\
20 & Redemption & 100 & \(1 /(1.025)^{20}\) & 61.03 \\
Value of bond & & & 123.38 \\
\end{tabular}
(b) When interest yields rise, bond prices fall.
(i) Valueofzerocouponbond \(=100 \times \frac{1}{(1.06)^{10}}\)
\[
\begin{aligned}
& =100 \times 0.5584 \\
& =55.84 .
\end{aligned}
\]
(ii) PVofannuity \(=4 \times \frac{1}{0.03}\left[1-\frac{1}{(1.03)^{20}}\right]\)
\(=133.33 \times[0.4463]\)
= 59.51
\begin{tabular}{llrrr} 
Period & Cash flow & \begin{tabular}{r} 
Discount factor \\
at 3\%
\end{tabular} & PV \\
\hline \(1-20\) & Interest & 4 & See above & 59.51 \\
20 & Redemption & 100 & \(1 /(1.03)^{20}\) & 55.37 \\
\cline { 3 - 5 } & & & & 114.88 \\
\cline { 4 - 4 }
\end{tabular}

\section*{25 EVA}

The adjusted capital investment is \(\$ 8,000,000\).
This is the equity investment. Development expenditure should be included in the total investment, and goodwill should be included at its current value (= cost less cumulative impairment). No adjustments to the amount of investment are therefore required for these items.

Net operating profit after tax \(=\$ 2,100,000-\$ 300,000-\$ 450,000=\$ 1,350,000\).
\(\mathrm{EVA}=\$ 1,350,000-(12 \% \times \$ 8,000,000)\)
\(=\$ 390,000\).

\section*{26 MM, gearing and company valuation}

Value of geared company \(=\) Value of company ungeared \(+(\) Value of debt \(\times\) Tax rate \()\)
\(\mathrm{V}_{\mathrm{g}}=\mathrm{V}_{\mathrm{u}}+\mathrm{Dt}\)
\(\mathrm{V}_{\mathrm{g}}=(4,000,000 \times \$ 10)+(\$ 15,000,000 \times 30 \%)\)
\(=\$ 44,500,000\)
\begin{tabular}{lr} 
& \$ million \\
\hline Total value of geared company (equity + debt) & 44.5 \\
Value of debt & \((15.0)\) \\
\cline { 2 - 2 } Therefore value of equity in geared company & 29.5 \\
\cline { 2 - 2 }
\end{tabular}

The total value of the equity in the geared company is lower than when the company was geared, but there are fewer shares left in issue and the value per share will be higher.

\section*{27 Acquisition}
(a) The earnings of Little next year are expected to be \(\$ 86,000\). A forward P/E multiple of 8.0 could be applied to this estimate, and the valuation of the equity shares in Little would be:
\(\$ 86,000 \times 8.0=\$ 688,000\).
(b) The cost of equity of Big is expected to be:
\(6 \%+1.60(11-6) \%=14 \%\).
The WACC of Big is expected to be:
\([35 \% \times 7.4(1-0.30)]+(65 \% \times 14)\)
\(=10.913 \%\).
(c) Since Little is in the same industry as Big, it is probably appropriate to use the WACC of Big to obtain a DCF-based valuation of Little. The WACC of \(10.913 \%\) will be rounded to \(11 \%\).

The cash flows from the acquisition of Little must be calculated.
\begin{tabular}{|c|c|c|c|}
\hline & Year 1 & Year 2 & Year 3 \\
\hline & \$ & \$ & \$ \\
\hline Sales & 200,000 & 280,000 & 320,000 \\
\hline Cash costs & \((120,000)\) & \((160,000)\) & \((180,000)\) \\
\hline & 80,000 & 120,000 & 140,000 \\
\hline Capital allowances & \((20,000)\) & \((30,000)\) & \((40,000)\) \\
\hline Interest & \((10,000)\) & \((10,000)\) & \((10,000)\) \\
\hline Taxable profit & 50,000 & 80,000 & 90,000 \\
\hline Tax at 30\% & \((15,000)\) & \((24,000)\) & \((27,000)\) \\
\hline Profit after tax & 35,000 & 56,000 & 63,000 \\
\hline Profit after tax & 35,000 & 56,000 & 63,000 \\
\hline Add back capital allowances & 20,000 & 30,000 & 40,000 \\
\hline & 55,000 & 86,000 & 103,000 \\
\hline Asset replacement & \((25,000)\) & \((30,000)\) & \((35,000)\) \\
\hline Cash flow & 30,000 & 56,000 & 68,000 \\
\hline
\end{tabular}

Cash flows will increase by \(4 \%\) each year from Year 4 onwards.
The dividend growth valuation model can be used to calculate the Year 3 value of these cash flows, using a growth rate of \(4 \%\) and a cost of capital of \(11 \%\) :
\[
\begin{aligned}
& \text { Year } 3 \text { value of cash flows from Year } 4=\frac{\$ 68,000(1.04)}{(0.11-0.04)} \\
& =\$ 1,010,286
\end{aligned}
\]

The expected cash flows can now be converted in to a present value:
\begin{tabular}{lrrr} 
Year & Cash flow & \begin{tabular}{r} 
Discount \\
factor at 11\%
\end{tabular} & PV \\
\hline & \(\$\) & & \(\$\) \\
1 & 30,000 & 0.901 & 27,030 \\
2 & 56,000 & 0.812 & 45,472 \\
3 & 68,000 & 0.731 & 49,708 \\
4 onwards & \(1,010,286\) & 0.731 & 738,519 \\
Total value & & & 860,729 \\
& & &
\end{tabular}

\section*{28 Takeover}

Cost of equity in Flat Company, using the CAPM \(=5 \%+1.20(11-5) \%=12.2 \%\)
WACC in Flat Company \(=(12.2 \times 75 \%)+(7(1-0.30) \times 25 \%)=10.375 \%\), say \(10.4 \%\)
Cost of equity in Slope Company, using the CAPM \(=5 \%+1.35(11-5) \%=13.1 \%\) WACC in Slope Company \(=(13.1 \times 60 \%)+(8(1-0.30) \times 40 \%)=10.1 \%\).

Free cash flow is defined here as EBIT less tax, plus tax-allowable depreciation minus replacement capital expenditure.

Free cash flows and valuation of Flat Company based on free cash flows
\begin{tabular}{|c|c|c|c|c|}
\hline Year & 1 & 2 & 3 & 4 \\
\hline & \$000 & \$000 & \$000 & \$000 \\
\hline Earnings before interest and tax & 1,918 & 2,014 & 2,115 & 2,221 \\
\hline Tax at 30\% & (575) & (604) & (635) & (666) \\
\hline & 1,343 & 1,410 & 1,480 & 1,555 \\
\hline Add back tax-allowable depreciation & 872 & 915 & 961 & 1,009 \\
\hline Less: Replacement capital spending & (966) & \((1,014)\) & \((1,065)\) & \((1,118)\) \\
\hline Free cash flow & 1,249 & 1,311 & 1,376 & 1,446 \\
\hline Discount factor at 10.4\% & 0.906 & 0.820 & 0.743 & 0.673 \\
\hline Present value & 1,132 & 1,075 & 1,022 & 973 \\
\hline \multicolumn{3}{|l|}{\multirow[t]{2}{*}{End-of-year 4 value of free cash flows from Year 5 onwards}} & = & 6 (1.03) \\
\hline & & & \multicolumn{2}{|r|}{(0.104-0.03)} \\
\hline \multicolumn{5}{|l|}{\(=(\) in \$000) 20,322} \\
\hline
\end{tabular}

Present value of Year 5 onward cash flows \((\) Year 0 value \()=20,322 \times 0.673=13,677\).

Total valuation of Flat Company equity, using the free cash flow method, is \((1,132+\) \(1,075+1,022+973+13,677)=\$ 17,879,000\).

Free cash flows and valuation of Slope Company based on free cash flows
\begin{tabular}{|c|c|c|c|c|}
\hline Year & 1 & 2 & 3 & 4 \\
\hline & \$000 & \$000 & \$000 & \$000 \\
\hline Earnings before interest and tax & 1,893 & 1,969 & 2,047 & 2,129 \\
\hline Tax at 30\% & (568) & (591) & (614) & (639) \\
\hline & 1,325 & 1,378 & 1,433 & 1,490 \\
\hline Add back tax-allowable depreciation & 728 & 757 & 787 & 819 \\
\hline Less: Replacement capital spending & (822) & (854) & (889) & (924) \\
\hline Free cash flow & 1,231 & 1,281 & 1,331 & 1,385 \\
\hline Discount factor at 10.1\% & 0.908 & 0.825 & 0.749 & 0.681 \\
\hline Present value & 1,118 & 1,057 & 997 & 943 \\
\hline \multicolumn{3}{|l|}{\multirow[t]{2}{*}{End-of-year 4 value of free cash flows from Year 5 onwards}} & \multicolumn{2}{|r|}{1,385 (1.02)} \\
\hline & & & & .02) \\
\hline \multicolumn{5}{|l|}{\(=(\) in \$000) 17,441} \\
\hline
\end{tabular}

Present value of Year 5 onward cash flows \((\) Year 0 value \()=17,441 \times 0.681=11,877\).

Total valuation of Slope Company equity, using the free cash flow method, is \((1,118\) \(+1,057+997+943+11,877)=\$ 15,992,000\).

\section*{Combined group WACC}
\begin{tabular}{llrcc} 
& & \begin{tabular}{c} 
Market \\
value \\
\(\$ m\)
\end{tabular} & \begin{tabular}{c} 
Cost of \\
capital
\end{tabular} & MV \(\times\) Cost \\
Flat equity & \((6 m \times 3.20)\) & 19.20 & 0.104 & 1.9968 \\
Flat debt & \((19.2 \mathrm{~m} / 0.75) \times\) & 6.40 & 0.049 & 0.3136 \\
Slope equity & \(25 \%\) & & 13.86 & 0.101 \\
Slope debt & \((9 m \times 1.54)\) & 0.24 & 0.056 & 1.3999 \\
& \((13.86 \mathrm{~m} / 0.60) \times\) & 0.5174 \\
& \(40 \%\) & \(\boxed{48.70}\) & & -4.2277 \\
\hline
\end{tabular}
\(\mathrm{WACC}=4.2277 / 48.70=0.068\) or \(8.68 \%\), say \(8.7 \%\).
Free cash flows and valuation of combined company based on free cash flows

Tax-allowable depreciation in the year just ended (combined) was 1,530 and replacement capital expenditure combined was 1,710.
\begin{tabular}{|c|c|c|c|c|}
\hline Year & 1 & 2 & 3 & 4 \\
\hline & \$000 & \$000 & \$000 & \$000 \\
\hline Earnings before interest and tax & 4,100 & 4,305 & 4,520 & 4,746 \\
\hline Tax at 30\% & \((1,230)\) & \((1,292)\) & \((1,356)\) & \((1,424)\) \\
\hline & 2,870 & 3,013 & 3,164 & 3,322 \\
\hline Add back tax-allowable depreciation & 1,607 & 1,687 & 1,771 & 1,860 \\
\hline Less: Replacement capital spending & \((1,860)\) & \((1,885)\) & \((1,980)\) & \((2,079)\) \\
\hline Free cash flow & 2,617 & 2,815 & 2,955 & 3,103 \\
\hline Discount factor at 8.7\% & 0.920 & 0.846 & 0.779 & 0.716 \\
\hline Present value & 2,408 & 2,381 & 2,302 & 2,222 \\
\hline \multicolumn{2}{|l|}{\multirow[t]{2}{*}{End-of-year 4 value of free cash flows from Year 5 onwards}} & \multirow[t]{2}{*}{\(=\)} & \multicolumn{2}{|l|}{3,103 (1.04)} \\
\hline & & & \multicolumn{2}{|l|}{\multirow[t]{2}{*}{(0.087-0.04)}} \\
\hline \multicolumn{3}{|l|}{\(=(\) in \$000) 68,662} & & \\
\hline
\end{tabular}

Present value of Year 5 onward cash flows \((\) Year 0 value \()=68,662 \times 0.716=49,162\).

Total valuation of equity in the combined company, using the free cash flow method, is \((2,408+2,381+2,302+2,222+49,162)=\$ 58.475\) million
\begin{tabular}{lc} 
Summary of free cash flow valuations & \(\$ \mathrm{~m}\) \\
Value of Flat Company equity & 17.879 \\
Value of Slope Company equity & 15.992 \\
& 33.871 \\
Value of equity in combined company & 58.475 \\
\hline Increase in equity value & 24.604 \\
\hline
\end{tabular}

On the basis of these estimates, the value of equity (as valued on a free cash flow basis) will increase by about \(72.6 \%\) as a result of the takeover.
(b)

The estimates of equity value might not be reliable, for several reasons.
(1) The WACC used for the combined company, based on current market values, is lower than the WACC used for each separate company valuation. This lower WACC is questionable, and if a WACC of over \(10 \%\) were used, the valuation of the company after the takeover would be much lower.
(2) The estimates for the increase in the combined Year 1 EBIT might be unrealistic, and the estimates of higher growth in sales and earnings should also be questioned.
(3) Valuations based on a dividend growth model, rather than a fre cash flow model, would produce a lower valuation.
(c)

Shareholders in Slope are being offered 2 shares in Flat (current value \$6.40) for every three shares they hold (current value \$4.62). On the basis of current market values, they are being offered a price that is \(38.5 \%\) above the current share price.

This is a very high premium in a takeover bid, and is likely to be very attractive to them.
For the same reason, shareholders in Flat might oppose the takeover bid, because 'value' is being given to the shareholders of Slope and a very high premium is being offered for the shares. The shareholders in Flat will only support the bid if they believe that it will 'unlock value' in the shares or result in substantial synergy gains through higher sales, cost savings or faster business growth.

\section*{29 Interest rate parity}
(a) \(\quad \mathrm{GBP} / \mathrm{USD}=1.8000 \times(1.035 / 1.05)=1.7743\)
(Note: the interest rate is lower for the dollar than for sterling, therefore the dollar should increase in value over time against sterling.)
\(\mathrm{GBP} / \mathrm{EUR}=1.5000 \times(1.025 / 1.05)=1.4643\)
\(\mathrm{EUR} / \mathrm{USD}=1.2000 \times(1.035 / 1.025)=1.2117\)
(b) \(\quad \mathrm{GBP} / \mathrm{USD}=1.8000 \times(1.035 / 1.05)^{3}=1.7240\)
\(\mathrm{GBP} / \mathrm{EUR}=1.5000 \times(1.025 / 1.05)^{3}=1.3954\)
\(\mathrm{EUR} / \mathrm{USD}=1.2000 \times(1.035 / 1.025)^{3}=1.2355\)

\section*{30 Foreign exchange}
(a) A hedge against the risk can be obtained by entering into a forward rate agreement to buy \(\$ 750,000\). The forward rate is the forward rate that favours the bank. This is 1.8535 (and not 1.8543).
The cost of buying the dollars will be \(\$ 750,000 / 1.8535=£ 404,639.87\).
(b) Subtract a premium, add a discount.
\begin{tabular}{lr} 
Spot rate & 1.3025 \\
Premium & \((0.0018)\) \\
Forward rate & 1.3007 \\
\end{tabular}

The \(\$ 450,000\) will be sold in exchange for \(€ 345,967.56\) (450,000/1.3007).

\section*{31 Money market hedge}
(a) The company will receive \(\$ 600,000\) in six months, and will want to receive sterling and pay dollars.
It can do this with a money market hedge by borrowing US dollars now. The interest rate for six months in dollars is \(3.5 \% \times 6 / 12=1.75 \%\). It will need to borrow now:
\(\$ 600,000 / 1.0175=\$ 589,680.59\).
It can immediately exchange these dollars into sterling at the spot rate of 1.8800, to obtain:
\(\$ 589,680.59 / 1.8800=£ 313,659.89\).

After six months, the dollar loan will be repayable with interest. The total repayment will be \(\$ 600,000\), and the payment can be made from the \(\$ 600,000\) received from the customer.
(b) The company can do anything with the sterling it receives now from the hedging transaction. If it chose to invest the cash for six months at \(5 \%\) per year ( \(2.5 \%\) for six months), the investment of \(£ 313,659.89\) would increase to:
\(£ 313,659.89 \times 1.025=£ 321,501.39\).
To avoid opportunities for arbitrage between the money markets and the forward FX markets, the six-month forward exchange rate would therefore need to be:
\(\$ 600,000 / £ 321,501.39=1.8662\).

\section*{32 \\ FRA}
(a) The company wants to borrow in three months' time for a period of six months; therefore to create a hedge with an FRA, it must buy a 3v9 FRA.
The interest rate for the FRA is \(3.97 \%\).
- The company will borrow in three months' time at the current LIBOR rate plus 0.50\%.
- The FRA will be settled in three months' time.
- If the six-month LIBOR rate is higher than \(3.97 \%\), the company will receive a payment from the bank to settle the FRA. The amount of this payment is the value of the difference between the FRA rate of \(3.97 \%\) and the LIBOR rate.
- If the six-month LIBOR rate is lower than \(3.97 \%\), the company will make a payment to the bank to settle the FRA, for the value of the difference between the two rates.

The effect of the FRA is therefore to 'lock in an effective interest rate of \(3.97 \%\) \(+0.50 \%=4.47 \%\).
(Tutorial note: For example, if the LIBOR rate in three months is \(5.5 \%\), the situation will be as follows:
\begin{tabular}{lr} 
& \(\%\) \\
\hline Company borrows at LIBOR \(+0.50 \%\) & 6.00 \\
Company receives from settlement of FRA \((5.50-3.97)\) & \((1.53)\) \\
\cline { 2 - 2 } & Effective interest cost
\end{tabular}

This is the FRA rate \(+0.50 \%\) ).
(b) An FRA works on the same principles as an interest rate coupon swap. The main difference is that an FRA is for one interest period only, although a company can arrange a series of FRAs. A coupon swap is longer-term, and covers several settlement dates.

\section*{33 Swap}

The company should enter into a four-year interest rate coupon swap in which it receives the floating rate and pays the fixed rate ( \(5.25 \%\) ).

The effective interest rate will change from floating rate to fixed rate, as follows:
\begin{tabular}{lr} 
& \(\%\) \\
\hline Bank loan interest & \((\) LIBOR +1.25\()\) \\
Swap & \((5.25)\) \\
Pay & LIBOR \\
Receive & \((6.50)\) \\
Effective rate
\end{tabular}

\section*{34 Credit arbitrage (1)}
\begin{tabular}{|c|c|c|c|}
\hline \multicolumn{3}{|c|}{Company X} & Company Y \\
\hline & \% & & \% \\
\hline Borrow: & & Borrow: & \\
\hline Fixed rate & (7.25) & Floating rate & \((\) LIBOR + 1.50) \\
\hline Swap & & Swap & \\
\hline Receive fixed & 6.27 & Pay fixed & (6.30) \\
\hline Pay floating & (LIBOR) & Receive floating & LIBOR \\
\hline Net cost & (LIBOR + 0.98) & Net cost & (7.80) \\
\hline Cost of variable rate borrowing & (LIBOR + 1.25) & Cost of fixed rate borrowing & (8.00) \\
\hline Saving in cost & 0.27 & Saving in cost & 0.20 \\
\hline
\end{tabular}

\section*{35 Credit arbitrage (2)}
\begin{tabular}{lr} 
& \(\%\) \\
\hline Entity A can borrow more cheaply at a fixed rate by \((7.25-6.35)\) & 0.90 \\
Entity A can borrow more cheaply at a floating rate by \((1.25-0.75)\) & 0.50 \\
Difference & 0.40 \\
Bank's profit & \(\underline{0.10}\) \\
Net benefit to share between the two entities & \(\mathbf{0 . 3 0}\)
\end{tabular}

If the entities share the benefit equally, each will be able to reduce its effective cost of borrowing by ( \(0.30 / 2\) ) \(0.15 \%\).
- Entity A wants to borrow at a floating rate. It can borrow directly at LIBOR + \(0.75 \%\). By borrowing at a fixed rate and swapping into a floating rate, its effective interest rate will be LIBOR \(+0.75 \%-0.15 \%=\) LIBOR \(+0.60 \%\).
- Entity B wants to borrow at a fixed rate. It can borrow directly at \(7.25 \%\). By borrowing at a floating rate and swapping into a fixed rate, its effective interest rate will be \(7.25 \%-0.15 \%=7.10 \%\).

For Entity A, the arrangement could be as follows:
\begin{tabular}{lr} 
& \(\%\) \\
\hline Borrow at a fixed rate & \((6.35)\) \\
Swap & \\
Pay & (LIBOR) \\
Receive (balancing figure) & 5.75 \\
\cline { 2 - 2 } Effective interest cost & \((\) LIBOR +0.60\()\) \\
\hline
\end{tabular}

For Entity A, the arrangement would be as follows:
\begin{tabular}{lr} 
& \(\%\) \\
\hline Borrow at a fixed rate & (LIBOR +1.25\()\) \\
Swap & \\
Pay (balancing figure) & \((5.85)\) \\
Receive & LIBOR \\
Effective interest cost & \((7.10)\) \\
\hline
\end{tabular}

The bank's profit would come from the difference between the fixed rate received from Entity B ( \(5.85 \%\) ) and the fixed rate paid to Entity A (5.75\%).

This assumes that the two Entities each arrange their swap with the bank, and not directly with each other.

\section*{36 Currency swap}

\section*{(a)}

Small company will want to borrow 3 million zants, but can borrow in sterling at a rate that is \(2 \%\) lower than the rate that the Zantland counterparty can obtain. The Zantland counterparty presumably wants to borrow in sterling (the equivalent of 3 million zants), but can can borrow in zants at a rate that is \(0.5 \%\) lower than the rate that Small Company can obtain.

This provides an opportunity for credit arbitrage of \(2 \%+0.5 \%=2.5 \%\).
The bank would take \(0.5 \%\) in fees, leaving \(2 \%\) of net credit arbitrage for the swap counterparties to share. Small Company would have three-quarters of this amount, which is \(1.5 \%\).

The swap arrangement might therefore be as follows:


Small company would pay \(1.5 \%\) less than by borrowing direct (at ZIBOR \(+2 \%\) ) and the Zantland counterparty would borrow at \(0.5 \%\) less than by borrowing sterling direct at \(8.5 \%\).
(b)

It is assumed that \(15 \%\) is the appropriate discount rate for evaluating the project's cash flows in sterling. (A DCF rate of \(15 \%\) would be very low for evaluating the cash flows in zants, considering the expected high rate of inflation in Zantland.)

It is also assumed that the swap will be undertaken, and in Year 0 Small Company will spend \(£ 333,333\) ( 3 million zants at the spot rate of 9.00 ). At the end of Year 3 , it is assumed that Small Company will receive the same amount \((£ 333,333)\) on the
termination of the currency swap, and a further 3,000,000 zants for the remainder of the sale price of the operations centre.

The project cash flows will therefore be as follows:
\(0 \quad £(333,333)\)
1 200,000 zants at the end of Year 1 spot rate
2 200,000 zants at the end of Year 2 spot rate
3 200,000 zants at the end of Year 3 spot rate
3 3,000,000 zants at the end of Year 3 spot rate
\(33,000,000\) zants at the swap rate of 9.00 , therefore \(£ 333,333\).
Year Spot rate
\begin{tabular}{cc} 
Best case & Worst case \\
\(10 \%\) inflation & \(50 \%\) inflation \\
9.00 & 9.00 \\
9.90 & 13.50 \\
10.89 & 20.25 \\
11.98 & 30.38
\end{tabular}
\begin{tabular}{|c|c|c|c|c|c|c|}
\hline \multirow[t]{3}{*}{Year} & \multirow[t]{2}{*}{Cash flow} & \multirow[t]{3}{*}{DCF factor at \(15 \%\)} & \multicolumn{2}{|c|}{Best case} & \multicolumn{2}{|c|}{Worst case} \\
\hline & & & Cash flow & PV & Cash flow & PV \\
\hline & zants & & £ & £ & £ & £ \\
\hline 0 & & 1.000 & \((333,333)\) & \((333,333)\) & \((333,333)\) & \((333,333)\) \\
\hline 1 & 200,000 & 0.870 & 20,202 & 17,576 & 14,815 & 12,889 \\
\hline 2 & 200,000 & 0.756 & 18,365 & 13,884 & 9,877 & 7,467 \\
\hline 3 & 3,200,000 & 0.658 & 267,112 & 175,760 & 105,332 & 69,308 \\
\hline 3 & 3,000,000 & 0.658 & 333,333 & 219,333 & 333,333 & 219,333 \\
\hline NPV & & & & + 93,220 & & \((24,336)\) \\
\hline
\end{tabular}

\section*{Conclusion}

On the basis of the assumptions used, the project would have a positive NPV if inflation in Zantland exceeds inflation in the UK by \(10 \%\) per year, but will have a negative NPV if inflation in Zantland exceeds inflation in the UK by \(50 \%\) per year.

There is consequently an element of risk in the project due to uncertainty about the spot exchange rate, and this risk element should be assessed more closely before a decision is taken about the investment.

\section*{\(37 \quad\) Currency futures}
(a) The company must make a payment in US dollars in May. It must therefore buy dollars to make the payment.
Using futures, the company will therefore buy dollars and sell euros. It will therefore sell euro/US dollar futures, which are for \(€ 125,000\) each.
At the futures price of 1.2800, the amount of euros to sell in exchange for \(\$ 640,000\) is:
\(\$ 640,000 / 1.2800=\$ 500,000\).
The number of contracts to sell is therefore: \(\$ 500,000 / \$ 125,000\) per contract \(=\) 4.0 contracts.

The company will sell 4 June contracts at 1.2800 .
(b) It will close its position in May, when the futures price is 1.2690 .

The value of 1 tick for this contract is \(125,000 \times \$ 0.0001=\$ 12.50\).
\begin{tabular}{lr} 
Original selling price & 1.2800 \\
Buying price to close the position & 1.2690 \\
\hline Gain per contract & 0.0110 \\
\hline
\end{tabular}

Total gain on futures position \(=4\) contracts \(\times 110\) ticks \(\times \$ 12.50=\$ 5,500\).
The French company must pay \(\$ 640,000\) to its supplier. It has \(\$ 5,500\) profit from closing the futures position. It therefore needs an additional (\$640,000 \(\$ 5,500)=\$ 634,500\).
It must buy these dollars at the spot rate of 1.2710 . The cost in euros will be \(\$ 634,500 / 1.2710=€ 499,213\).

The effective exchange rate for the payment of \(\$ 640,000\) is therefore:
\(\$ 640,000 / € 499,213=\) US\$1.2820/€1.
This is close to the price at which the futures were originally sold. However, the hedge is not perfect because the position was closed before the settlement date for the contract.

\section*{38 More currency futures}
(a) The US company must make a payment in sterling in January. It will sell the sterling it receives in exchange for dollars.

Using futures, the company will therefore sell sterling and buy dollars. It will therefore sell sterling/US dollar futures, which are for \(£ 62,500\) each.

The number of contracts to sell is therefore: \(£ 400,000 / £ 62,500\) per contract \(=\) 6.4 contracts.

The company will therefore sell either 6 or 7 March contracts at 1.8600.
In the answer in (b), it is assumed that the company will sell 6 March sterling/US dollar futures.
(b) The US company will close its position in January, when the futures price is 1.8420.

The value of 1 tick for this contract is \(62,500 \times \$ 0.0001=\$ 6.25\).
\begin{tabular}{lr} 
Original selling price & 1.8600 \\
Buying price to close the position & 1.8420 \\
Ger contract & 0.0180 \\
\hline
\end{tabular}

Total gain on futures position \(=6\) contracts \(\times 180\) ticks \(\times \$ 6.25=\$ 6,750\).
The US company will receive \(£ 400,000\) which it will sell at the spot rate of 1.8450 .
\begin{tabular}{lr} 
& \(\$\) \\
\hline From sale of \(£ 400,000\) spot at \(\$ 1.8450 / £ 1\) & 738,000 \\
Profit on futures position & 6,750 \\
\cline { 2 - 2 } Total income & 744,750 \\
\hline
\end{tabular}

The effective exchange rate for the \(£ 400,000\) received is therefore:
\(\$ 744,750 / £ 400,000=\) US \(\$ 1.8619 / £ 1\).
This is close to the price at which the futures were originally sold. However, the hedge is not perfect because the position was closed before the settlement date for the contract.

\section*{39 \\ Basis}
(a) On 1st March: Days to settlement of the June futures contracts \(=31+30+31+\) \(60=122\) days .
\begin{tabular}{lr} 
On 1 March & \\
\hline Spot rate & 1.8540 \\
Futures price & 1.8760 \\
\cline { 2 - 2 } Basis & 0.0220 \\
\cline { 2 - 2 }
\end{tabular}

The basis is 220 points, with the futures rate higher than the spot rate.
The basis at the end of June when the futures reach settlement will be 0 .
It is assumed that basis will decrease to zero at a constant rate per day. The basis will therefore reduce by ( 220 points \(/ 122\) days) \(=1.80328\) points per day.
At close of trading on 30th April, there are \((31+30) 61\) days remaining to the settlement of the June futures. The expected basis at this date is therefore:
1.80328 points per day \(\times 61\) days \(=110\) points.

At the end of 30th April
\begin{tabular}{ll} 
Spot rate & 1.8610 \\
Expected basis & 0.0110 \\
\cline { 2 - 3 } Expected futures price (higher) & 1.8720 \\
\hline
\end{tabular}
(b) At close of trading on 15th June, there are 15 days remaining to the settlement of the June futures. The expected basis at this date is therefore:
1.80328 points per day \(\times 15\) days \(=27\) points.

At the end of 30th April
\begin{tabular}{lr} 
Spot rate & 1.8690 \\
Expected basis & 0.0027 \\
\cline { 2 - 2 } Expected futures price (higher) & 1.8717 \\
\hline
\end{tabular}

\section*{40 Imperfect hedge and basis}
(a) There is a loss on the underlying currency exposure, because sterling weakens in value between 20th April and 20th July.
\begin{tabular}{lr}
\hline At 20th April: expected value of \(£ 625,000\) receivable (at 1.8050) & \(1,128,125\) \\
At 20th July: actual value of \(£ 625,000\) received (at 1.7700 ) & \(1,106,250\) \\
\hline Loss on underlying currency exposure & 21,875 \\
\hline
\end{tabular}

The futures position is opened on 20th April by selling futures contracts (selling British pounds and buying dollars). The US company should sell 10 contracts ( \(£ 625,000 / £ 62,500\) per contract). When the position is closed on 20th July, there is a gain on the position.
\begin{tabular}{lr} 
& \(\$\) \\
\hline 20th April: Open position - Sell at & 1.7800 \\
20th July: Close position - Buy at & 1.7600 \\
\cline { 2 - 2 } Gain on underlying currency exposure & 0.0200 \\
\cline { 2 - 2 }
\end{tabular}

Total gain (10 contracts) \(=10\) contracts \(\times 200\) ticks per contract \(\times £ 6.25\) per tick \(=\$ 12,500\).

The futures position has failed to provide a perfect hedge, resulting in a net 'loss' of \$9,375.
\begin{tabular}{lr} 
Effective exchange rate & \(\$\) \\
\hline Revenue from sale of \(£ 625,000\) spot on 20th July (at 1.7600) & \(1,100,000\) \\
Gain on futures position & 12,500 \\
\hline Total dollar income & \(1,112,500\) \\
\hline
\end{tabular}

Effective exchange rate \(=\$ 1,112,500 / £ 625,000=\$ 1.7800\).
(b) The reason why the hedge is not perfect in this case is explained by the existence of basis. When the futures position was opened, the basis was 250 points (1.8050-1.7800). When the position was closed, the basis was 100 points ( \(1.7700-1.7600\) ). The spot price has moved in value during the three months by more than the movement in the futures price, by 150 points. The value of this difference is \(\$ 9,375\) ( 10 contracts \(\times 150\) ticks per contract \(\times £ 6.25\) per tick).

\section*{41 Currency hedge}
(a) Hedging with a forward exchange contract

Only the net exposure should be hedged. This is a net payment of \(€(2,650,000\) \(-540,000)=€ 2,110,000\).

The entity will need to buy euros in three months' time. The three-month forward rate for the contract would be 1.4443 (the rate more favourable to the bank).

Cost in sterling \(=€ 2,110,000 / 1.4443=£ 1,460,915\).
(b) Money market hedge

The company must pay \(€ 2,110,000\) in three months' time. To create a money market hedge, it must therefore buy euros spot and invest them for three months at \(3.4 \%\) per year. The amount of euros invested, plus accumulated interest, must be worth \(€ 2,110,000\) after three months.

It is assumed that the three-month investment rate for euros is \(3.4 \% \times 3 / 12=\) \(0.85 \%\).

The amount of euros to invest now is therefore \(€ 2,110,000 / 1.0085=€ 2,092,216\). These must be purchased spot at 1.4537 , and the cost in sterling will be:
\(€ 2,092,216 / 1.4537=£ 1,439,235\).

With a forward FX contract, the payment of \(£ 1,460,915\) will be made in three months' time. With a money market hedge, the payment of \(£ 1,439,235\) would happen immediately. It can therefore be argued that an additional cost of a money market hedge is the loss of interest (opportunity cost) from investing \(£ 1,439,235\) for three months at \(5.6 \%\) per year. The lost interest would be \(£ 1,439,235 \times 5.6 \% \times 3 / 12=£ 20,149\).

The overall cost of a money market hedge would therefore be \(£ 1,439,235+\) \(£ 20,149=£ 1,459,384\).
(c) Currency futures hedge

The company must pay euros. It needs to buy euros to make the payments. The futures are denominated in euros; therefore the company will buy futures.
The number of contracts required \(=€ 2,110,000 / € 100,000\) per contract \(=21.1\) contracts. The company should probably buy 21 contracts.
The payments are due in October. The company should therefore buy futures with the next settlement date following. It should buy December contracts at 0.6929 .

The remaining \(€ 10,000\) that is not hedged by futures can be purchased forward at 1.4443 , at a cost of \(£ 6,924\).

If the basis is 0 when the futures position is closed in October, the effective exchange rate for the \(€ 2,100,000\) will be \(£ 0.6929=€ 1\), or \(£ 1=€ 1.4432\).
The net cost in sterling will be:
\begin{tabular}{lr} 
& \(£\) \\
\hline\(€ 2,100,000\) at \(\$ 1.4432 / £ 1\) & \(1,455,100\) \\
\(€ 10,000\) at \(€ 1.4443 / £ 1\) & 6,924 \\
\cline { 2 - 2 } Total cost in sterling & \(1,462,024\) \\
\cline { 2 - 2 }
\end{tabular}

The money market hedge is the cheapest method of hedging.

\section*{42 Hedging with STIRs}

The company wants a hedge against the risk of higher interest rates. It should therefore sell short-term interest rate futures.

The borrowing period will begin in February; the company should therefore buy March futures, which have the next settlement date following the start of the loan period.

The planned borrowing period is 5 months, but with futures, the notional deposit period is only 3 months. To get round this difficulty, the number of futures contracts should be adjusted.

The number of March futures to sell \(=\frac{£ 4.5 \text { million }}{£ 500,000} \times \frac{5 \text { months }}{3 \text { months }}\) \(=15\) contracts.

\section*{Conclusion}

The company should sell 15 March short sterling futures.

\section*{43 More hedging with STIRs}
(a) The company wants a hedge against the risk of higher interest rates. It should therefore sell short-term interest rate futures.

The borrowing period will begin in February; the company should therefore buy March futures, which have the next settlement date following the start of the loan period.
The planned borrowing period is 4 months, but with futures, the notional deposit period is only 3 months. To get round this difficulty, the number of futures contracts should be adjusted.

The number of March futures to sell \(=\frac{\$ 12 \text { million }}{\$ 1 \text { million }} \times \frac{4 \text { months }}{3 \text { months }}\)
\(=16\) contracts.

\section*{Conclusion}

The company should sell 16 March eurodollar futures at 93.70.
(b) When the futures are sold, the basis is:
\begin{tabular}{lr} 
Spot LIBOR rate \((100-5.5)\) & 0.9450 \\
Futures price & 0.9370 \\
\hline Basis & 0.0080 \\
\hline
\end{tabular}

It is now the end of October. The March futures will reach settlement date in five months' time.
If we assume that the basis will reduce from 80 points at the end of October to 0 by the end of March at an equal amount each month, by the end of January the basis should be:
\(\frac{2 \text { months to settlement }}{5 \text { months original time to settlement }} \times 80\) points \(=32\) points

The futures price is lower than the spot price.
At the beginning of February, if the three-month LIBOR rate is \(7.5 \%\), the futures price should be:
\begin{tabular}{lr} 
Spot LIBOR rate \((100-7.5)\) & 0.9250 \\
Basis & 0.0032 \\
\cline { 2 - 2 } Futures price & 0.9218 \\
\hline
\end{tabular}

The futures position will be closed out, as follows:
\begin{tabular}{lr} 
Selling price in October & 0.9370 \\
Buying price to close & 0.9218 \\
& 0.0152 \\
\hline
\end{tabular}

The gain is 152 points or \(1.52 \%\)
The futures hedge is a perfect hedge, and the effective cost of borrowing can therefore be calculated as follows:
\begin{tabular}{lr} 
& \(\$\) \\
\hline Borrow \(\$ 12\) million at & 7.50 \\
Gain on futures position & \((1.52)\) \\
\cline { 2 - 2 } Net effective borrowing cost & 5.98 \\
\cline { 2 - 2 }
\end{tabular}

The net effective borrowing cost is \(5.98 \%\).
(Note: This differs from the rate in the futures contracts sold in October. In October, the interest rate in the sold futures was \(6.30 \%\) ( \(100-93.70\) ). The difference is 32 points, which is the amount of the basis risk. Here, the company has benefited from the basis to obtain a lower borrowing cost).

\section*{44 \\ FRAs and futures}
(a)

\section*{FRAs}

The company can use an FRA to fix the interest rate receivable on \(£ 8.2\) million. A 4 v 9 FRA is required, and the bank will offer a rate of \(4.52 \%\).

The company will therefore fix a rate of \(4.52 \%\) for LIBOR and if it can invest at LIBOR \(+0.40 \%\) this means that the effective investment rate will be \(4.92 \%\).

\section*{Futures}

The company wants to fix an interest rate for income, so it should buy futures. The money for investment will be received at the end of July, so September futures should be used.

The company should buy ( \(£ 8.2\) million \(/ £ 500,000) \times(5\) months \(/ 3\) months \()=27.33\) futures contracts, say 27 futures, and the price is 95.35 .
(b)

\section*{FRAs}

At the end of July when the \(£ 8.2\) million, the company will invest the money for 5 months. If it can still obtain a rate of LIBOR \(+0.40 \%\), it will invest the money at \(4.65 \%\).

The FRA contract must also be settled, as follows:
\begin{tabular}{lr} 
& \(\%\) \\
Pay LIBOR & \((4.25)\) \\
Receive & 4.52 \\
Profit on settlement & 0.27 \\
\cline { 2 - 2 }
\end{tabular}

The company will receive a payment on settlement of the FRA equivalent to \(0.27 \%\) in interest, which means that its effective interest income from investing the money for 5 months will be \(4.65 \%+0.27 \%=4.92 \%\).

\section*{Futures}

When the futures contracts were purchased on 1st April, basis was \(95.35-95.00=\) 0.35 . This is 0.05833 per month ( \(0.35 / 6\) months). Assuming basis changes by a constant amount each month, the expected basis at the end of July ( 2 months to the end of September) is 0.117.

The expected futures price at the end of July if LIBOR is \(4.25 \%\) is therefore:
\(95.75+0.117=95.867\).
The futures position will be closed as follows:
\begin{tabular}{lr} 
Close: sell at & 95.867 \\
Purchase price & 95.350 \\
Profit & \(\underline{0.517}\) \\
\hline
\end{tabular}

Total profit on 27 contracts \(=51.7 \times £ 12.50 \times 27=£ 17,449\).
The company can invest \(£ 8.2\) million \(+£ 17,449\) for 5 months at \(4.65 \%\) (LIBOR + \(0.40 \%\) ) and total interest will be \(£ 159,213(£ 8,217,449 \times 4.65 \% \times 5 / 12\) ).

On the money received of \(£ 8.2\) million, this represents an effective interest rate of \((£ 159,213 / £ 8.2\) million \() \times(12 / 5)=0.047\) or \(4.7 \%\).

In this particular case, FRAs would be a better way of hedging the interest rate risk than futures.

\section*{45 Traded equity options}
(a) The investor should buy put options on TBA shares.
\begin{tabular}{llrr}
\begin{tabular}{l} 
Strike \\
price
\end{tabular} & Cost per option & & \begin{tabular}{r} 
Number of options \\
purchased with £12,000
\end{tabular} \\
\hline & & \(£\) & \\
950 & \((2,000\) shares \(\times \$ 0.15)\) & 300 & 40 \\
1,000 & \((2,000\) shares \(\times \$ 0.50)\) & 1,000 & 12
\end{tabular}

The investor could purchase 12 put options at a strike price of \(1,000(£ 10)\) or 40 put options at a strike price of 950 ( \(£ 9.50\) ).
He is more likely to buy the out-of-the-money options at 950 .
(b) If the share price is 910 at expiry and the investor still holds the options, the options will be exercised. It is assumed that he buys the options at 950.
Traded equity options are settled by physical delivery. The investor would need to buy shares at 910 and exercise the option to sell them at 950 .
\begin{tabular}{lr} 
& \(£\) \\
\hline Buy \(40 \times 2,000\) shares at 910 & 728,000 \\
Buy \(40 \times 2,000\) shares at 950 & 760,000 \\
\cline { 2 - 2 } Profit on exercise & 32,000 \\
Cost of options & 12,000 \\
\cline { 2 - 2 } Net profit on speculative investment & 20,000 \\
\cline { 2 - 2 } &
\end{tabular}
(Note: This calculation of the profit ignores the time value of money. The options are paid for when they are bought, but the profit is made only when the options are exercised).
Traded equity options can be bought and sold on the exchange, and the investor is likely to sell the put options before they expire, making a profit on the sale. As the options become increasingly in-the-money, their market value will increase.

\section*{46 \\ Currency options}

In September, the UK company will want to sell dollars and buy sterling (to convert its dollar receipts into sterling).

Since it wants to buy sterling in six months' time, it should buy call options with a September expiry.

If the strike price is \(\$ 1.8500\), the sterling equivalent of \(\$ 2\) million \(=£ 1,081,081\) (2,000,000/1.85).

The contract size is \(£ 31,250\); therefore for a perfect hedge, the company would want to buy 34.6 contracts \((1,081,081 / 31,250)\).

Since this is not possible, it should buy either 34 or 35 contracts. In the answer that follows, it is assumed that the company will buy 35 options.

Premium cost \(=35 \times 31,250 \times \$ 0.019=\$ 20,781.25\).
To pay the premium, the company would have to buy \(\$ 20,781.25\) spot, at a sterling cost of \(£ 11,340.38\).

At the expiry date, the options are in-the-money if the spot exchange rate is 1.9200 . They will therefore be exercised, at a profit of \(\$ 0.07\) per \(£ 1\) (1.9200 - 1.8500).

Gain on exercise of 35 option contracts \(=35 \times 31,250 \times \$ 0.07=\$ 76,562.50\).
The total dollar income of the company will therefore be \(\$ 2,076,562.50\).
This can be exchanged into sterling at the spot rate, to obtain \(£ 1,081,543\) (\$2,076,562.50/1.9200).

The option premium cost was \(£ 11,340\), therefore ignoring the time value of money, the net revenue for the company is \(£ 1,081,543-£ 11,340=£ 1,070,203\).

This gives an effective exchange rate for the \(\$ 2\) million dollar receipts of 1.8688 (2,000,000/1,070,203).

\section*{47 Interest rate hedge}

\section*{(a) Futures}

The company wants a hedge against the risk of a rise in two-month interest rates. It should therefore sell futures. Since the interest period will be 2 months and futures are for a three-month deposit, the quantity of futures sold should be:
( \(£ 21\) million \(/ £ 500,000) \times 2 / 3=28\) contracts.
The loan period will begin in mid-June; therefore sell 28 June contracts, at a price of 94.610.

\section*{Options on futures}

The company will want options to sell futures; therefore it should buy put options on 28 June futures. The premium cost will depend on the strike price chosen. (Since the options are needed for two months, apply a factor of \(\times\) \(2 / 12\) ).
\begin{tabular}{llr} 
Strike price & & Premium \\
\hline 94750 & \((£ 21,000,000 \times 0.500 \% \times 2 / 12)\) & \(£ 17,500\) \\
95000 & \((£ 21,000,000 \times 0.850 \% \times 2 / 12)\) & \(£ 29,750\)
\end{tabular}

\section*{FRA}

The company should buy a \(3 v 5\) FRA, for a notional principal amount of \(£ 21\) million. The FRA rate will be \(5.38 \%\).
(Note: The FRA rate is more favourable than the futures rate of 5.39\% (100 94.610). The company would therefore prefer to buy an FRA than sell futures. However, it might prefer to buy put options on futures rather than buy an FRA).
In mid-June, the company will borrow \(£ 21\) million for two months. If the LIBOR rate is \(6 \%\), it will borrow at \(6.75 \%\) (LIBOR \(+0.75 \%\) ) for two months.

\section*{Futures}

The futures price in mid-June can be estimated as follows:
\begin{tabular}{lr} 
June futures price in mid-March & 94.610 \\
LIBOR rate in mid-March & 95.000 \\
\hline Basis in mid-March & 0.390 \\
\hline
\end{tabular}

In mid-March, there were 3.5 months to settlement of the June futures. In midJune, there are 0.5 months remaining to settlement. Basis is assumed to have reduced in size by mid-June to:
\((0.5 / 3.5) \times 39\) points \(=5.6\) points, say 5 points .
\begin{tabular}{lr} 
LIBOR rate in mid-June & 94.000 \\
Basis in mid-June & 00.005 \\
\hline Estimated futures price in mid-June & 93.995 \\
\hline
\end{tabular}

The company will close its position by buying 28 June futures at 93.995.

The total gain on the futures position:
\(=28\) contracts \(\times 61.5\) ticks per contract \(\times £ 12.50\) per tick
\(=£ 21,525\).
(The gain on the futures position offsets the cost of the increase in the interest rate above the rate fixed by the futures contract ( \(6 \%-5.39 \%=0.61 \%\) ). The extra borrowing cost is \(£ 21\) million \(\times 0.61 \% \times 2 / 12=£ 21,350\). The difference of \(£ 175\) is due to the basis risk).

\section*{Options on futures}

The company will exercise its options to sell futures at the strike price for the option, and will then close the futures position, giving a gain on closing the futures position.
\begin{tabular}{|c|c|c|}
\hline & Strike price 94750 & Strike price 95000 \\
\hline Option strike price to sell & 94.750 & 95.000 \\
\hline Buying price to close & 93.995 & 93.995 \\
\hline Gain (ticks) & 0.755 & 1.005 \\
\hline Total gain: & & \\
\hline \(28 \times 75.5\) ticks \(\times £ 12.50\) & £26,425 & \\
\hline \(28 \times 100.5\) ticks \(\times £ 12.50\) & & £35,175 \\
\hline
\end{tabular}

However, after taking into account the cost of the option premiums, the net gain is reduced.

\section*{FRA}

The company's FRA bank will make a payment equivalent to \((6 \%-5.38 \%)=\) \(0.62 \%\) per year on \(£ 21\) million for two months, to settle the FRA.
The gain on the FRA will offset the higher interest cost of borrowing.

\section*{48 Black-Scholes}

\section*{Step 1: Calculate \(\mathrm{d}_{1}\)}
\(\mathrm{d}_{1}=\frac{\ln \left(\frac{870}{900}\right)+(0.05 \times 0.5)}{\sqrt{0.09} \times \sqrt{0.5}}+0.5 \mathrm{c} \sqrt{0.09} \times \sqrt{0.5}\)
\(\ln (870 / 900)=\ln 0.96667\). Using a calculator, the natural logarithm of 0.96667 can be obtained. This is -0.0339 .
\(\mathrm{d}_{1}=\frac{-0.0339+0.025}{0.30 \times 0.7071}+0.5 \times 0.30 \times 0.7071\)
\(=(-0.0089 / 0.21213)+0.106\)
\(=-0.042\), say- 0.04

\section*{Step 2: Calculate \(\mathbf{d}_{\mathbf{2}}\)}
\(\mathrm{d}_{2}=\mathrm{d}_{1}-\sigma \sqrt{\mathrm{t}}=-0.042-(\sqrt{0.09} \times \sqrt{0.5})\)
\(=-0.042-0.212\)
\(=-0.254\), say -0.25

\section*{Step 3: Obtain values for \(N\left(d_{1}\right)\) and \(N\left(d_{2}\right)\)}
(a) \(\mathrm{N}\left(\mathrm{d}_{1}\right)\). When \(\mathrm{d}_{1}=0.042\), the corresponding value in the normal distribution table is 0.0160 . The value of \(d_{1}\) is negative, so subtract from \(0.50 . N\left(d_{1}\right)=0.5000\) \(-0.0160=0.4840\).
(b) \(\mathrm{N}\left(\mathrm{d}_{2}\right)\). When \(\mathrm{d}_{2}=0.25\), the corresponding value in the normal distribution table is 0.0987 . The value of \(\mathrm{d}_{2}\) is negative, so subtract from \(0.50 . \mathrm{N}\left(\mathrm{d}_{2}\right)=0.5000\) \(-0.0987=0.4013\).

\section*{Step 4: Calculate the option price}

\section*{The value of \(\mathrm{e}^{-\mathrm{rt}}\)}

The value of rt is \((0.05)(0.5)=0.025\).
The value of \(\mathrm{e}^{-0.025}\) is the same as \(1 / \mathrm{e}^{0.025}\). Using a calculator, the value of \(\mathrm{e}^{0.025}\) can be obtained: this is 1.0253 . Therefore \(\mathrm{e}^{-0.025}=1 / 1.0253=0.9753\).
\[
\begin{array}{rlrl}
\text { Option price } & = & \mathrm{P}_{\mathrm{S}} \mathrm{~N}(\mathrm{~d} 1) & -\mathrm{Xe}^{-\mathrm{rt}} \mathrm{~N}(\mathrm{~d} 2) \\
& =870(0.4840) & -900(0.9753)(0.4013) \\
& =421-352 &
\end{array}
\]

The option price is 69 .

\section*{\(49 \quad\) Put-call parity}

Value of put option + Current share price \(=\) Value of call option + PV of exercise price.

The PV of the exercise price \(=600 \times 1 /(1.04)^{0.5}=600 \times 0.9806=588\)
Value of put \(+582=33+588\)
Value of put \(=39\)

\section*{50 Delta hedge}
(a) The bank should hold one share in TER for every 8 call options \((1 / 0.125)\) that it writes.
(Alternatively, if it buys call options, it should sell one share forward for settlement on the option expiry date for every 8 options that it buys).
(b) Calculate \(\mathrm{d}_{1}\)
\[
\mathrm{d}_{1}=\frac{\ln \left(\frac{258}{260}\right)+(0.04 \times 0.5)}{\sqrt{0.16} \times \sqrt{0.5}}+0.5 \times \sqrt{0.16} \times \sqrt{0.5}
\]
\(\ln (258 / 260)=\ln 0.9923\). Using a calculator, the natural logarithm of 0.9923 can be obtained. This is -0.0077 .
\[
\begin{aligned}
& \mathrm{d}_{1}=\frac{-0.0077+0.02}{0.40 \times 0.7071}+0.5 \times 0.40 \times 0.7071 \\
& =(0.0123 / 0.2828)+0.1414 \\
& =0.185
\end{aligned}
\]

The bank should hold one share in BVZ for every 5.4 call options \((1 / 0.185)\) that it writes.
(Alternatively, if it buys call options, it should sell one share forward for settlement on the option expiry date for every 5.4 options that it buys).

\section*{51 Bonus scheme}

\section*{Bonus linked to EVA}

It is assumed that research and development costs should be capitalised. The EVA in the year just ended is therefore as follows.

Sales
Cost of sales
Depreciation
Tax: \(44+[(14+35) \times 30 \%]\)
NOPAT
\$ million
683.0
(421.0)
(58.7)
138.3

Interest should be calculated on the capital employed at the beginning of the year, adjusted to include the research and development costs in the previous three years that should be capitalised for the purpose of EVA calculations.

Capital employed \((\) in \(\$\) million \()=525+(14 \times 3)=567\)
Notional interest at \(9 \%=\$ 51.03\) million, say \(\$ 51.0\) million.

NOPAT
Notional interest
EVA
\$ million
138.3
(51.0)
87.3

If the bonus payment is \(0.5 \%\) of EVA, on the basis of the previous year's EVA the executive could expect to receive \(\$ 436,500\) as a cash bonus.

\section*{Share options}

The share options would be call options. Their value can be estimated from the value of a similar put option, using the put-call parity theorem.

Value of put \(=\) Value of call - Current market value + Strike price \(\times \mathrm{e}^{-\mathrm{rT}}\).
\(30=\) Value of call \(-140+140 \times \mathrm{e}^{-(0.05 \times 1)}\)
\(30=\) Value of call \(-140+(140 \times 0.951229)\)
Value of call \(=36.83\) cents.
The value of at-the-money call options on 2 million shares, with an exercise date in one year, is therefore about \(\$ 736,600\).

\section*{52 Delta neutral}

\section*{(a)}

Barn plc could either buy put options on Door plc shares, or could write call options. With put options, the company would pay a premium to secure a worstpossible share price. With call options, the company would receive income from selling the options but would be exposed to a risk of a rise in the share price above the exercise price.

The company should choose November options, and might consider a strike price of either 600 or 650 .

If Barn plc buys November put options, it will buy 4,000 contracts.
(1) If it buys put options at 600 , the option premium payable will be 22.0. If the share price at the end of November is 580 , the put options will be exercised for a gain of \(20(600-580)\). However, allowing for the premium, the net loss will be 2.0 pence per share or \(£ 80,000\) in total compared with a strategy of not hedging the risk with options and selling the shares at the current market price of 580 .
(2) If it buys put options at 650 , the option premium payable will be 47.5 . If the share price at the end of November is 580 , the put options will be exercised for a gain of \(70(650-580)\). Allowing for the premium, the net gain will be 22.5 pence per share or \(£ 900,000\) in total compared with a strategy of not hedging the risk with options and selling the shares at the current market price of 580 .
(b)

To establish a delta neutral position, the company should sell call options. If the share price goes up, the value of the investment in 4 million shares will rise, but this will be offset by the loss on the call options, leaving the net value of the investment unchanged. If the share price falls, the loss in the value of the shares will be offset by a gain on the options position.
The option delta for the relevant option is 0.46 , which means that for every 1 share held, the company would sell \(1 / 0.46=2.1739\) call options on the shares, or \(8,695,600\) call options in total.

A problem with establishing a delta neutral hedge is that ideally at-the-money options should be sold, but these are not normally available as traded options.

Another problem is that the option delta is only applicable over a fairly narrow range of prices for a share. If the share price moves by more than a small amount, the option delta changes and options must be cancelled or written in order to reestablish a delta neutral hedge. The position therefore needs continual monitoring and adjusting if the share price is volatile.

\section*{Paper P4}

Advanced Financial Management

\section*{i}

\section*{Index}
A
Accounting: economic model ..... 47
Acquisitions ..... 230
Adjusted present value method (APV) ..... 186
Agency theory ..... 22
All-share consideration ..... 271
Alpha factor ..... 136
Altman's Z score ..... 277
Annual volatility of a project ..... 103
Annuities ..... 59
Asset beta ..... 173
Audit committee ..... 29
B
Bank for International Settlements (BIS) ..... 439
Bankers' acceptances ..... 295
Base case NPV ..... 187
Base rates ..... 292
Basis ..... 357
Basis risk ..... 358
Beta factor of a portfolio ..... 129
Beta factor of a security ..... 133
Beta factor of a small portfolio ..... 135
Bid rate ..... 313
Bill of exchange ..... 295
Black-Scholes option pricing model ..... 403
Grabbe variant ..... 410
Board committees ..... 28
Board of directors ..... 27
Bond duration ..... 67
Bond futures ..... 375
Book value ..... 240
Borrowers' options ..... 391
Bounded rationality ..... 24
Business reorganisation ..... 282
C
Calculating the NPV of a project ..... 57
Call options ..... 379
Capital allowances ..... 76
Capital asset pricing model (CAPM) ..... 132
advantages and disadvantages ..... 138
formula ..... 133
Capital rationing ..... 195
Capital reconstruction ..... 279
Caps ..... 393
Carbon neutrality ..... 45
Carbon trading ..... 52
Cash flow forecasting ..... 298
Cash management ..... 298
Certificate of deposit (CDs) ..... 296
Closing positions ..... 354
Coefficient of variation ..... 116
Collars ..... 395
Commercial paper (CP) ..... 296
Commodity futures ..... 350
Commodity option ..... 378
Common markets ..... 432
Communication with shareholders ..... 30
Competition law ..... 235
Corporate code of ethics ..... 35
Corporate failure prediction models ..... 276
Corporate governance: improvement ..... 27
Corporate governance: other countries ..... 30
Corporate social responsibility ..... 21
Cost of capital and gearing ..... 158
Cost of debt capital ..... 148
Cost of equity ..... 144
CAPM method ..... 147
Coupon-bearing instruments ..... 294
Coupon swap ..... 339
Countertrade ..... 306
Credit arbitrage ..... 343
Credit ratings ..... 153
Credit risk ..... 300
Cross rates ..... 320
Currency futures ..... 362
Currency options ..... 386
Currency risk ..... 211
Currency swaps ..... 345
Currency transaction exposures ..... 308
D
DCF and inflation ..... 81
DCF and taxation ..... 75
DCF appraisal: investment in a ..... 222
developing country
Delta ..... 419
Delta hedging ..... 416
Demergers ..... 282
Directors' remuneration ..... 29
Discount instruments ..... 294
Discount tables ..... 58
Discounted cash flow: net present ..... 56
value method
Discounted payback period ..... 63
Disposal value ..... 239
Distribution tables ..... 105
Divestments ..... 283
Dividend growth model method ..... 145
Dividend policy ..... 38
monitoring ..... 39
Dividend valuation model method ..... 144
constant annual dividends ..... 242
constant rate of growth in annual ..... 244dividends
Documentary credits ..... 304
Duration ..... 65
E
Ecological footprint ..... 44
Economic model: accounting ..... 47
Economic value added (EVA) ..... 256
Efficient portfolio ..... 125
Environmental audits ..... 47, 50
Environmental footprint ..... 44
Environmental reporting ..... 47
Environmental risk ..... 44, 46
Equity: cost ..... 144
Estimating future spot exchange rates ..... 217
Exercise price ..... 380
Exchange controls ..... 212
Exchange rate risk ..... 211
Exchange rates and volatility ..... 309
Exercising options ..... 383
Expected value ..... 91
Expiry date ..... 379
Export factoring ..... 305
Financial futures ..... 350
Financial management: ethical issues ..... 34
Financial option ..... 378
Financial risk management ..... 300
Financial strategy formulation ..... 32
Financial synergies ..... 234
Fiscal risk ..... 94, 215
95, 216reduction
Fixed exchange rate policy ..... 311
Floating rate debt ..... 148
Floors ..... 395
Foreign exchange risk ..... 308
Foreign investment decisions: affecting ..... 210
factors
Forfaiting ..... 305
Forward contracts ..... 316
Forward rate agreement (FRA) ..... 333
Forward rates ..... 316
Free cash flow ..... 253
Free floating ..... 310
Free trade ..... 432
Futures exchanges ..... 350
Futures prices ..... 351
FX risk ..... 211
FX swaps ..... 346
G
Gamma ..... 420
Gearing and cost of capital ..... 158
Going private ..... 286
Gordon growth model ..... 38
Gordon's growth approximation ..... 145
Greeks, The ..... 419
H
Hedging risk ..... 321
Hedging with options ..... 385
Hostile takeover bid ..... 235
Imperfect hedges ..... 360, 388
Inflation and DCF ..... 81
Inter-bank market ..... 291
Interest costs and taxation ..... 75
Interest rate option ..... 390
Interest rate parity theory ..... 219
Interest rate risk ..... 330
Interest rate swap ..... 338
Internal control and risk management ..... 30
Internal rate of return (IRR) ..... 69
International Bank for Reconstruction ..... 438
and Development (IBRD)
International bond markets ..... 435
International CAPM ..... 139
International Development Association ..... 438
(IDA)
International equity ..... 436
International Fisher effect ..... 219
International lending ..... 435
International money markets ..... 436
International Monetary Fund (IMF) ..... 438
International sources of finance ..... 228
Investment portfolio: choice ..... 124
Investment risk ..... 114
Investor preferences: indifference ..... 125
theory
Irredeemable fixed rate debt ..... 148
KMV/Moody's model278
L
Lenders' options ..... 392
Letters of credit ..... 304
Libor ..... 291
\begin{tabular}{lr} 
Managed floating & 311 \\
Management buyouts/ buy-ins & 284 \\
Margin & 352 \\
Market portfolio & 126 \\
Market premium & 128 \\
Market timing theory & 184 \\
Market value added (MVA) & 261 \\
Mergers & 230 \\
Modified internal rate of return (MIRR) & 72 \\
Modigliani-Miller & 159 \\
Money cost of capital & 81 \\
Money market hedge & 325 \\
Money market instruments & 294 \\
Money markets & 290 \\
Monte Carlo simulation & 96 \\
capital investment appraisal & 99 \\
Multinational companies & 422 \\
double taxation agreement & 425 \\
tax planning & 427 \\
tax rates & 425 \\
withholding tax & 425 \\
Multi-period capital rationing & 196 \\
Multivariate discriminant analysis & 276
\end{tabular}

\section*{N}

Net present value method of investment 56 appraisal
Netting 321
Notional loans and deposits 335
Normal distribution 105

\section*{0}

Offer rate 313
Open positions 354
Opportunism 24
Options 378
Organisation: objectives 20
P
Payback period ..... 63
Pecking order theory ..... 183
Perpetual bonds ..... 148
Perpetuities ..... 62
Preference shares ..... 151
Primary corporate objective ..... 20
Protectionism ..... 431
Political risk ..... 212
Portfolio theory ..... 118
Premiums and discounts ..... 318
Present value of the tax shield ..... 189
Private equity ..... 287
Project value at risk ..... 103
Project volatility ..... 101
Purchase consideration ..... 270
Purchasing power parity (PPP) theory ..... 217
Put call parity ..... 408
Put options ..... 379
Q

Quoting exchange rates315
R

\section*{R}
Real cost of capital ..... 81
Real option ..... 203, 378
valuation ..... 207
Real options theory ..... 204
Redeemable fixed rate bonds ..... 149
Redeemable fixed rate debt ..... 149
Reducing balance method ..... 77
Regulatory framework for mergers ..... 234
and acquisitions
Repo market ..... 292
Retained earnings ..... 39
Revenue synergies ..... 233

Revenue synergies 233
Risk ..... 90
Risk and return in investments ..... 114
Risk management and internal control ..... 30
Risk management framework ..... 34
Risk modelling ..... 94
S
Scenario analysis ..... 303
Sensitivity analysis ..... 93
Settlement dates ..... 351
Share buy-back ..... 285
Share repurchase scheme ..... 285
Shareholder value added ..... 252
Short-term interest rate future (STIR) ..... 367
Single period capital rationing ..... 196
Social footprint ..... 45
Sources of debt ..... 300
Sources of equity ..... 299
Sources of short-term finance ..... 300
Spot rates ..... 314
Spreads ..... 152
Standard normal distribution table ..... 107
Stakeholder interests: conflict ..... 20
resolution ..... 26
Static trade-off theory ..... 182
Stakeholder theory ..... 25
Straight line method ..... 76
Stress testing ..... 303
Sustainability reporting ..... 47
Swap rates ..... 339
Swaption ..... 392
Synergy ..... 233
Synthetic agreement for forward ..... 323
exchange (SAFE)
Systematic risk ..... 130
securities ..... 132
\(T\)
Takeover Code ..... 235
Takeovers ..... 230
Taxation and DCF ..... 75
Taxation and international investment ..... 214
Tax-allowable depreciation ..... 76
Tax exhaustion ..... 79
Term structure of interest rates ..... 151
Theta ..... 420
Ticks ..... 356
Transaction cost economics (TCE) ..... 23
Transaction cost theory ..... 23
Transaction risk ..... 211
Translation risk ..... 212
Treasury bills ..... 294
Treasury function ..... 298
Triple bottom line reports ..... 48
Two-asset portfolio ..... 119
U
Uncertainty ..... 90
Unsystematic risk ..... 130
V
Valuation assumptions ..... 264
Valuation methods ..... 239
Value at Risk models (VaR) ..... 302
Variable rate debt ..... 148
Vega ..... 420
Venture capital finance ..... 284
W
Weighted avergae cost of capital ..... 142
calculation ..... 155
capital investment appraisal ..... 143
market value ..... 157
World bank ..... 437
World Trade Organisation (WTO) ..... 433
Y
Yield curve ..... 151
Z
Z scores (Zeta scores) ..... 276
Zero cost dollars ..... 395

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